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Grover on *SIMON*

Ravi Anand[1] · **Arpita Maitra**[2,3] · **Sourav Mukhopadhyay**[1]

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Abstract
For any symmetric key cryptosystem with n-bit secret key, the key can be recovered in $O(2^{n/2})$ exploiting Grover search algorithm, resulting in the effective key length to be half. In this direction, subsequent work has been done on AES and some other block ciphers. On the other hand, lightweight ciphers like $SIMON$ was left unexplored. In this backdrop, we present Grover's search algorithm on all the variants of $SIMON$ and enumerate the quantum resources to implement such attack in terms of NOT, CNOT and Toffoli gates. We also provide the T-depth of the circuits and the number of qubits required for the attack. We show that the number of qubits required for implementing Grover on $SIMON\ 2n/mn$ is $O(2nr + mn)$, where r is the number of chosen plaintext–ciphertext pairs. We run a reduced version of $SIMON$ in IBMQ quantum simulator and the 14-qubit processor as well. We found that where simulation supports theory, the actual implementation is far from the reality due to the infidelity of the gates and short decoherence time of the qubits. The complete codes for all version of $SIMON$ have also been presented.

Keywords Lightweight cryptography · Quantum cryptanalysis · Quantum circuits · Grover's algorithm · Feistel ciphers

1 Introduction

The last two decades witnessed an enormous proliferation in the domain of quantum computation and communication. Due to the two pioneering quantum algorithms, Shor's algorithm [21] and Grover's search algorithm [6], the security of currently

✉ Arpita Maitra
arpita76b@gmail.com

1 Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur, West Bengal 721302, India

2 TCG Centre for Research and Education in Science and Technology, Kolkata, West Bengal 700091, India

3 CR Rao Advanced Institute of Mathematics, Statistics and Computer Science, Hyderabad, India

deployed cryptosystems is under a threat. As a consequence, in recent time, a lot of symmetric constructions are being evaluated in quantum settings. For example, one can mention the key recovery attacks against Even–Mansour constructions and distinguishers against three-round Feistel constructions [16]. Not only that, the key recovery attacks against multiple encryptions [12], forgery attacks against CBC-like MACs [13] have also been studied. The list is expanding considering Quantum meet-in-the-middle attack on Feistel constructions [9], key recovery attacks against FX constructions [18] etc. Researchers have also tried to convert the existing classical attacks to quantum settings [10,13,14,19].

Very recently, Bonnetain et al. [5] proposed a novel methodology for quantum cryptanalysis on symmetric ciphers. They exploited the algebraic structure of certain classical cryptosystems to bypass the standard quantum superposition queries.

In case of symmetric ciphers or hash function, Grover's algorithm provides a quadratic speed up in exhaustive key search. So a conservative rule of thumb is to double the security parameter, i.e., at least double the size of the key or double the size of the output of hash function. However, this does not rule out the need of analyzing the cost of Grover's algorithm on symmetric ciphers. In this direction, subsequent efforts have been made to derive cost estimation for applying Grover's search algorithm on all variants of AES [7,11,17,28]. The cost of applying Grover's search algorithm as a pre-image search attack on hash functions has also been studied [2].

Fault-tolerant commercialized quantum computers are still elusive. However, several companies are providing simulation facilities through the web. Along with the simulation, IBM provides facility to run the program in their small-scale actual quantum processors. Based on this state-of-the-art situation, this is very important to explore actual implementation issues of all these quantum cryptanalysis procedures.

$SIMON$ is a family of lightweight block ciphers released by NSA in June 2013. It is a balanced Feistel structured block cipher. $SIMON$ is optimized for performance in hardware implementations. In October 2018, the $SIMON$ and Speck ciphers have been standardized by ISO as a part of the RFID Air Interface Standard, International Standard ISO/29167-21 (for $SIMON$) and International Standard ISO/29167-22 (for Speck), making them available for use by commercial entities [31]

As these are comparatively new ciphers, quantum cryptanalysis on those ciphers remained unexplored. In this backdrop, in the current effort, we study the cost of Grover search on all the variants of $SIMON$ and try to implement that in publicly available IBM quantum processors. Due to the limitation of the qubits, we could not run the full-scale cipher; instead, we run the algorithm for a reduced version. We found that whereas simulation meets theory, actual implementation has been masked with error.

Our contribution One may argue that it is already well known that Grover search provides quadratic speedup over classical exhaustive key search. In this direction, we like to emphasize that for implementing Grover algorithm on a symmetric cipher, one requires a reversible implementation of that cipher which is a hard task. In this regard, we design the reversible version of all the variants of $SIMON$ [4] so that one can successfully implement Grover oracle and Grover diffusion for key search on all of those variants. We also provide the full implementation code for the cipher in QISKIT [30]. We estimate the resources in terms of NOT, CNOT and Toffoli gates

Table 1 $SIMON$ parameters

Block size ($2n$)	Key size ($k = mn$)	Word size (n)	Keywords (m)	Rounds (T)
32	64	16	4	32
48	72,96	24	3,4	36,36
64	96,128	32	3,4	42,44
96	96, 144	48	2,3	52,54
128	128,192,256	64	2,3,4	68,69,72

required to attack the cipher. We provide the T-depth of the circuits and the number of qubits needed to implement the attack, too. We tested our circuits with existing classical test vectors to make sure that the implementations are correct. The code for all the variants is given in [26]. This is because when the full-scale quantum computers arrive, one may implement the code immediately. For independent verification of our results with the state-of-the-art IBM quantum simulator and processors, we add all the QASM and QISKIT codes for a reduced $SIMON$. To the best of our knowledge, this is the first full implementation and resource estimation on $SIMON$ in quantum settings.

