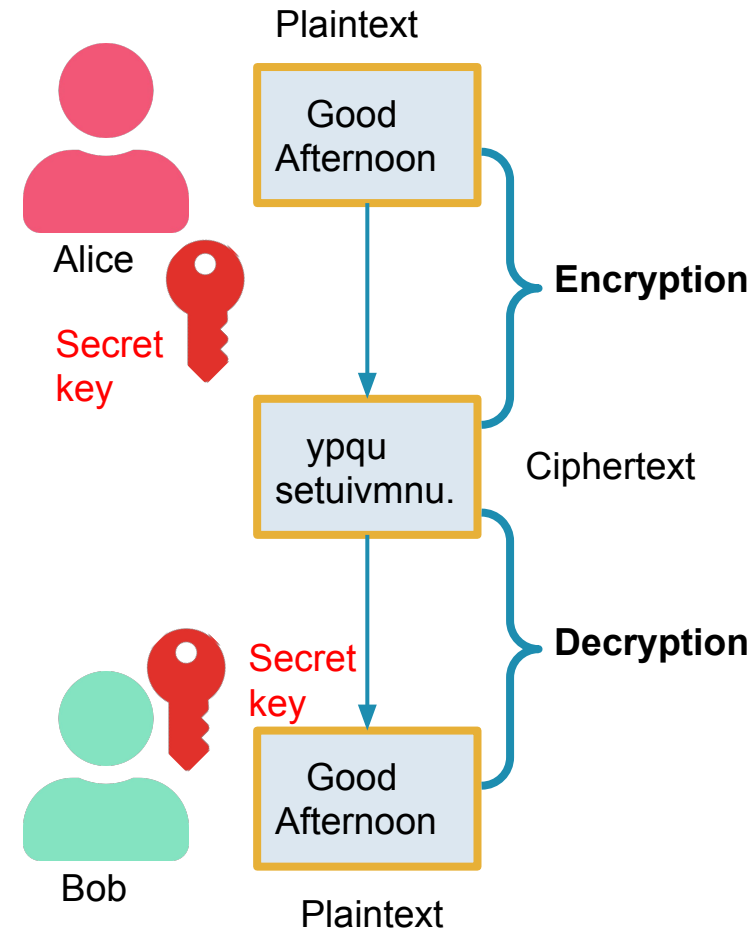


# Quantum secure public-key cryptography: history and current developments

Suparna Kundu

# Symmetric key cryptography

- Before 1970 people only used Symmetric key Cryptography
- Symmetric key scheme
  - A very strong algorithm
  - Both party shares same **secret key**

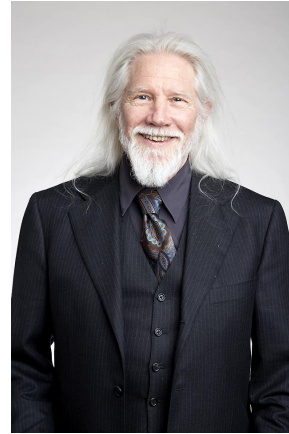


Symmetric-key scheme

# Public key cryptography

**1976:** Whitfield Diffie and Martin Hellman introduce the concept of Public-key Cryptography

- Deffie-Hellman<sup>1</sup> (DH) protocol



**1978:** Ron Rivest, Adi Shamir and Leonard Adleman proposed first successful PKC **RSA**<sup>2</sup>

**1985, 1987:** Elliptic curve cryptography (ECC) introduced by **Victor Miller**<sup>3</sup> and **Neal Koblitz**<sup>4</sup>

<sup>1</sup> Diffie, Whitfield; Hellman, Martin E. (November 1976). "New Directions in Cryptography". *IEEE Transactions on Information Theory*. **22** (6): 644–654.

<sup>2</sup> Rivest, R.; Shamir, A.; Adleman, L. (February 1978). "A Method for Obtaining Digital Signatures and Public-Key Cryptosystems" (PDF). *Communications of the ACM*. **21** (2): 120–126.

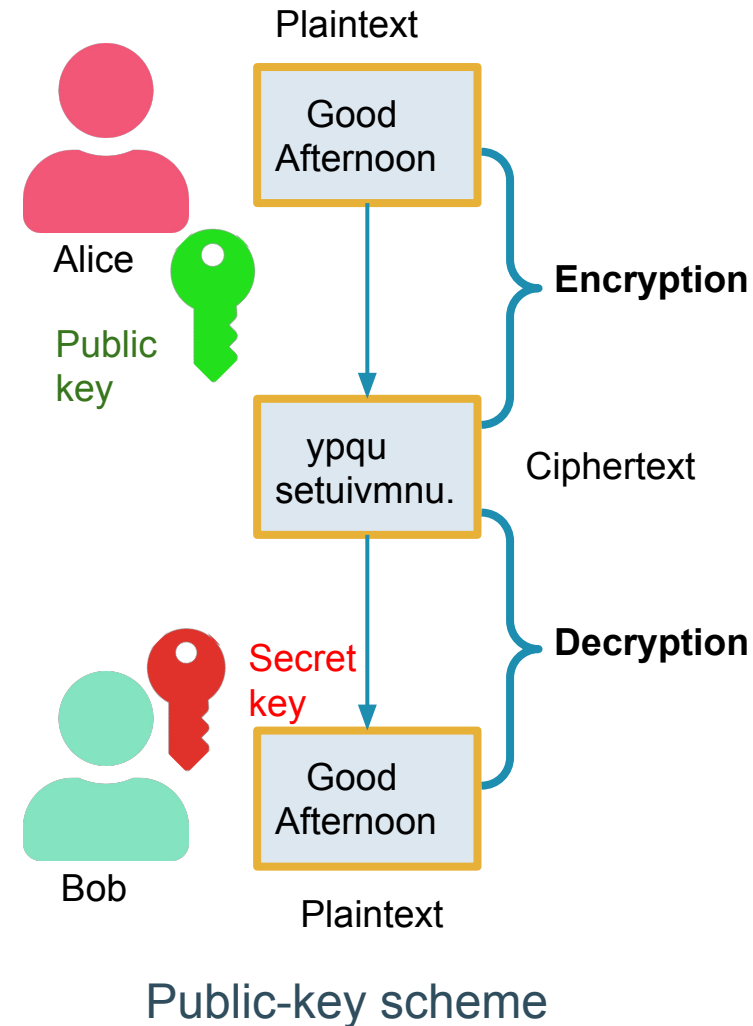
<sup>3</sup> Miller, V. (1985). "Use of elliptic curves in cryptography". *Advances in Cryptology — CRYPTO '85 Proceedings*. *CRYPTO*. Lecture Notes in Computer Science. **85**. pp. 417–426.

<sup>4</sup> Koblitz, N. (1987). "Elliptic curve cryptosystems". *Mathematics of Computation*. **48** (177): 203–209.

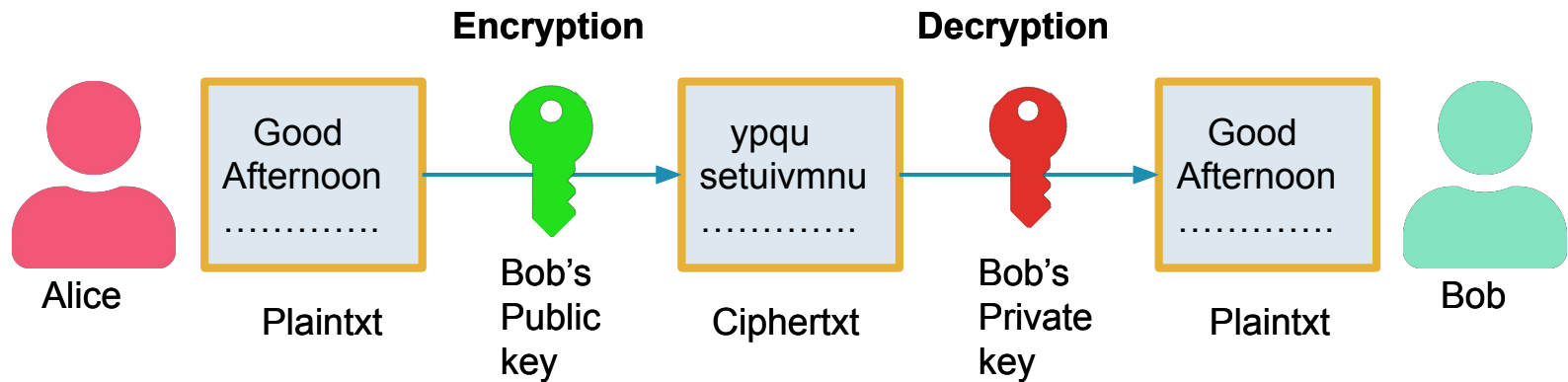
# Public key cryptography

Public key crypto system uses:

