CrystalBall: **Design a SAT Solver as you need it**

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Institute for Advancing Intelligence

tcg crest

School of Computing

The SAT problem

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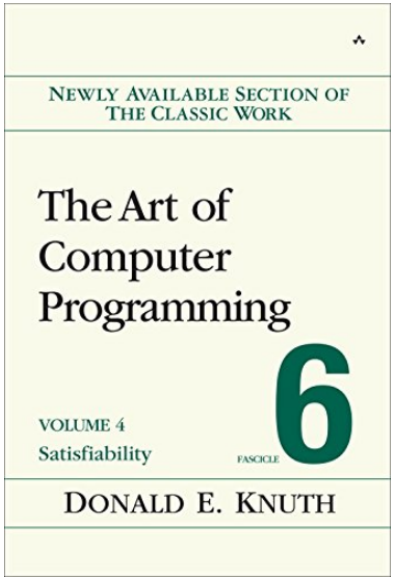
– To show it's difficuilty

21st century : Reduce a problem to SAT

– To solve it in practice



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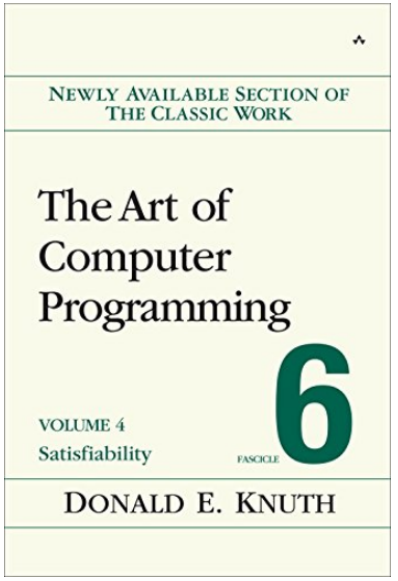
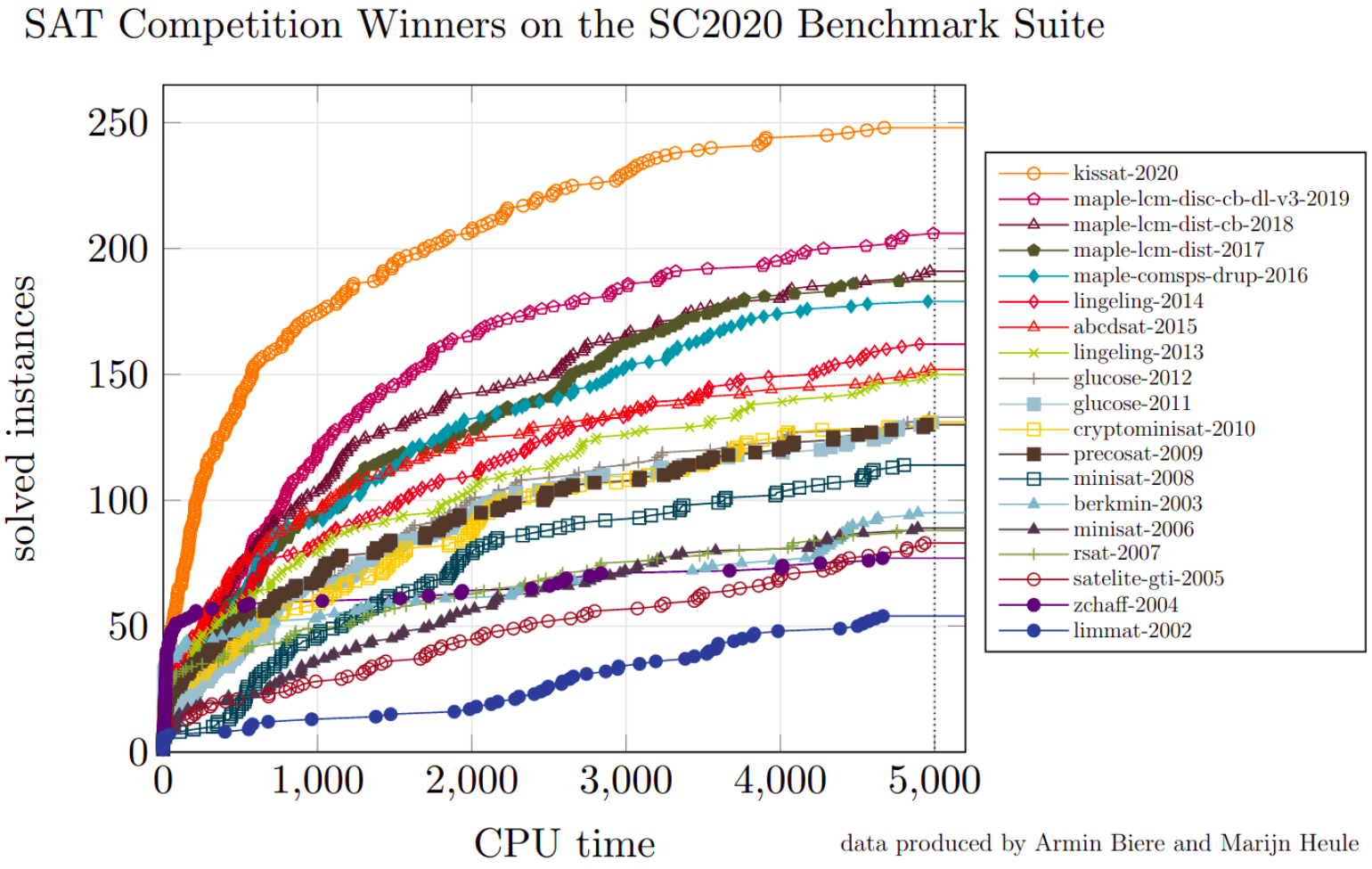