2 Preliminaries

2.1 Brief summary on *SIMON*

$SIMON$ is a family of balanced Feistel structured lightweight block ciphers with ten different block sizes and key sizes (Table 1). The round function used in the Feistel structure of $SIMON$ block ciphers consists of circular shift, bitwise AND and bitwise XOR operations. The state update function is defined as,

$$F(x, y) = \left(y \oplus S^1(x)S^8(x) \oplus S^2(x) \oplus k, x\right). \tag{1}$$

The structure of one round $SIMON$ encryption is depicted in Fig. 1, where S^j represents a left circular shift by j bits, L_i and R_i are n-bit words which constitute the state of $SIMON$ at the ith round and k_i is the round key which is generated by key scheduling algorithm described below.

The different variants of $SIMON$ are denoted by $SIMON\,2n/mn$, where $2n$ denotes the block size of the variant and mn is the size of the secret key. Here, n can take values from 16, 24, 32, 48 or 64 and m from 2, 3 or 4. For each combination of (m, n), the corresponding round number T is adopted.

The key schedule of $SIMON$ has three different procedures depending on the key size. The first m round keys are initialized directly from the main key. The remaining $(T - m)$ round keys are generated by the following procedure:

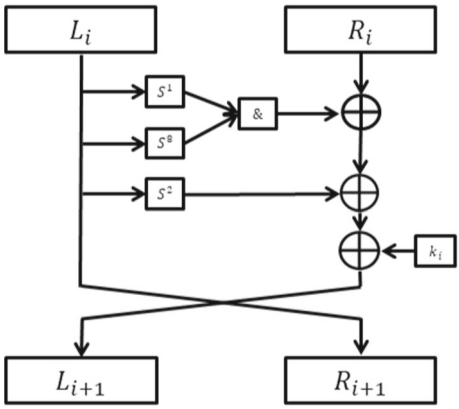


Fig. 1 $SIMON$ round function

$$k_{i+m} = \begin{cases} c_i \oplus k_i \oplus S^{-3}(k_{i+1}) \oplus S^{-4}(k_{i+1}) & m = 2, \\ c_i \oplus k_i \oplus S^{-3}(k_{i+2}) \oplus S^{-4}(k_{i+2}) & m = 3, \\ c_i \oplus k_i \oplus S^{-1}(k_{i+1}) \oplus S^{-3}(k_{i+3}) \oplus S^{-4}(k_{i+3}) & m = 4, \end{cases} \quad (2)$$

where c_i are the round dependent constants and $S^{-j}(x)$ denotes right rotation by j times on x.

2.1.1 Encryption

The input to the encryption oracle is a $2n$-bit plaintext block P. This block is divided into n-bit subblocks $P = (L_0, R_0)$ which is the initial state of the cipher. The encryption consists of T applications of the round function with the respective round key produced by the key schedule. The ciphertext obtained is a $2n$-bit block $C = (L_{T-1}, R_{T-1})$

2.1.2 Decryption

Decryption of the ciphertext $C = (L_{T-1}, R_{T-1})$ consists of first swapping L and R part of the block cipher, i.e., the input to the decryption oracle is (R_{T-1}, L_{T-1}). Then, T-round functions with round keys in reverse order (i.e., round keys $k_{T-1}, \ldots k_0$) are applied followed by a final swapping of the two subblocks.

2.1.3 Existing cryptanalysis of SIMON

To the best of our knowledge, no successful attack on full-round Simon of any variant is known. As is typical for iterated ciphers, reduced-round variants have been successfully attacked. Some of the results are summarized in Table 2.

For a more detailed description of $SIMON$, the readers are referred to [4].

Table 2 Summary of existing cryptanalysis results on $SIMON$

Variant rounds	Attacked	Time complexity	Data complexity	Attack type
Simon32/64	21/32	2^{63}	2^{31}	Integral [23]
Simon48/72	20/36	$2^{59.7}$	2^{48}	Zero correlation [23]
Simon48/96	21/36	$2^{72.63}$	2^{48}	Zero correlation [23]
Simon64/96	26/42	$2^{63.9}$	2^{63}	Differential [1]
Simon64/128	26/44	2^{94}	2^{63}	Differential [1]
Simon96/96	35/52	$2^{93.3}$	$2^{93.2}$	Differential [1]
Simon96/144	35/54	2^{101}	$2^{93.2}$	Differential [1]
Simon128/128	46/68	$2^{125.7}$	$2^{125.6}$	Differential [1]
Simon128/192	46/69	2^{142}	$2^{125.6}$	Differential [1]
Simon128/256	46/72	2^{206}	$2^{125.6}$	Differential [1]

2.2 Grover's algorithm

Grover's algorithm [6] searches through a space of N elements for a solution. We can assume that $N = 2^n$ and each state be represented by the indices in $\{0, 1\}^n$. Let us assume that there exists a state y such that

$$f(x) = \begin{cases} 1 & \text{if x=y ,} \\ 0 & \text{otherwise.} \end{cases}$$

We also assume that f is easily and effectively computable. The *oracle* f is provided as a black box. It finds the state y. Grover's algorithm needs only $O(2^{n/2})$ oracle calls as compared to $O(2^n)$ oracle calls needed classically.

Grover's algorithm can be summarized in the following steps:

1. Apply Hadamard gate on the initial state $|00...0\rangle$ bit by bit to obtain the following superposition

$$|\psi\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} |x\rangle.$$

2. The second step makes $\lfloor \frac{\pi}{4} 2^{n/2} \rfloor$ calls to Grover's iteration. Grover's iteration comprises two subroutines.