- Two keys
  - **Public key** → **Encryption**
  - **Secret key** → **Decryption**
- Cryptographic algorithms are based on mathematical problem (**One-way function**)
  - **RSA**: Large integer factorization (Given  $N=p*q$ , find  $p$  and  $q$ )
  - **ECC**: Elliptic curve discrete logarithm problem (Given  $xP$  and  $P$ , find  $x$ )



# Public key cryptography



## Public-key cryptography

used in

SSL/  
TLS

Secure key  
exchange



Crypto-  
currencies



Digital  
signature



G Pay  
Digital  
payment

# Quantum Computer and its uses

1980: Paul Benioff<sup>1</sup> proposed quantum mechanical model of the Turing machine



1994: Peter Shor<sup>2</sup> developed a quantum algorithm for factoring integers

- It can break **RSA** scheme

<sup>1</sup>Benioff, Paul (1980). "The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines". *Journal of Statistical Physics*. **22** (5): 563–591.

## Algorithms for Quantum Computation: Discrete Logarithms and Factoring

Peter W. Shor  
AT&T Bell Labs  
Room 2D-149  
600 Mountain Ave.  
Murray Hill, NJ 07974, USA

### Abstract

*A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor. It is not clear whether this is still true when quantum mechanics is taken into consideration. Several researchers, starting*

[1, 2]. Although he did not ask whether quantum mechanics conferred extra power to computation, he did show that a Turing machine could be simulated by the reversible unitary evolution of a quantum process, which is a necessary prerequisite for quantum computation. Deutsch [9, 10] was the first to give an explicit model of quantum computation. He defined both quantum Turing machines and quantum circuits and investigated some of their properties.

<sup>2</sup>Mermin, David (March 28, 2006). "Breaking RSA Encryption with a Quantum Computer: Shor's Factoring Algorithm" (PDF). *Cornell University, Physics 481-681 Lecture Notes*. Archived from the original (PDF) on 2012-11-15.

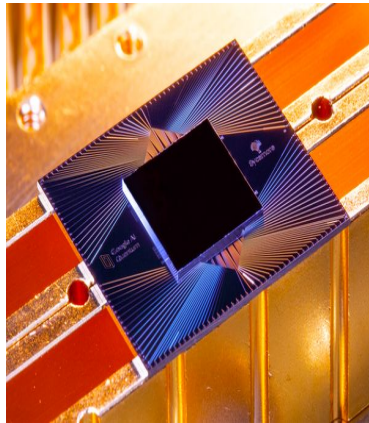
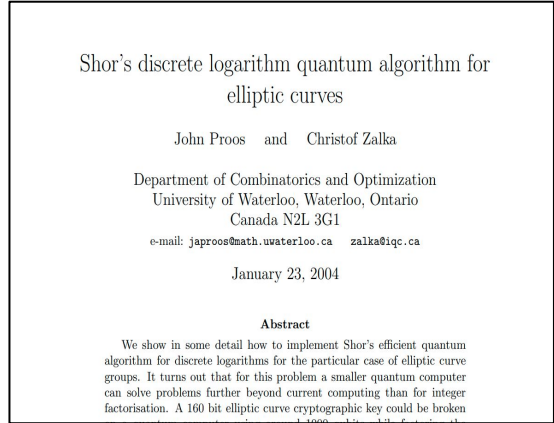
# Quantum Computer and its uses

**2004:** Proos and Zalka<sup>1</sup> presented an algorithm to solve the elliptic curve discrete logarithm problem in polynomial time

**2019:** Google AI, with NASA, claimed to achieve 54-qbit quantum supremacy<sup>2</sup>

**2020:** IBM claimed to achieve 65-qbit quantum supremacy<sup>3</sup>

**Target<sup>3</sup>:** 1000-qbit on 2023



<sup>1</sup> Proos, John & Zalka, Christof. (2003). Shor's Discrete Logarithm Quantum Algorithm for Elliptic Curves. Quantum Information & Computation. 3.

<sup>2</sup> <https://ai.googleblog.com/2019/10/quantum-supremacy-using-programmable.html>

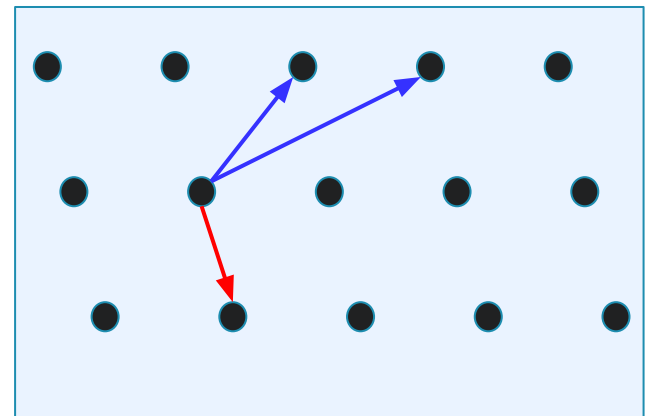
<sup>3</sup> <https://research.ibm.com/blog/ibm-quantum-roadmap>

# Lattice based Cryptography

1996: Miklós Ajtai<sup>1</sup> introduced the first lattice-based cryptographic construction

**SVP:** Given a lattice  $L$ , find the shortest non-zero vector

**Hard Problem:** Shortest vector problem (SVP)



<sup>1</sup>Ajtai, Miklós (1996). "Generating Hard Instances of Lattice Problems". *Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing*. pp. 99–108.

Basis vectors in blue  
Shortest vector in red



# Lattice based Cryptography

1998: Jeffrey Hoffstein, Jill Pipher,  
and Joseph H. Silverman<sup>1</sup>  
introduced

- the first lattice-based public-key encryption scheme
- known as NTRU
- This version wasn't provably secure

The logo for NTRU consists of a large, bold, light green letter 'N' followed by the lowercase letters 't', 'r', and 'u' in a dark green, cursive-style font. A horizontal bar is positioned above the 'u'.

Nth Degree Truncated  
Polynomial Ring Units  
( $R = \mathbb{Z}[X]/(X^N - 1)$ )

<sup>1</sup> Hoffstein, Jeffrey; Pipher, Jill; Silverman, Joseph H. (1998). "NTRU: A ring-based public key cryptosystem". *Algorithmic Number Theory. Lecture Notes in Computer Science*. **1423**. pp. 267–288.