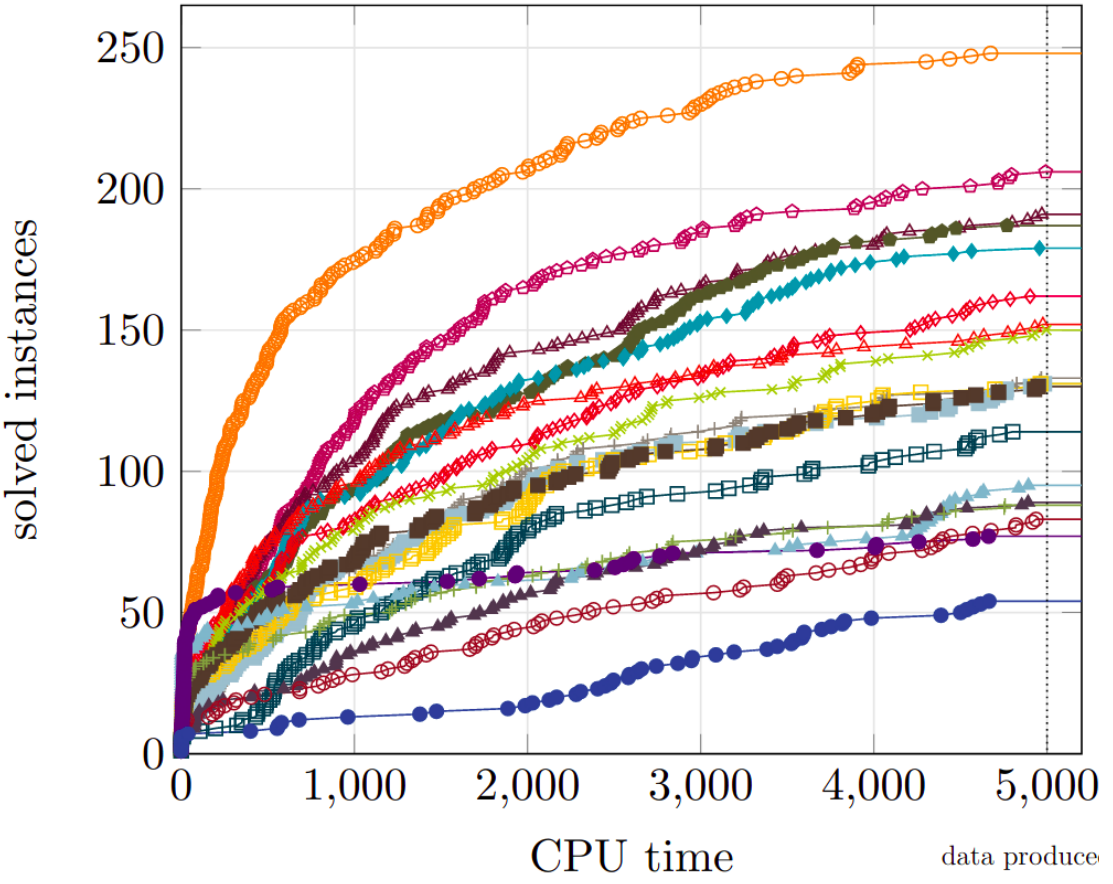
Industrial SAT solvers : Software/Hardware Verification, cryptography, AI planning, genome rearrangement, constrained scheduling...