   The first subroutine makes use of the operator U_f which evaluates the Boolean function $f : \{0, 1\}^n \to \{0, 1\}$, which marks the solutions of the search problem, *i.e.*, $f(x) = 1$ if and only if the element corresponding to x is a solution. When we apply the Grover oracle U_f to a state $|x\rangle|z\rangle$, where $|x\rangle$ is a n-qubit state and $|z\rangle$ is a single qubit, it acts as $U_f : |x\rangle|z\rangle \to |x\rangle|z \oplus f(x)\rangle$. If $|z\rangle$ is chosen to be $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, then we have $U_f : |x\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \to (-1)^{f(x)}|x\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$,

which means that the oracle applies a phase shift only to the solution indices while leaving the remaining indices unaltered. Each call to U_f involves two calls to a reversible implementation of f and one call to a comparison circuit that checks if x is a solution or not.

The second subroutine implements the transformation $2|0\rangle\langle 0| - I$, also known as the diffusion operator. This routine flips the amplitude of the states about its mean, thus amplifying the amplitude of the solution. This involves single-qubit gates and one t-fold controlled NOT gate. So in the second step, the following two steps are repeated $O(2^{n/2})$ times:

(a) For any state $|x\rangle$ in the superposition $|\psi\rangle$, rotate the phase by π radians if $f(x) = 1$ and leave the system unaltered otherwise.
(b) Apply the diffusion operator.

3. Measure the resulting superposition and obtain the solution with the probabilities determined by the amplitudes of the states.

2.3 Block cipher key search using grover

Let E be a block cipher with block size n and key size k. For any key $K \in \{0, 1\}^k$, let $E_K(M)$ be the encryption of plaintext M under the key K. For a given plaintext–ciphertext pair (M, C) with $C = E_K(M)$, we can apply Grover's algorithm to determine the key K [25]. The steps involved are:

1. Define a Boolean function f for Grover's oracle which takes the key K as input,

$$f(K) = \begin{cases} 1 & \text{if } E_{K_0}(M) = C, \\ 0 & \text{otherwise.} \end{cases}$$

2. Initialize the system by making a superposition of all the possible keys with same amplitude,

$$|\mathcal{K}\rangle = \frac{1}{2^{K/2}} \sum_{j=0}^{2^K - 1} |K_j\rangle.$$

3. Iterate $2(a)$, (b) as described in Sect. 2.2 for $O(2^{K/2})$ times.
4. Measure the system and observe the state $K = K_0$ with probability atleast $(\frac{1}{2})$.

As a matter of fact, there may be more than one key that satisfies $C = E_{K_0}(M)$. To ensure that the key obtained is unique, we may require more than one plaintext–ciphertext pairs under the same key. Let us consider that we have r such pairs (M_i, C_i). In this case, the Boolean function for Grover's oracle would be defined as,

$$f(K) = \begin{cases} 1 & \text{if } E_K(M_i) = C_i, 0 \le i \le r, \\ 0 & \text{otherwise.} \end{cases}$$

2.4 Attack model

We mount known plaintext attack using Grover's algorithm [6] for all the variants of $SIMON$. We consider that the adversary has access to certain pairs of plaintexts and corresponding ciphertexts. Then, he finds the secret key using Grover's search using quantum resources. In this regard, we need to design a quantum circuit for all the variants of $SIMON$. In the following section, we describe the circuit.

3 Quantum circuit for *SIMON*

In this section, we develop a reversible quantum circuit for $SIMON$. We analyze our circuits based on the number of qubits, NOT gates, *CNOT* gates and *Toffoli* gates. Grover search will be executed on this circuit under the known plaintext attack model, i.e., when pairs of plaintext and the corresponding ciphertext are already known.

The circuits described in this section are implemented in QISKIT [30]. The circuit is reversibly computable and needs no ancilla qubits. We also estimate the T-depth of the circuit.

The internal state size of $SIMON$ varies from 32 bits to 128 bits as described in Table 1. $SIMON$ consists of two subroutines, the round function and the key expansion. We describe both these routines first and then show how they can be used simultaneously in the whole cipher construction.

3.1 Circuit for round update function

The round function F is defined as,

$$F(x, y) = \left(y \oplus S^1(x)S^8(x) \oplus S^2(x) \oplus k, x\right),$$

where $S^i(x)$ denotes left rotation by i times on x.

Now, we assume that we have k-qubits reserved for the key, K and, n-qubits each for L and R. Let (L_0, R_0) be the initial state and the state propagate as $(L_0, R_0), (L_1, R_1), (L_2, R_2), \cdots, (L_j, R_j)$ in j rounds.
Now, due to the construction of $SIMON$, we can write

$$\begin{aligned} R_2(i) = L_1(i) \; &= R_0(i) \oplus K_0(i) \oplus L_0((i+1)mod(n/2)) \,\&\, L_0((i+8)mod(n/2)) \\ &\oplus L_0((i+2)mod(n/2)), 0 \le i \le (n/2). \end{aligned}$$

Note that here i denotes the position of the bit in L and R of a round. If we consider two-round $SIMON$, then each bit of R_2 will be the XORing of each bit of R_0, $F(L_0)$ and K_0, where $F(x) = S^1(x)S^8(x) \oplus S^2(x)$. Similarly, each bit of L_2 will be the XORing of each bit of L_0, $F(R_2)$ and K_1. So the qubits reserved for R_0 can be used to store the values for R_2. Similarly, the qubits reserved for L_0 can be used to store the value of L_2, hence no need for extra qubits.