# Lattice based Cryptography

2005: Oded Regev<sup>1</sup> introduced

- 1st provably secure lattice-based public-key encryption scheme
- Learning with errors (LWE) problem

$$\underbrace{\begin{pmatrix} a_{1,1} & \cdots & a_{n,1} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix}}_{\mathbf{s}} + \underbrace{\begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix}}_{\mathbf{e}} = \underbrace{\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}}_{\mathbf{b}}$$

Problem: Find  $\mathbf{s}$  in presence of  $\mathbf{e}$

Search problem:

- Choose  $\mathbf{s} \leftarrow Z_q^n$
- Oracle generates  $\mathbf{a} \leftarrow Z_q^n$  & small  $\mathbf{e} \leftarrow \chi$
- Oracle outputs  $\mathbf{a}, \mathbf{b} = \mathbf{a} \cdot \mathbf{s} + \mathbf{e} \bmod q$
- repeat with fresh  $\mathbf{a}$  and  $\mathbf{e}$

<sup>1</sup> Regev, O.: On lattices, learning with errors, random linear codes, and cryptography. In: STOC '05. pp. 84–93. ACM (2005), <http://doi.acm.org/10.1145/1060590.1060603>

# Lattice based Cryptography

LWE decision problem:

LWE distribution	Uniform distribution
Fixed $\mathbf{s} \leftarrow \mathcal{U}(\mathbb{Z}_q^n)$	
$\mathbf{a}_i \leftarrow \mathcal{U}(\mathbb{Z}_q^n) \quad \mathbf{e}_i \leftarrow \chi$	$\mathbf{a}_i \leftarrow \mathcal{U}(\mathbb{Z}_q^n) \quad b_i \leftarrow \mathcal{U}(\mathbb{Z}_q^n)$
$\mathbf{a}_1, b_1 = \mathbf{a}_1 \cdot \mathbf{s} + \mathbf{e}_1 \text{ mod } q$	$\mathbf{a}_1, b_1$
• • •	• • •
$\mathbf{a}_n, b_n = \mathbf{a}_n \cdot \mathbf{s} + \mathbf{e}_n \text{ mod } q$	$\mathbf{a}_n, b_n$

Problem: Distinguishing the distribution

# Lattice based Cryptography

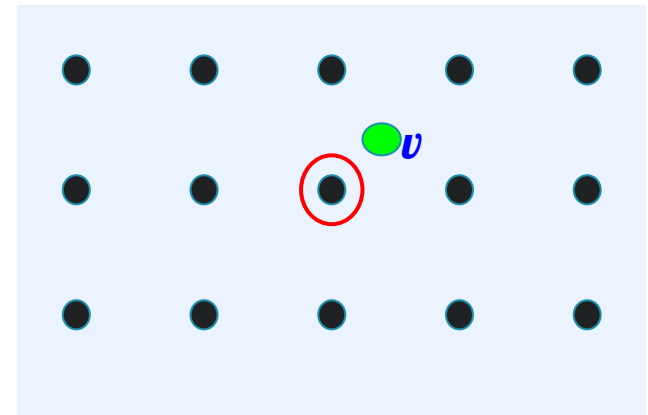
## Relation with lattice:

- Given  $\mathbf{A} \in \mathbb{Z}_q^{n \times n}$  &  $\mathbf{b} \in \mathbb{Z}_q^n$  with  $\mathbf{b} = \mathbf{A} \cdot \mathbf{s} + \mathbf{e}$
- small  $\mathbf{e}$  is in  $[-q/2, q/2]$
- The set  $\mathcal{L}(\mathbf{A}) = \{\mathbf{y} \in \mathbb{Z}_q^n : \mathbf{y} = \mathbf{A} \cdot \mathbf{x}, \text{ where } \mathbf{x} \in \mathbb{Z}_q^n\}$
- $\mathcal{L}(\mathbf{A})$  forms a lattice
  - if  $y_1, y_2 \in \mathcal{L}(\mathbf{A})$  then  $y_1 - y_2 \in \mathcal{L}(\mathbf{A})$
- If  $\mathbf{e} \neq 0$  then  $\mathbf{b} \notin \mathcal{L}(\mathbf{A})$  but close to it

# Lattice based Cryptography

## Closest vector problem (CVP):

Given lattice  $\mathcal{L} \subset V$  and  $v \in V$  ( may not be in  $\mathcal{L}$ ) then find the closest vector of  $v$  in  $\mathcal{L}$



## Bounded distance decoding (BDD):

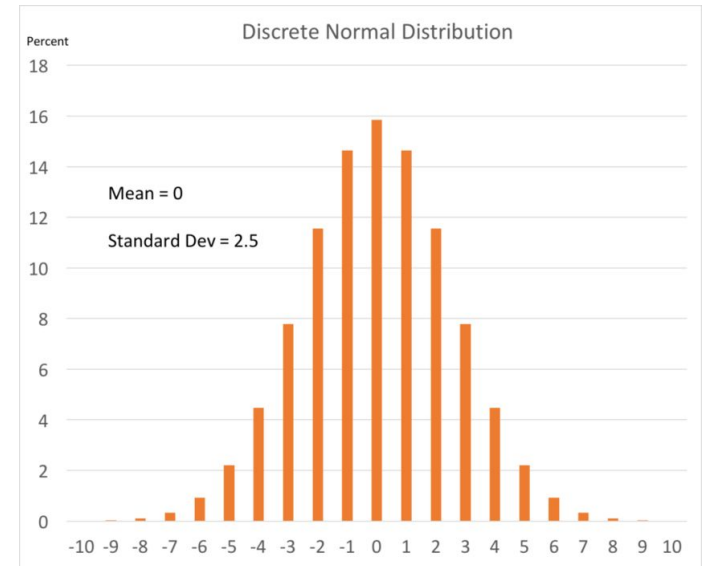
Maximum distance of vector  $v$  from  $\mathcal{L}$  is  $\lambda(\mathcal{L})/2$ , where  $\lambda(\mathcal{L})$  is the shortest non-zero vector in  $\mathcal{L}$

By solving BDD where  $|e| \leq \lambda(\mathcal{L})/2$  we can find  $s$  of  $b = A \cdot s + e$

# Lattice based Cryptography

## Few properties of LWE problem:

- The error distribution  $\chi$  usually discrete Gaussian distribution  $(0, \sigma)$  over  $\mathbb{Z}$
- Oded Regev<sup>1</sup> and Chris Peikert<sup>2</sup> proved that LWE problem is as hard as worst case lattice problems
  - For this  $\sqrt{2\pi}\sigma > \sqrt{n}$
- Search LWE problem and Decision LWE problem is equivalent



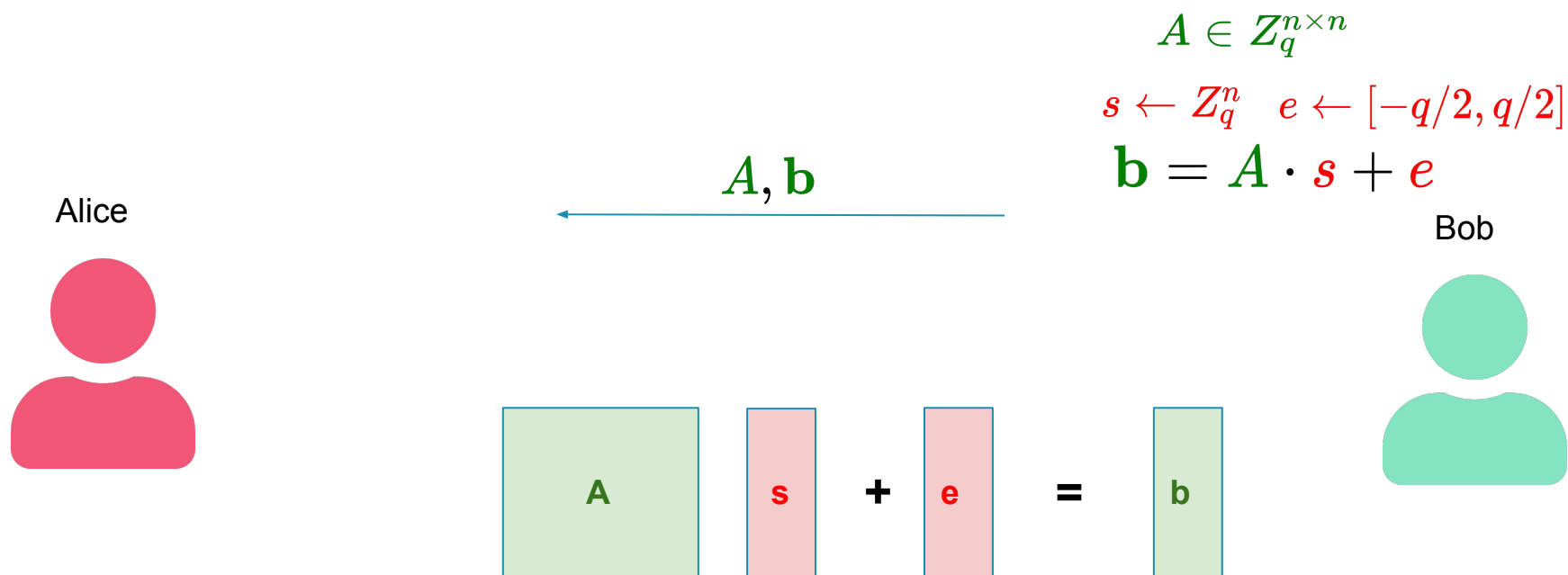
Taken from: <https://www.researchgate.net/profile/Sebastian-Schneeweiss>

<sup>1</sup> O. Regev. New lattice-based cryptographic constructions. Journal of the ACM, 51(6):899–942, 2004. Preliminary version in STOC’05.

<sup>2</sup> C. Peikert. Public-key cryptosystems from the worst-case shortest vector problem. In Proc. 41st ACM Symp. on Theory of Computing (STOC), pages 333–342. 2009.

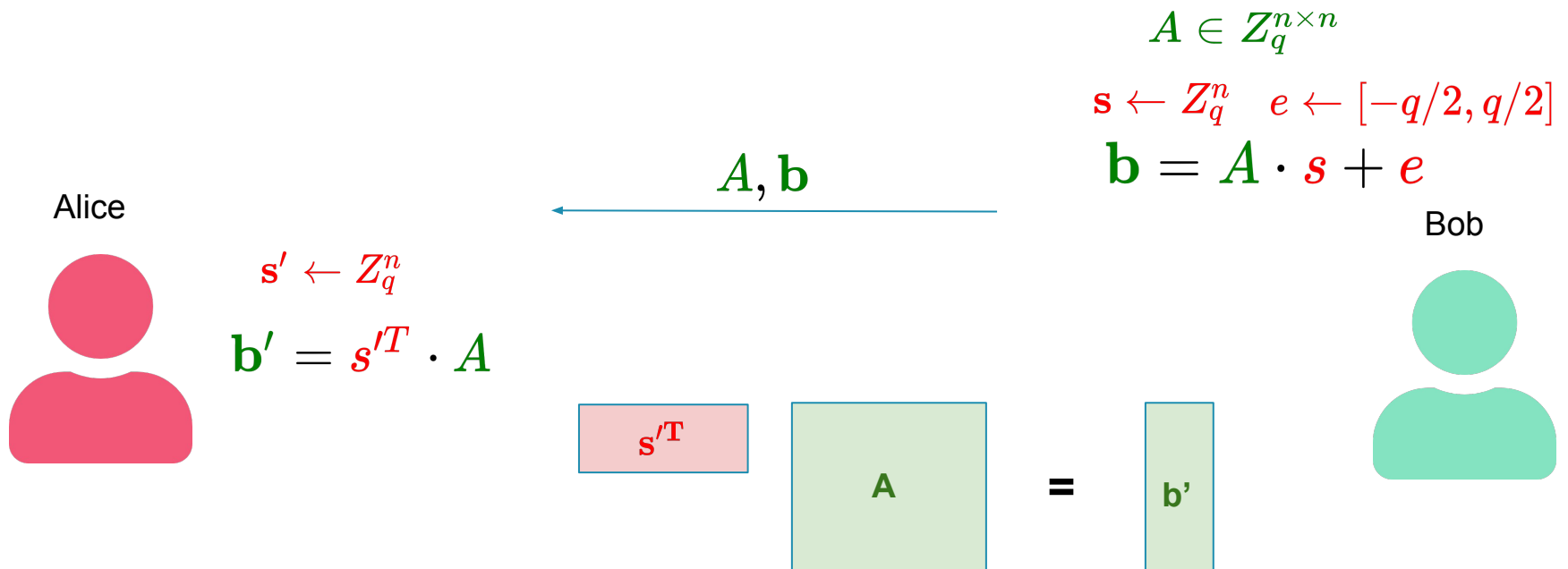
# Lattice based Cryptography

## LWE based Encryption: Key generation



# Lattice based Cryptography

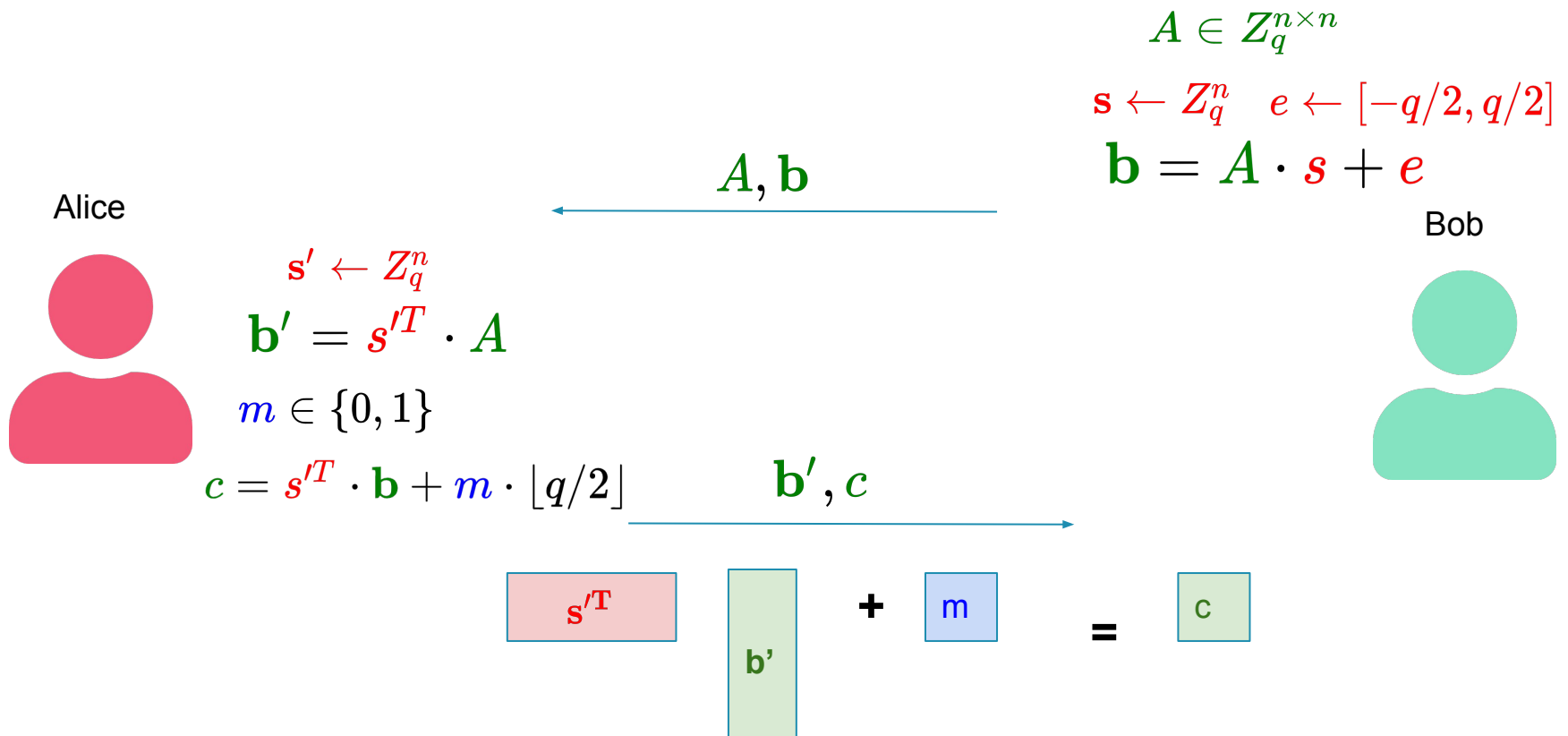
## LWE based Encryption: Encryption





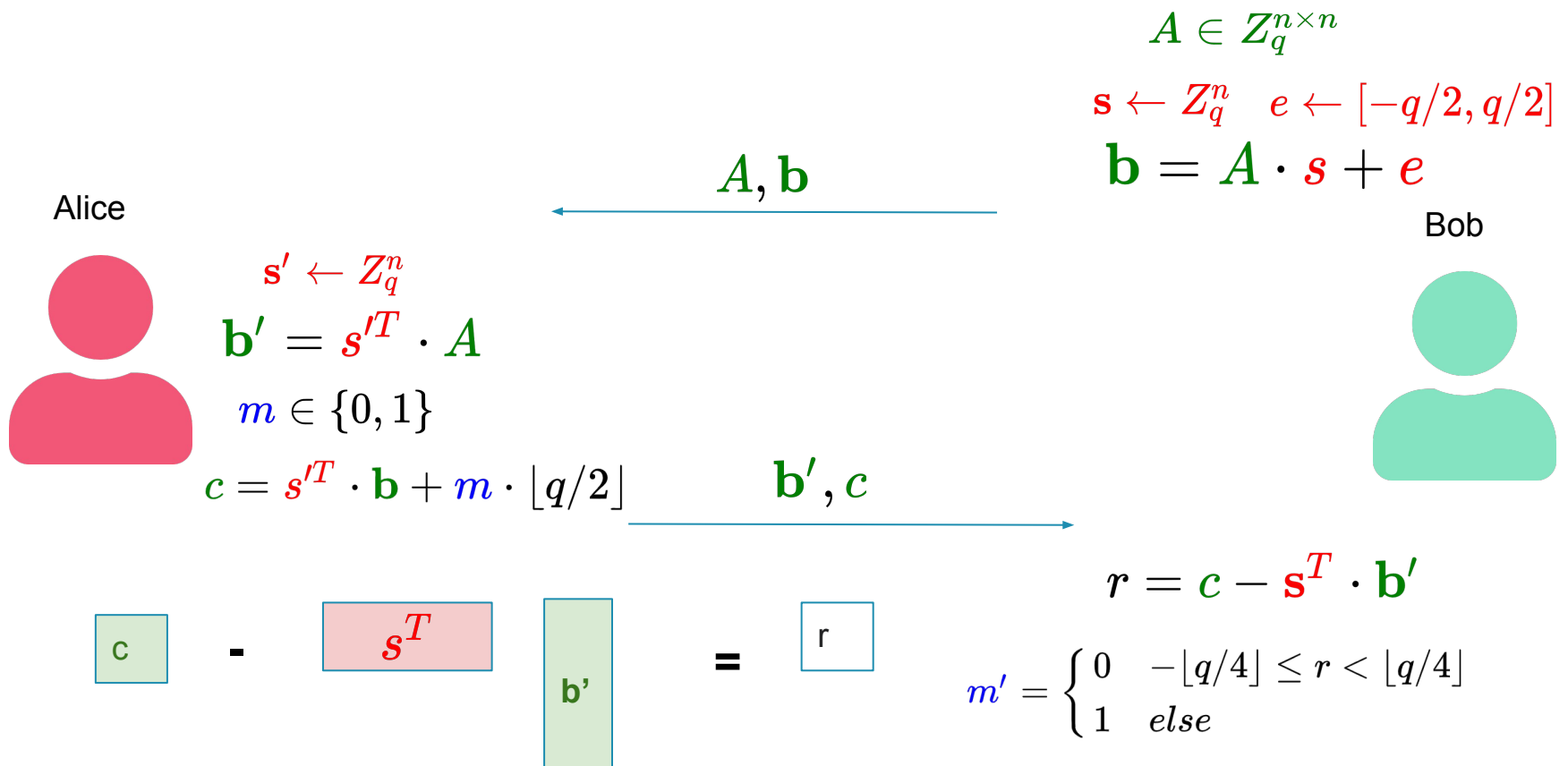
# Lattice based Cryptography

## LWE based Encryption: Encryption



# Lattice based Cryptography

## LWE based Encryption: Decryption



# Lattice based Cryptography

2012: Jintai Ding<sup>1</sup> first proposed

- the idea of key exchange by using LWE problem and
- Variant of LWE **Ring-LWE** problem
- Uses the ring  $\mathbf{R}_q^n = \mathbf{Z}_q[x]/f(x)$
- $f(x)$  is n degree polynomial

**Ring-LWE problem:**  
Matrix-vector multiplication replaced by polynomial multiplication

$$\left( \leftarrow a \in \mathbf{R}_q^n \rightarrow \right) \cdot \begin{pmatrix} \uparrow \\ s \in \mathbf{R}_q^n \\ \downarrow \end{pmatrix} + \begin{pmatrix} \uparrow \\ e \in \mathbf{R}_q^n \\ \downarrow \end{pmatrix}$$

$$\left( \leftarrow a \in \mathbf{R}_q^n \rightarrow \right) \\ ax^3+bx^2+cx+d$$

a	b	c	d
b	c	d	-a
c	d	-a	-b
d	-a	-b	-c

= **A**

<sup>1</sup>Ding, Jintai; Xie, Xiang; Lin, Xiaodong (2012). *A Simple Provably Secure Key Exchange Scheme Based on the Learning with Errors Problem*

# Lattice based Cryptography

## Cryptosystems based on LWE

- **Inefficient** due to matrix-vector multiplication
- But have **strong** security

## Cryptosystems based on Ring-LWE

- **Fast** polynomial multiplication
- Some researchers are skeptical about hardness of hard lattice problems in ideal lattices

# Lattice based Cryptography

2016: Erdem Alkim, Léo Ducas, Thomas Pöppelmann, and Peter Schwabe jointly proposed

- practical key-exchange mechanism (KEM) **NewHope**<sup>1</sup>