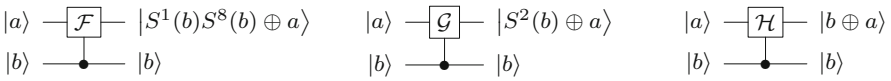


Fig. 2 Subroutines comprising the round function F

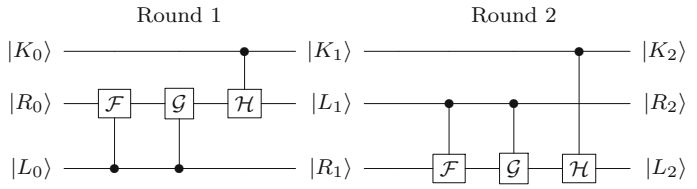


Fig. 3 Circuit for two rounds

Each bit of R_2, i.e., $R_2(i)$, is computed using the following three steps:

1. $Toffoli(L_0((i+1)mod(n/2)), L_0((i+8)mod(n/2)), R_0(i))$,
2. $CNOT(L_0((i+2)mod(n/2)), R_0(i))$,
3. $CNOT(K_0(i), R_0(i))$.

L_2 can be implemented in similar way. Proceeding sequentially, we can build a circuit for as many rounds as required.

Now, it is easy to calculate that for 1 round we require n *Toffoli*gates and $2n$ *CNOT* gates. So, for j rounds we need jn *Toffoli*gates and $2jn$ *CNOT* gates.

Let us now define three functions $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$, shown in Fig. 2, for easy understanding of the circuit construction.

Here, $|a\rangle = |a_1a_2...a_n\rangle$, $|b\rangle = |b_1b_2...b_n\rangle$ are n length quantum states and $(+)$ is addition modulo n, i.e., $(i+8) = (i+8)mod(n)$.
The circuit for the two rounds is shown Fig. 3, where K_j, L_j, R_j represent quantum states of size n for a round j.

Now, consider the circuit in Fig. 4. This is the IBMQ [29] implementation of the circuit for two rounds of a reduced version of $SIMON$. The assumed state size is 16 and the key size is 16 and $m = 2$. The state is split into L, R each of size 8, and the key is split into two round keys (k_0, k_1) also of size 8. The state update function is assumed to be $F(x, y) = (y \oplus S^1(x)S^4(x) \oplus S^2(x) \oplus k, x)$. The circuit here describes the two rounds of the cipher, i.e., if we measure the L and R states, we would get the values of the state after two rounds. We can extend these circuits for more than two rounds described earlier.

One should note that this implementation works for all variants of $SIMON$ except $SIMON$ 128/192. The problem arises for the $SIMON$ 128/192 as the number of rounds is 69. This can be solved by implementing the last round such that the state R is modified according to the state update function and L state is left as it is. Then, we apply a swap function on the states L and R, which increase the number of *CNOT* gates in the circuit by an amount of $64 \times 3 = 192$.

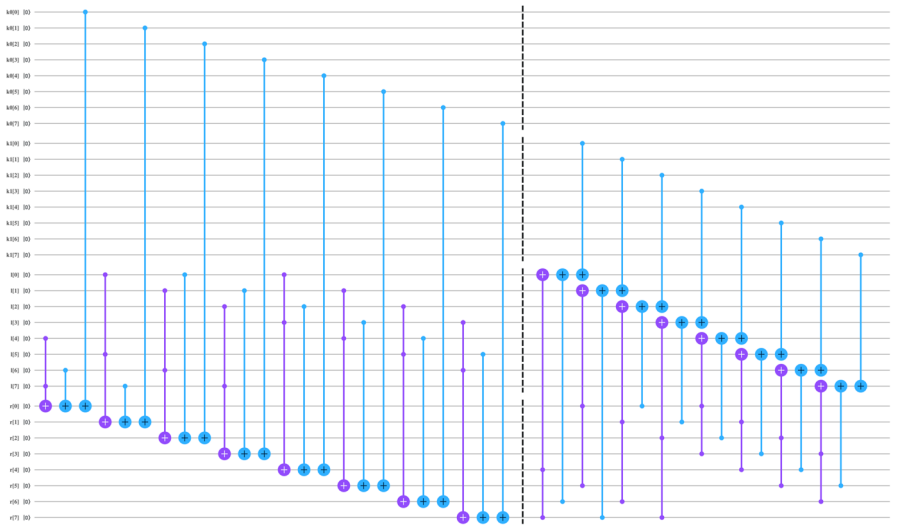


Fig. 4 Circuit for two rounds of reduced version of $SIMON$. Two-qubit controlled gates represent $SIMON$

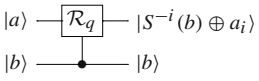
3.2 Key expansion

The key expansion routine is linear and invertible. It is defined in Eq. 2. We can implement an in-place construction of key expansion without the use of any ancilla qubits. We here assume that the round constants are implemented in the circuits using an adequate number of NOT gates. This number will depend on the round constant values. Here, we show the construction for all the three cases $m = 2, 3, 4$.

Let us define a subroutine $\mathcal{R}_q$ on two states $|a\rangle = |a_1 a_2 ... a_n\rangle$ and $|b\rangle = |b_1 b_2 ... b_n\rangle$ such that

$$\mathcal{R}_q(a, b) = (a \oplus S^{-i}(b), b).$$

That is, the state $|b\rangle$ remains unchanged and each qubit in $|a\rangle$ gets modified as $|a_j\rangle = CNOT\,(b_{(j-i)mod(n)}, a_j)$. So each application of $\mathcal{R}_q$ on $|a\rangle$, $|b\rangle$ involves n $CNOT$ gates, where n is the size of $|a\rangle$, $|b\rangle$.



For $m = 2$, we have two key words k_0, k_1 which are used for the first and second rounds of encryption. k_0, k_1 are states of size n, so we need $2n$ qubits to store this value.