- Security depends on hardness of the **Ring-LWE** problem
- Followed **Ding**'s idea with modification
- Selected for **Google's** post-quantum experiment



<sup>1</sup> Erdem Alkim, Philipp Jakubeit, Peter Schwabe: NewHope on ARM Cortex-M. SPACE 2016: 332-349

# Lattice based Cryptography

## NewHope<sup>1</sup>:

- Used ring  $\mathbf{R}_q^n = \mathbf{Z}_q[x]/f(x)$  with  $q$  prime and  $f(x) = (1+x^n)$ ,  $n$  power-of-2
- One of the most costly operations is polynomial multiplication
- Number theoretic transformation (NTT) method is used for this
- $res = a * b$  as
- The complexity is  $O(n \log n)$
- They achieved a very good efficiency due to their design choices

$$\mathbf{b} = \mathbf{A} \cdot \mathbf{s} + \mathbf{e}$$

$$\mathbf{b}' = \mathbf{s}'^T \cdot \mathbf{A}$$

$$c = \mathbf{s}'^T \cdot \mathbf{b} + m \cdot \lfloor q/2 \rfloor$$

$$res = NTT^{-1}(NTT(a) * NTT(b))$$

<sup>1</sup> Erdem Alkim, Philipp Jakubeit, Peter Schwabe: NewHope on ARM Cortex-M. SPACE 2016: 332-349

# NIST's contribution in post-quantum cryptography

**NIST**

**National Institute of  
Standards and Technology**  
U.S. Department of Commerce

2016:

initiated a  
post-quantum PKC standardization process

Categories:

- KEM / Encryption
- Digital signature

## NIST Released NISTIR 8105, Report on Post-Quantum Cryptography

“.....The goal of post-quantum cryptography (also called quantum-resistant cryptography) is to develop cryptographic systems that are secure against both quantum and classical computers, and can interoperate with existing communications protocols and networks. In recent years, there has been a substantial amount of research on quantum computers. If large-scale quantum computers are ever built, they will be able to break many of the public-key cryptosystems currently in use. This would seriously compromise the confidentiality and integrity of digital communications on the Internet and elsewhere.....”