The third round key k_2 can be computed on the same qubits which will store k_0. Each bit $k_2(j)$ can be computed from (k_0, k_1) by applying the following three steps:

1. $CNOT(k_1(j-3)mod(n/2), k_0(j))$,

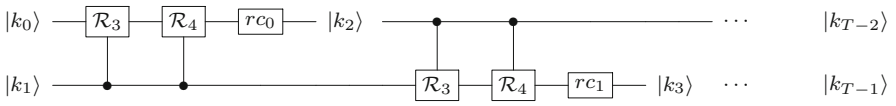


Fig. 5 Circuit for key expansion with two keywords. rc_i represents the round dependent constants

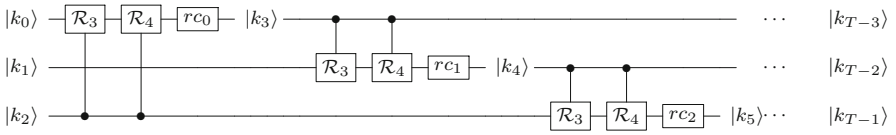


Fig. 6 Circuit for key expansion with three keywords. rc_i represents the round dependent constants

2. $CNOT(k_1(j-4)mod(n/2), k_0(j))$,
3. $NOT(k_0(j))$ only if the value of round constant rc(j) is 1,

where $0 \leq j \leq (n/2)$. The first and the second step is represented by $\mathcal{R}_3$ and $\mathcal{R}_4$ in Fig. 5, respectively.
After k_2 has been computed, we can compute each bit $k_3(j)$ applying the following steps:

1. $CNOT(k_2(j-3)mod(n/2), k_1(j))$,
2. $CNOT(k_2(j-4)mod(n/2), k_1(j))$,
3. $NOT(k_1(j))$ only if the value of round constant rc(j) is 1,

where $0 \leq j \leq (n/2)$. Similarly, we can proceed to compute the further round keys sequentially.

For $m = 3$, we have three key words k_0, k_1, k_2 each of size n. The round keys for further round can be computed as explained above for $m = 2$, and the details are shown in Fig. 6.

For $m = 4$, we have three key words k_0, k_1, k_2, k_3 each of size n. For the extra round keys, we will require an extra step. $k_4(j)$ is computed applying the following steps:

1. $CNOT(k_1(j-1)\text{mod}(n/2), k_0(j))$,
2. $CNOT(k_3(j-3)\text{mod}(n/2), k_0(j))$,
3. $CNOT(k_3(j-4)\text{mod}(n/2), k_0(j))$,
4. $NOT(k_0(j))$ only if the value of round constant rc(j) is 1,

where $0 \leq j \leq (n/2)$. The first step is the extra step required for $m = 4$. The circuit is described in Fig. 7.

It can be easily calculated that for 1 round of key expansion, we need mn $CNOT$ gates and n' NOT gates (where $0 \leq n' \leq n$ depending on the number of 1's in the round constant). In the complete implementation of $SIMON$, $(T-m)$ round keys are generated as the first m key words are used as first m round keys. So, for $(T-m)$ rounds of key expansion we need $(T-m)(mn)$ $CNOT$ gates and $(T-m)n'$ NOT gates.

Consider the circuit in Fig. 8. This circuit represents reduced version of key expansion of $SIMON$ with key words 2, as defined in Eq. 2. The two round constants are

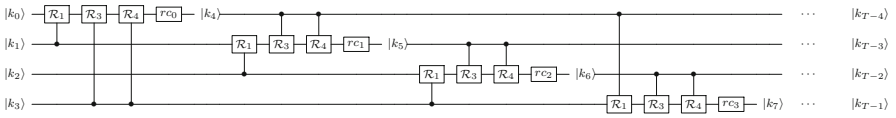


Fig. 7 Circuit for key expansion with four keywords. rc_i represents the round dependent constants

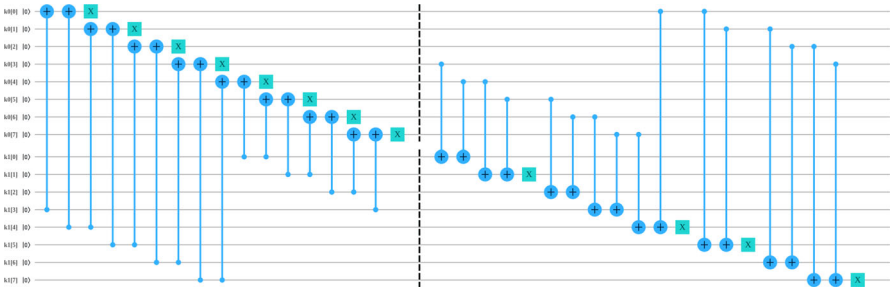


Fig. 8 Key expansion for $m = 2$ and key size 16 split into two round keys of size 8 each.

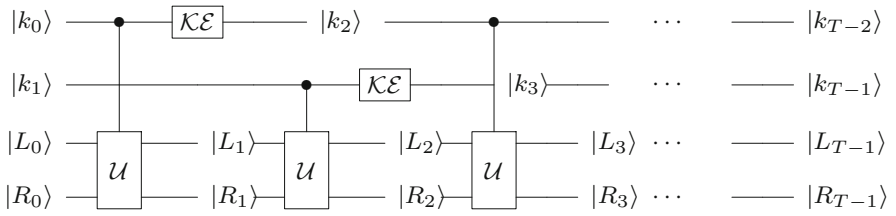


Fig. 9 The circuit for implementing $SIMON$ with two key words.

assumed to be $c_0 = 11111111$ and $c_1 = 01001101$, which are implemented in the circuit by using NOT gates. Barrier separates k_3 from k_4.

3.3 Circuit for *SIMON*

We implement $SIMON$ as a reversible circuit as reversibility is necessary for the cipher to be useful as a subroutine in Grover search. Using the circuits developed for round function and key expansion, we can now construct the circuit for full round $SIMON$.