<sup>1</sup> <https://csrc.nist.gov/News/2016/NIST-Released-NISTIR-8105,-Report-on-Post-Quantum>

# NIST's contribution in post-quantum cryptography

## 2017: First round:

	Digital Signature	KEM/Encryption	Total
Lattice-based	5	21	29
Code-based	3	16	19
Multi-variate	7	2	9
Hash-based	3	0	3
Others	2	6	8



# Kyber<sup>1</sup> (2017)

Kyber<sup>1</sup> team introduced another new variant of **LWE** problem



- **Module-LWE problem:**

- **Trade off** between ring and standard LWE
- **Fast** operations
- **Strong** security

$$\begin{pmatrix} a_{1,1} \in \mathbb{R}'_q & \cdots & a_{1,k} \in \mathbb{R}'_q \\ \vdots & \ddots & \vdots \\ a_{k,1} \in \mathbb{R}'_q & \cdots & a_{k,k} \in \mathbb{R}'_q \end{pmatrix} \cdot \begin{pmatrix} s_1 \in \mathbb{R}'_q \\ \vdots \\ s_k \in \mathbb{R}'_q \end{pmatrix} + \begin{pmatrix} e_1 \in \mathbb{R}'_q \\ \vdots \\ e_k \in \mathbb{R}'_q \end{pmatrix}$$

- Modulus  $q$  of underlying ring  $\mathbb{R}'_q = \mathbb{Z}_q[X]/(X^n + 1)$  is **prime** and  $n = 256$
- Polynomial multiplication
  - One of most costly operation
  - Used **NTT**

<sup>1</sup> Joppe W. Bos, Léo Ducas, Eike Kiltz, Tancrede Lepoint, Vadim Lyubashevsky, John M. Schanck, Peter Schwabe, Gregor Seiler, Damien Stehlé: CRYSTALS - Kyber: A CCA-Secure Module-Lattice-Based KEM. EuroS&P 2018: 353-367

# Saber<sup>1</sup> (2018)

Learning with rounding (LWR) problem:

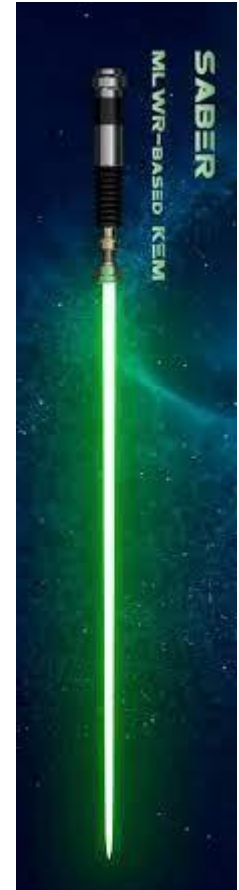
Given samples  $(\mathbf{a}, b = \lfloor \mathbf{a} \cdot \mathbf{s} \rfloor_p)$  for  $\mathbf{a}, \mathbf{s} \in \mathbb{Z}_q^n$ , and rounding modulus  $p$ , find secret  $\mathbf{s}$

Errors are generated *inherently* → less randomness

At least as hard as LWE (Banerjee et al.<sup>2</sup>, Rosen et al.<sup>3</sup>)

Similar to LWE, for LWR can define

- Ring-LWR
- Module-LWR



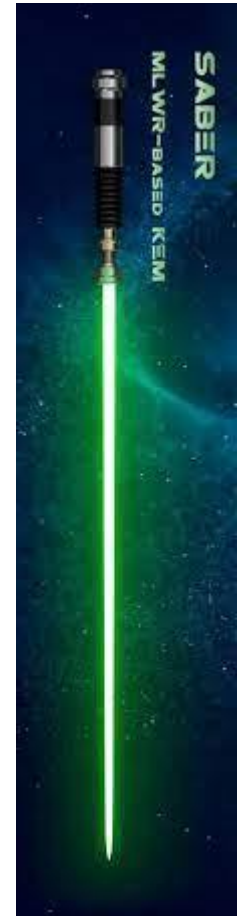
<sup>1</sup>Jan-Pieter D'Anvers, Angshuman Karmakar, Sujoy Sinha Roy, Frederik Vercauteren: Saber: Module-LWR Based Key Exchange, CPA-Secure Encryption and CCA-Secure KEM. AFRICACRYPT 2018: 282-305

<sup>2</sup>Banerjee, A., Peikert, C., Rosen, A.: Pseudorandom functions and lattices. In: EUROCRYPT 2012. pp. 719–737 (2012), [https://doi.org/10.1007/978-3-642-29011-4\\_42](https://doi.org/10.1007/978-3-642-29011-4_42)

<sup>3</sup>Alperin-Sheriff, J., Apon, D.: Dimension-preserving reductions from LWE to LWR. Cryptology ePrint Archive, Report 2016/589 (2016)

# Saber<sup>1</sup> (2018)

- The hard problem is **Module-LWR**
- Underlying ring is  $R_q = \mathbb{Z}_q[X]/(X^n + 1)$ ,  $n=256$
- Note that there is **two** modulus
  - ring modulus **q** and rounding modulus **p**
  - both are **power-of-2**
- Polynomial multiplication
  - As **q** is not **prime**
  - Can't use **NTT**
  - But use hybrid multiplication
    - a combination of Toom-Cook, karatsuba and schoolbook
  - Approximately as **efficient** as **NTT**



# Round 3 finalists of NIST's competition

2020: Third round:

	Digital Signature	KEM/Encryption	Total
Lattice-based	2	3	5
Others	1	1	2

	KEM/Encryption	Schemes
Lattice-based	3	Kyber, NTRU, Saber

# Brief overview of our work

## Motivation:

- After NIST's PQC standardization process many new results published
  - cryptanalysis<sup>5</sup>, mathematical results<sup>3</sup>,
  - implementation design<sup>1,2</sup> and new techniques<sup>4</sup>
- Improving existing schemes incorporating these results
- Creating new schemes focused on
  - Improved efficiency
  - Without degrading security

<sup>1</sup>Jose Maria Bermudo Mera, Furkan Turan, Angshuman Karmakar, Sujoy Sinha Roy, Ingrid Verbauwhede: Compact domain-specific co-processor for accelerating module lattice-based key encapsulation mechanism. IACR Cryptology ePrint Archive 2020: 321 (2020), <https://eprint.iacr.org/2020/321>

<sup>2</sup>Angshuman Karmakar, Jose Maria Bermudo Mera, Sujoy Sinha Roy, Ingrid Verbauwhede: Saber on ARM CCA-secure module lattice-based key encapsulation on ARM. IACR Trans. Cryptogr. Hardw. Embed. Syst. 2018(3): 243-266 (2018), <https://doi.org/10.13154/tches.v2018.i3.243-266>

<sup>3</sup>Le H.Q., Mishra P.K., Duong D.H., Yasuda M. (2018) Solving LWR via BDD Strategy: Modulus Switching Approach. In: Camenisch J., Papadimitratos P. (eds) Cryptology and Network Security. CANS 2018. Lecture Notes in Computer Science, vol 11124. Springer, Cham, [https://doi.org/10.1007/978-3-030-00434-7\\_18](https://doi.org/10.1007/978-3-030-00434-7_18)

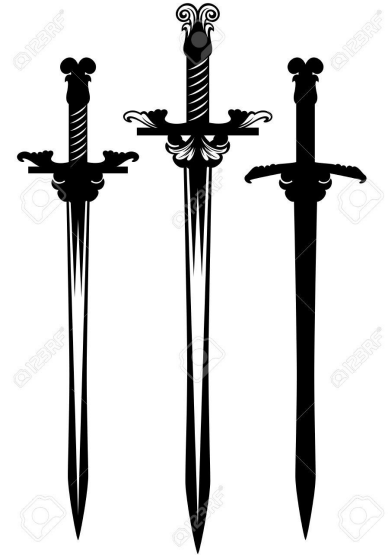
<sup>4</sup><https://estimate-all-the-lwe-ntru-schemes.github.io/docs/>

<sup>5</sup>Martin R. Albrecht and Benjamin R. Curtis and Amit Deo and Alex Davidson and Rachel Player and Eamonn W. Postlethwaite and Fernando Virdia and Thomas Wundere : Estimate all the {LWE, NTRU} schemes! , Cryptology ePrint Archive, Report 2018/331 (2018), <https://eprint.iacr.org/2018/331/>

# Brief overview of our work

## Scabbard<sup>1</sup>:

- A suite of efficient **LWR** based **KEM**
- Both modulus **q** and **p** here **power-of-2**
- Contains **3** different schemes
  - Florete
  - Sable
  - Espada

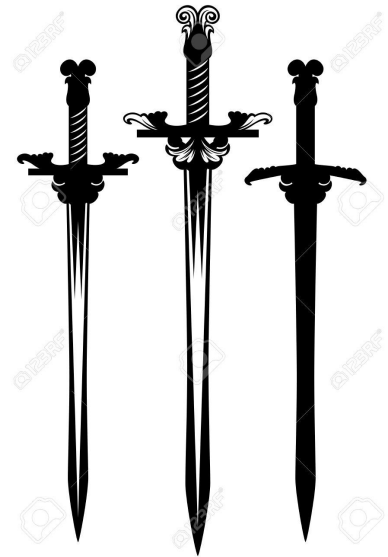


<sup>1</sup>Jose Maria Bermudo Mera, Angshuman Karmakar, Suparna Kundu, Ingrid Verbauwhede: Scabbard: a suite of efficient learning with rounding key-encapsulation mechanisms. IACR Cryptol. ePrint Arch. 2021: 954 (2021)

# Brief overview of our work

## Florete<sup>1</sup>:

- Underlying ring is  $\mathbb{R}_q^n = \mathbb{Z}_q[X]/(X^{768} - x^{384} + 1)$
- Hardness depends on **Ring-LWR**
  - Secure assuming **Ring-LWE** is hard
  - **No error sampling** is needed
- **Polynomial multiplication**:
  - hybrid multiplication
  - But more efficient than Saber
  - due to the hard problem choice
- Another costly operation: Pseudo random number sampling
  - Need less due to smaller parameter size than Saber
- One of the **fastest** KEM



<sup>1</sup>Jose Maria Bermudo Mera, Angshuman Karmakar, Suparna Kundu, Ingrid Verbauwhede: Scabbard: a suite of efficient learning with rounding key-encapsulation mechanisms. IACR Cryptol. ePrint Arch. 2021: 954 (2021)

# Brief overview of our work

## Sable, alternate Saber<sup>1</sup>:

- Underlying ring  $R_q = \mathbb{Z}_q[X]/(X^n + 1)$ ,  $n=256$
- Hardness depends on **Module-LWR**
- Better Performance and less memory needed than **Saber**
  - 2 ring moduli with few other parameters are smaller than Saber
  - Less pseudo random number generation
- It was possible due to fine-grain security analysis and few recent works



<sup>1</sup>Jose Maria Bermudo Mera, Angshuman Karmakar, Suparna Kundu, Ingrid Verbauwhede: Scabbard: a suite of efficient learning with rounding key-encapsulation mechanisms. IACR Cryptol. ePrint Arch. 2021: 954 (2021)



# Brief overview of our work

## Espada<sup>1</sup>:

- Small base ring  $R_q = \mathbb{Z}_q[X]/(X^n + 1)$ ,  $n=64$ 
  - Helps to reduce memory footprint
  - Suitable for low memory devices
- Hardness depends on **Module-LWR**
- One of the **lowest** stack memory used KEM
- Downside:
  - Need a lot pseudo random number generation
  - Which affect the performance
- Solution: **Faster pseudo random number generator helps**

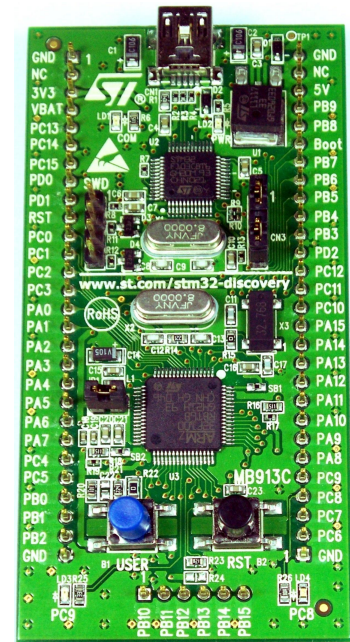


<sup>1</sup>Jose Maria Bermudo Mera, Angshuman Karmakar, Suparna Kundu, Ingrid Verbauwhede: Scabbard: a suite of efficient learning with rounding key-encapsulation mechanisms. IACR Cryptol. ePrint Arch. 2021: 954 (2021)

# Brief overview of our work

## Cortex-M4:

- Huge applications of Internet of Things (IoT)
- ARM Cortex-M4 is a small resource constrained device
- It is used in many IoT devices
- Preferred choices to demonstrate the usefulness of the schemes for IoT applications
- [KRSS]<sup>1</sup> invests effort to construct a framework for **PQC KEM**



<sup>1</sup> Matthias J. Kannwischer, Joost Rijneveld, Peter Schwabe, and Ko Stoffelen. PQM4: Post-quantum crypto library for the ARM Cortex-M4.

# Brief overview of our work

## Results: Performance (X1000 clock cycles)

- Cortex-M4: STM32F4DISCOVERY board running at 24 MHz
- By using the framework provided by [KRSS]<sup>1</sup>

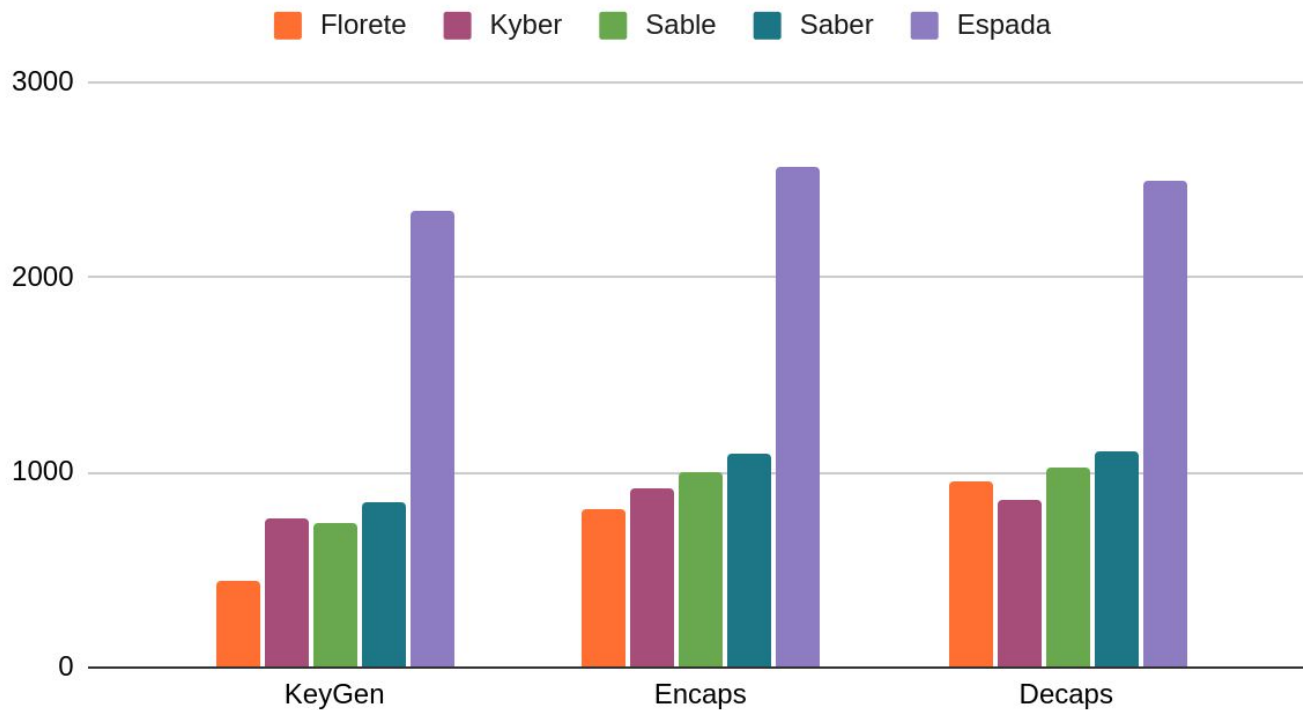
Scheme	KeyGen	Encaps	Decaps
Florete	439 (48%)	814 (25%)	953 (14%)
Sable	745 (11%)	1004 (8%)	1028 (7%)
Espada	2343 (-63%)	2568 (-57%)	2497 (-55%)
Saber	846	1098	1112
Kyber	763	923	862

<sup>1</sup>Matthias J. Kannwischer, Joost Rijneveld, Peter Schwabe, and Ko Stoffelen. PQM4: Post-quantum crypto library for the ARM Cortex-M4.