The input to the circuit is the key K and the plaintext split into two halves L_0, R_0. The output of the circuit is the ciphertext L_{T-1}, R_{T-1}, where T is the number of rounds. The size of K, L, R are mn, n, n, respectively, where m is either 2 or 3 or 4. In Fig. 9, we draw the circuit considering $m = 2$. Similar construction can be made for variants with $m = 3$ and $m = 4$. $\mathcal{U}$ is the round update function which consists of the three subroutines $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$ and $\mathcal{KE}$ is the key expansion routine described in Sect. 3.2.

Figure 10 gives an implementation of the reduced version of $SIMON$, with two key words. First, we run the circuit in IBMQ Simulator [29]. We consider the key size and state size to be 6 and the number of rounds to be 4. The state update function

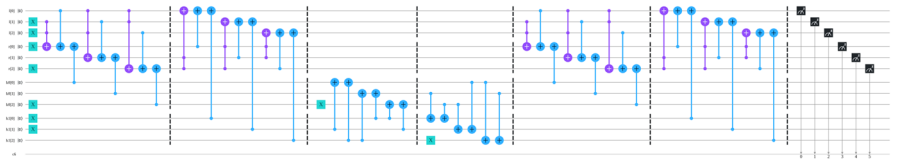


Fig. 10 Circuit for 4 rounds reduced $SIMON$ with two key words.

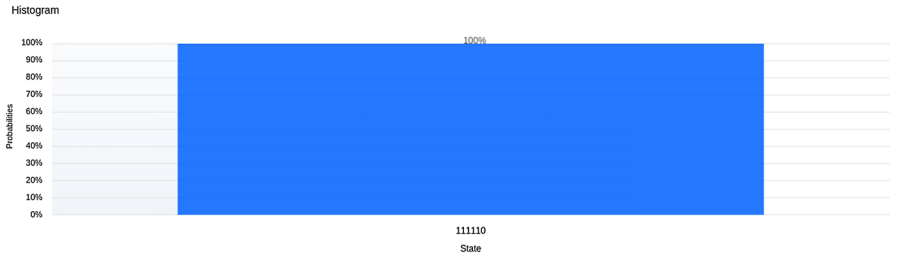


Fig. 11 Measurement of circuit in Fig. 10 in IBMQ simulator. As expected, the output after four rounds is [011111]

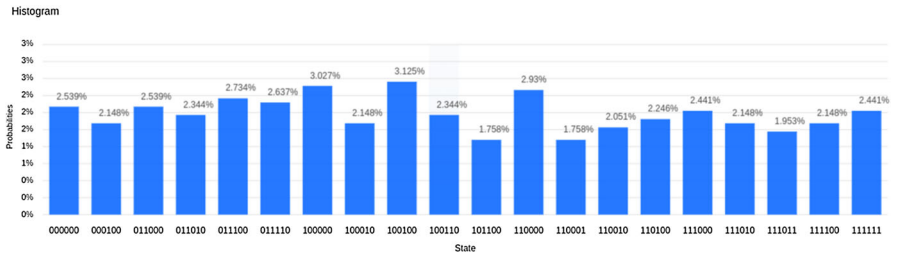


Fig. 12 Measurement of circuit in Fig. 10 in ibmq_melbourne, the 14-qubit actual processor

is defined as $((L_{j+1}, R_{j+1}) = (R_j \oplus (S^1(L_j)\&S^2(L_j)) \oplus S^0(L_j) \oplus k_j), L_j)$, and the key expansion is defined as $k_{j+2} = c_j \oplus k_j \oplus S^{-1}(k_{j+1}) \oplus S^{-2}(k_{j+1})$ where c_j are round-dependent constants [0, 0, 1] and [0, 0, 1] for the third and fourth rounds, respectively. Let the plaintext be $L_0 = [0, 1, 1]$, $R_0 = [1, 0, 1]$ and the key words be $k_0 = [0, 0, 1]$, $k_1 = [1, 1, 0]$. Then, after four rounds the ciphertext will be $L_4 = [0, 1, 1]$, $R_4 = [1, 1, 1]$. In IBMQ circuit, one should read the ciphertext as $L_4 = [L_4(0), L_4(1), L_4(2)]$ and $R_4 = [R_4(0), R_4(1), R_4(2)]$. In Fig. 11 ,the output is shown as $[R_4(2), R_4(1), R_4(0), L_4(2), L_4(1), L_4(0)]$.

Then, we run the circuit (Fig. 10) in ibmq_melbourne, the 14-qubit actual processor. In contrary to the simulation (Fig. 11), we observed a huge error in the result. Figure 12 shows the histogram we have obtained after running the circuit in ibmq_melbourne for 1024 shots. The reason behind this is the infidelity of the gates used in the circuit and the decoherence time of the qubits used in the actual processor, which has been pointed out in several works like [8,22] and also in the IBMQ's official site.

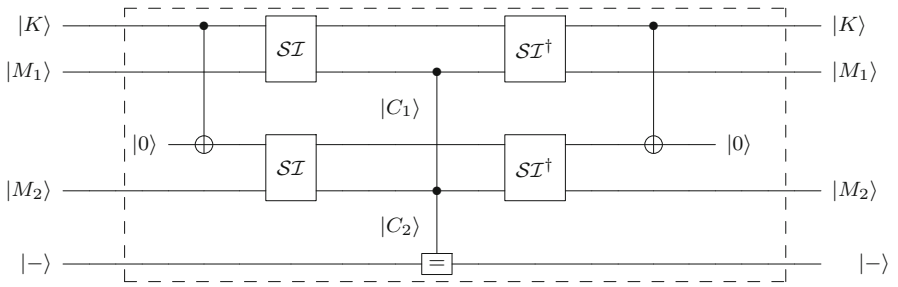


Fig. 13 Grover's oracle for $SIMON$ using two plaintext–ciphertext pairs. Here, $\mathcal{SI}$ represents the $SIMON$ encryption as described in Sect. 3.3. The (=) operator compares the output of the $\mathcal{SI}$ with the given ciphertexts and flips the target qubit if they are equal

3.4 Grover's oracle

In this subsection, we will discuss the implementation of Grover search on a block cipher under known plaintext attack. Let r pairs of plaintext and ciphertext be sufficient to successfully extract the unique solution. In this regard, we have to design an oracle that encrypts the given r plaintexts under the same key and then computes a Boolean value which determines whether all the resulting ciphertexts are equal to the given classical ciphertexts. This can be done by running the block cipher circuit r times in parallel. Then the resultant ciphertexts are compared with the classical ciphertexts. The target qubit will be flipped if the ciphertexts match. This is called Grover oracle. In Fig. 13, the construction of such an oracle is given for two instances of plaintext–ciphertext pairs considering $SIMON$ block cipher.

We implement the above idea, i.e., Grover oracle for our reduced version of $SIMON$ (Fig. 10) in the IBMQ simulator. We have not run the circuit for Grover in ibmq_melbourne, as the output of the reduced $SIMON$ in this processor has already been masked with a huge amount of error (Fig. 12).

Now, in this case, the plaintext is $M = [0, 1, 1, 1, 0, 1]$ and the key is $K = [0, 0, 1, 1, 1, 0]$. Then, after four rounds the ciphertext will be $C = [0, 1, 1, 1, 1, 1]$. In theory, this key $K = [0, 0, 1, 1, 1, 0]$ will be obtained as the output of Grover oracle. Figure 14a shows the outcome of Grover applied to this circuit. Note that in the histogram, two peaks appear: one for the exact key K and another for an unknown key $K' = [1, 1, 1, 0, 0, 0]$. This is because the unknown key K' also encrypts the plaintext M onto the ciphertext C. To find the unique key, we use another plaintext–ciphertext pair, $M_1 = [0, 0, 1, 1, 0, 1]$, $C_1 = [1, 1, 0, 0, 1, 1]$ under the same key. Figure 14b shows the outcome of Grover applied to this circuit. In this histogram also, we got two peaks. Similar to Fig. 14a, one peak is for the exact key K and another is for an unknown key $K'' = [0, 0, 1, 0, 0, 1]$ as K'' encrypts M_1 to C_1 too. Note that the key common to both the pairs is $K = [0, 0, 1, 1, 1, 0]$ which is the exact key. The full implementation is described in [26]. The functions Grover_oracle and Grover_Diffusion used in our code are taken from [15] by importing the file Our_Qiskit_Functions.

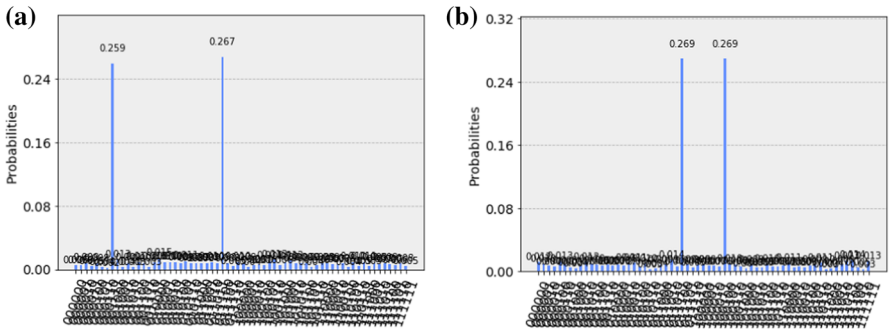


Fig. 14 Histogram obtained after running Grover's on the reduced $SIMON$ described in Fig. 10

Table 3 Cost of implementing $SIMON$ variants

$SIMON\ 2n/mn$	# NOT	# $CNOT$	# $Toffoli$	# qubits	Depth
$SIMON\ 32/64$	448	2816	512	96	946
$SIMON\ 48/72$	792	3312	864	120	1062
$SIMON\ 48/96$	768	4800	864	144	1597
$SIMON\ 64/96$	1248	5184	1344	160	1674
$SIMON\ 64/128$	1216	7396	1408	192	2643
$SIMON\ 96/96$	2400	9792	2496	192	4785
$SIMON\ 96/144$	2448	10080	2592	240	3282
$SIMON\ 128/128$	4224	17152	4352	256	8427
$SIMON\ 128/192$	4224	17472	4416	320	5656
$SIMON\ 128/256$	4352	26624	4608	384	8848

3.5 Resource estimation

3.5.1 Cost of implementing *SIMON*

We first estimate the cost of implementing $SIMON$ as a circuit including the key expansion as well as the round function. As discussed above, the round constants are implemented to the key expansion function using an adequate number of NOT gates. We have not included the number of NOT required to initialize the plaintext in our estimates as it depends on the given plaintext. Table 3 gives the cost estimates of implementing all $SIMON$ variants.