# Brief overview of our work

## Results: Performance (X1000 clock cycles)

### KeyGen, Encaps and Decaps



# Brief overview of our work

## Results: Memory in bytes

- Cortex-M4: STM32F4DISCOVERY board running at 24 MHz
- By using the framework provided by [KRSS]<sup>1</sup>

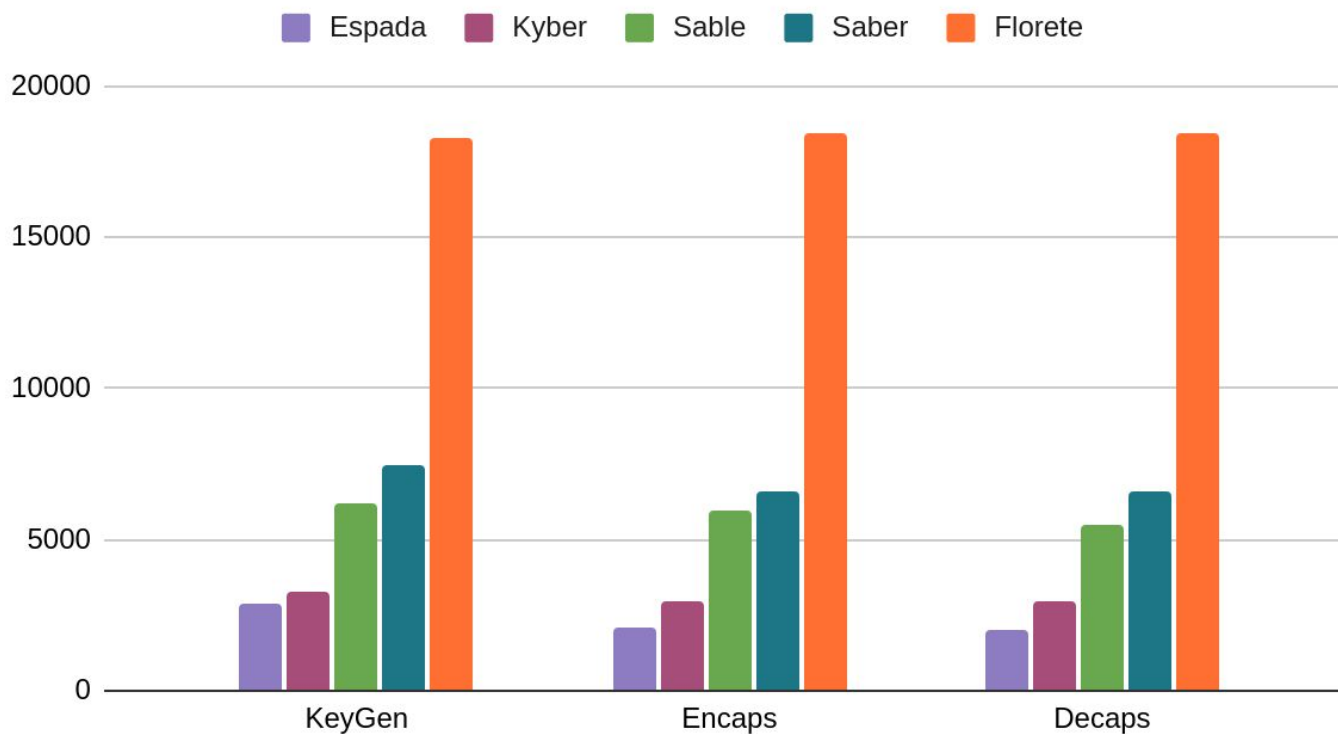
Scheme	KeyGen	Encaps	Decaps
Florete	18252 (2x)	18420 (3x)	18420 (3x)
Sable	6184 (.8x)	5992 (.9x)	5496 (.8x)
Espada	2896 (.4x) (61%)	2120 (.3x) (67%)	2000 (.3x) (69%)
Saber	7488	6560	6568
Kyber	3276	2964	2988

<sup>1</sup>Matthias J. Kannwischer, Joost Rijneveld, Peter Schwabe, and Ko Stoffelen. PQM4: Post-quantum crypto library for the ARM Cortex-M4.

# Brief overview of our work

## Results: Memory in bytes

### KeyGen, Encaps and Decaps



# Future works

- **Most of lattice-based schemes are using variants of LWE**
  - Well studied problems of mathematics
  - Thanks to NIST's competition, few more variants has published
    - e.g: Middle-product LWE
  - With suitable parameters and hard problem and newly published results
  - More efficient schemes may be constructed
- **Polynomial multiplication is a costly operation**
  - Little improvement can make a scheme efficient
- **Side channel attack resistant implementation**
  - Exploit weakness of implementation
  - Masking, hiding technique helps
  - Costs performance degradation
  - New technique needed

*ANY QUESTIONS?*



Thank You!

# Results: C and AVX implementation

Platform: Intel Core i7 with hyper-threading, Turbo-Boost and multi-core support disabled

Compiled with: gcc -o3

Scheme	C ( X1000 clock cycles )			AVX ( X1000 clock cycles )		
	KeyGen	Encaps	Decaps	KeyGen	Encaps	Decaps
Florete	86 (45%)	147 (26%)	193 (10%)	58 (45%)	87 (26%)	97 (13%)
Sable	152 (4%)	186 (7%)	207 (3%)	80 (25%)	95 (19%)	89 (20%)
Espada	334(-52%)	354 (-43%)	350 (-38%)	258 (-58%)	273 (-56%)	267 (-58%)
Saber	159	201	215	107	118	112
Kyber	252	324	365	537	594	557

# Results: Cortex-M4 implementation

Cortex-M4: STM32F4DISCOVERY board running at 24 MHz

By using the framework provided by [KRSS]<sup>1</sup>

Scheme	Performance (X1000 clock cycles)			Memory (X1000 clock cycles)		
	KeyGen	Encaps	Decaps	KeyGen	Encaps	Decaps
Florete	439 (48%)	814 (25%)	953 (14%)	18252 (2x)	18420 (3x)	18420 (3x)
Sable	745 (11%)	1004 (8%)	1028 (7%)	6184 (.8x)	5992 (.9x)	5496 (.8x)
Espada	2343 (-63%)	2568 (-57%)	2497 (-55%)	2896 (.4x) (61%)	2120 (.3x) (67%)	2000 (.3x) (69%)
Saber	846	1098	1112	7488	6560	6568
Kyber	763	923	862	3276	2964	2988

<sup>1</sup>Matthias J. Kannwischer, Joost Rijneveld, Peter Schwabe, and Ko Stoffelen. PQM4: Post-quantum crypto library for the ARM Cortex-M4.

# Accepted in TCHES 2021 issue 4

Jose Maria Bermudo Mera, Angshuman Karmakar, Suparna Kundu, Ingrid Verbauwhede  
“Scabbard: a suite of efficient learning with rounding key-encapsulation mechanisms”

# Module-Learning with errors (LWE)

$$\mathbf{A} \cdot \mathbf{s} + \mathbf{e} = \mathbf{b}$$

$$R_q = \mathbb{Z}_q[x] / f(x)$$

$$\begin{pmatrix} a_{1,1}(x) & \cdots & a_{l,1}(x) \\ \vdots & \ddots & \vdots \\ a_{l,1}(x) & \cdots & a_{l,l}(x) \end{pmatrix} \cdot \begin{pmatrix} s_1(x) \\ \vdots \\ s_l(x) \end{pmatrix} + \begin{pmatrix} e_1(x) \\ \vdots \\ e_l(x) \end{pmatrix} = \begin{pmatrix} b_1(x) \\ \vdots \\ b_l(x) \end{pmatrix}$$

↓  
polynomial of  
degree  $n$

$$a_{i,j}(x) \leftarrow \mathcal{U}(R_q) \quad s_i(x), e_i(x) \leftarrow \chi(R_q) \text{ with s.d. } \sigma$$

Hard Problem (Decision): Given  $(\mathbf{A}, \mathbf{b})$

$\mathbf{b}$ : sampled uniformly /  $\mathbf{b}$ : LWE sample

LWR problem:  $\mathbf{b} = \lfloor \mathbf{A} \cdot \mathbf{s} \rfloor_p$  and  $\mathbf{e}$  is rounding error (deterministic)