3.5.2 Cost of Grover oracle

Following [11], we assume that $r = \lceil k/(2n) \rceil$ known plaintext–ciphertext pairs are sufficient to give us a unique solution, where $2n$ is the block size and $k = mn$ is the key size of the cipher. Here, one should mention that if $r = \lceil k/(2n) \rceil$ is an integer, then consider its next integer. Grover's oracle consists of comparing the $2n$-bit outputs of the $SIMON$ instances with the given r ciphertexts. This can be done using a

Table 4 Cost of Grover oracle

$SIMON\ 2n/k$	r	# Clifford gates	# T gates	T-depth	full depth	# qubits
$SIMON\ 32/64$	3	19,840	24,492	12,288	27,180	161
$SIMON\ 48/72$	2	16,560	27,180	13,824	28,440	169
$SIMON\ 48/96$	3	33,792	40,812	20,736	45,860	241
$SIMON\ 64/96$	2	25,620	41,644	21,504	44,988	224
$SIMON\ 64/128$	3	52,184	65,196	33,792	74,994	321
$SIMON\ 96/96$	2	48,768	75,948	39,936	89,028	289
$SIMON\ 96/144$	2	50,400	78,636	41,472	86,104	337
$SIMON\ 128/128$	2	85,760	129,964	69,632	15,1564	385
$SIMON\ 128/192$	2	87,168	131,756	70,656	146,272	449
$SIMON\ 128/256$	3	186,880	205,740	110,592	246,624	641

$(2n \cdot r)$-controlled *CNOT* gates. (We neglect some NOT gates which depend on the given ciphertexts.) Following [24], we estimate the number of T gates required to implement a t-fold controlled NOT gates as $(32 \cdot t - 84)$.

We use the decomposition of *Toffoli* gates to 7 T-gates plus 8 Clifford gates, a T-depth of 4 and total depth of 8 as in [3]. To estimate the full depth and the T-depth, we only consider the depths of the $SIMON$ instances ignoring the multi controlled NOT gate used in comparing the ciphertexts. We also need $(2 \cdot (r-1) \cdot k)$ *CNOT* gates to make the input key available to all the $SIMON$ instances in the oracle. The total number of Clifford gates is the sum of the Clifford gates used in the implementation of $SIMON$ and the $(2 \cdot (r-1) \cdot k)$ *CNOT* gates needed for input key. The cost estimates for all $SIMON$ variants are presented in Table 4.

3.5.3 Cost of exhaustive key search

Using the estimates in Table 4 of Grover's oracle for the various variants, we provide the cost estimates for the full exhaustive key search on all variants in Table 5. We consider $\lfloor \frac{\pi}{4} 2^{k/2} \rfloor$ iterations of Grover's operator. As in [17], we do not consider the depth of implementing the two multi-controlled NOT gates while calculating the T-depth and overall depth.

4 Conclusion

In this work, we presented an implementation of Grover's search algorithm on $SIMON$. We first provided a reversible circuit of all variants of $SIMON$. Then, these circuits were used to estimate the cost of attacking $SIMON$ using Grover's algorithm. The plausible values of the overall circuit depth suggested by NIST [27] are between 2^{40} and 2^{96}, and we assume that this depth is an upper bound on the total depth of a quantum attack. The overall circuit depth of all the implementations presented in this work except for $SIMON 128/192$ and $SIMON 128/256$ lies in this

Table 5 Cost estimates of Grover's algorithm with $\lfloor \frac{\pi}{4} 2^{k/2} \rfloor$ oracle iterations with a success probability negligibly close to 1

$SIMON\ 2n/k$	# Clifford gates	# T gates	T-depth	Full depth	# qubits
$SIMON\ 32/64$	$1.35 \cdot 2^{45.5}$	$1.27 \cdot 2^{46}$	$1.18 \cdot 2^{45}$	$1.05 \cdot 2^{46.3}$	161
$SIMON\ 48/72$	$1.01 \cdot 2^{49.65}$	$1.03 \cdot 2^{50.45}$	$1.01 \cdot 2^{49.4}$	$1.05 \cdot 2^{50.37}$	169
$SIMON\ 48/96$	$1.02 \cdot 2^{62.66}$	$1.02 \cdot 2^{63.05}$	$1.01 \cdot 2^{61.97}$	$1.02 \cdot 2^{63.11}$	241
$SIMON\ 64/96$	$1.02 \cdot 2^{62.27}$	$1.01 \cdot 2^{63.08}$	$1.10 \cdot 2^{61.9}$	$1.07 \cdot 2^{63}$	224
$SIMON\ 64/128$	$1.03 \cdot 2^{79.27}$	$1.02 \cdot 2^{79.7}$	$1.06 \cdot 2^{78.6}$	$1.03 \cdot 2^{79.8}$	321
$SIMON\ 96/96$	$1.02 \cdot 2^{63.2}$	$1.04 \cdot 2^{63.85}$	$1.02 \cdot 2^{62.9}$	$1.02 \cdot 2^{64}$	289
$SIMON\ 96/144$	$1.05 \cdot 2^{87.2}$	$1.06 \cdot 2^{87.9}$	$1.22 \cdot 2^{86.7}$	$1.03 \cdot 2^{88}$	337
$SIMON\ 128/128$	$1.03 \cdot 2^{80}$	$1.14 \cdot 2^{80.5}$	$1.17 \cdot 2^{79.51}$	$1.12 \cdot 2^{80.7}$	385
$SIMON\ 128/192$	$1.04 \cdot 2^{112}$	$1.17 \cdot 2^{112.5}$	$1.19 \cdot 2^{111.51}$	$1.08 \cdot 2^{112.7}$	449
$SIMON\ 128/256$	$1.05 \cdot 2^{145.1}$	$1.11 \cdot 2^{145.2}$	$1.07 \cdot 2^{144.3}$	$1.12 \cdot 2^{145.4}$	641

range. This work is aimed at using the minimal number of qubits; in the future, it would be interesting to re-estimate the cost by reducing the depth of the implementations at the expense of some extra qubits by using other choices of decomposition of *Toffoli*gates, e.g., as in [20] which has a T-depth of 1 and an overall depth of 7 but uses 18 Clifford gates and 4 ancilla qubits.

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