SAT Revolution



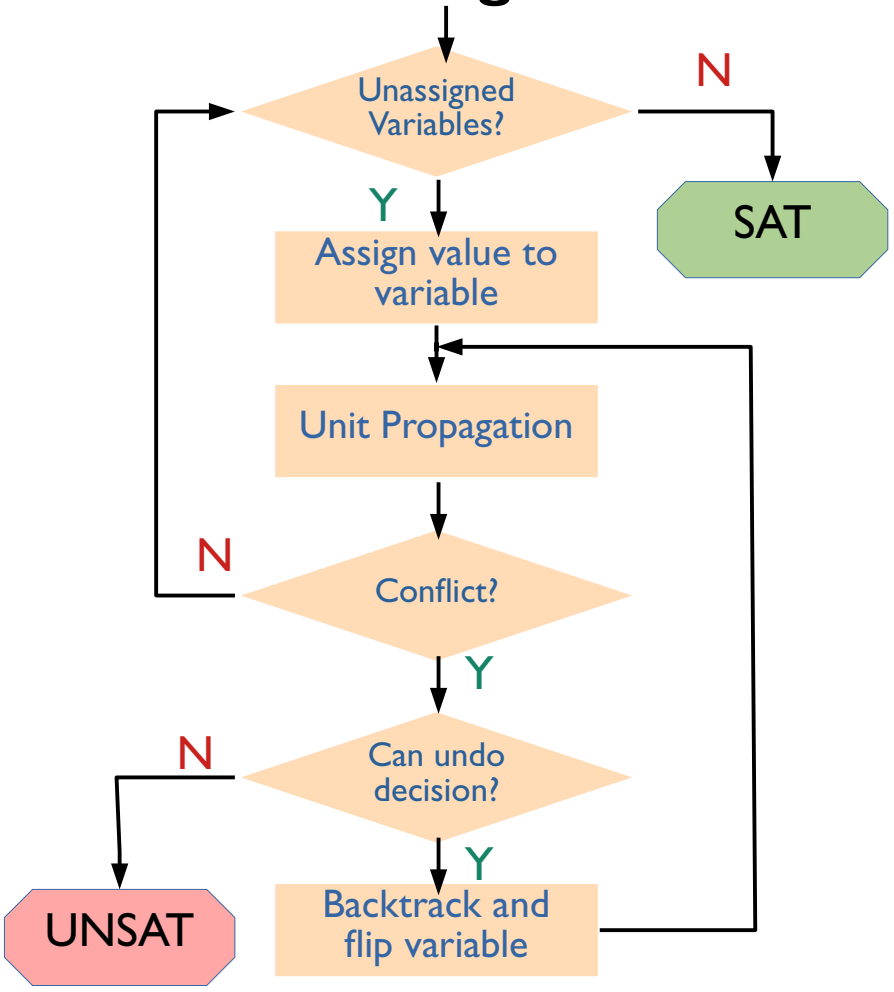
The Price of Success

- SAT is NP-complete yet solvers tend to solve problems involving millions of variables
- The solvers of today are very complex
- We understand very little why SAT solvers work!



The DPLL Algorithm

[DL60, DLL62]



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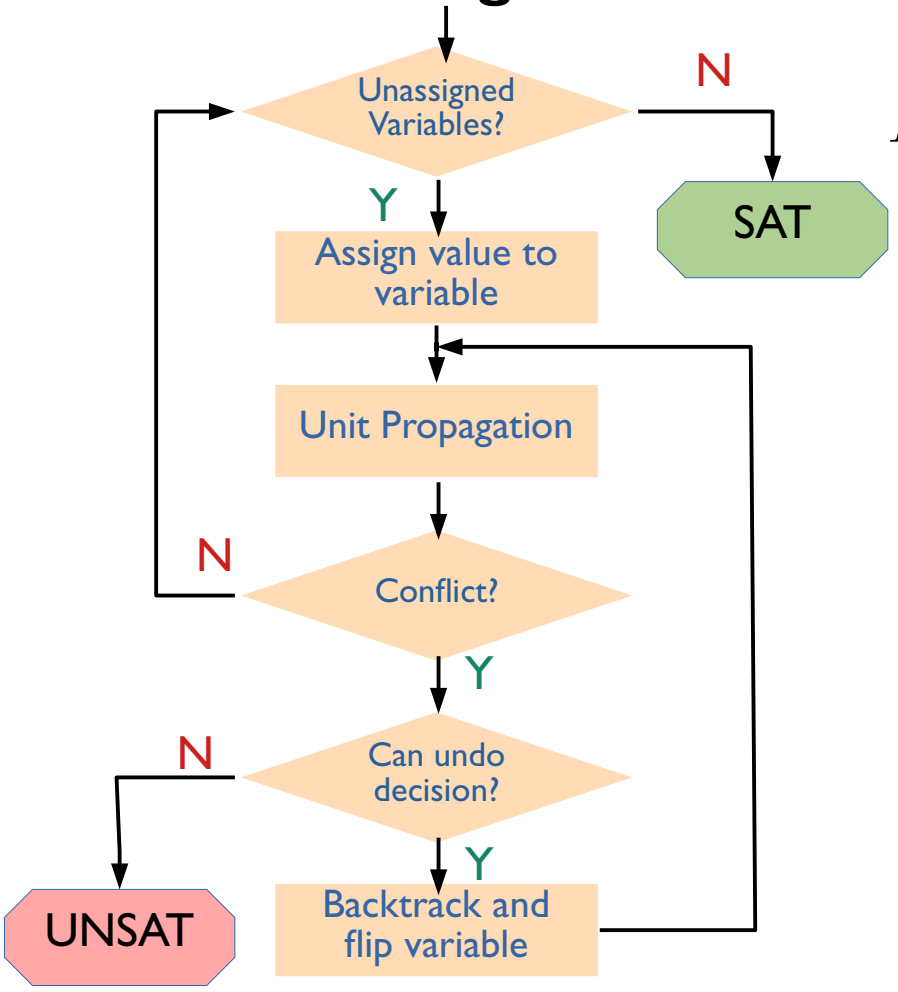

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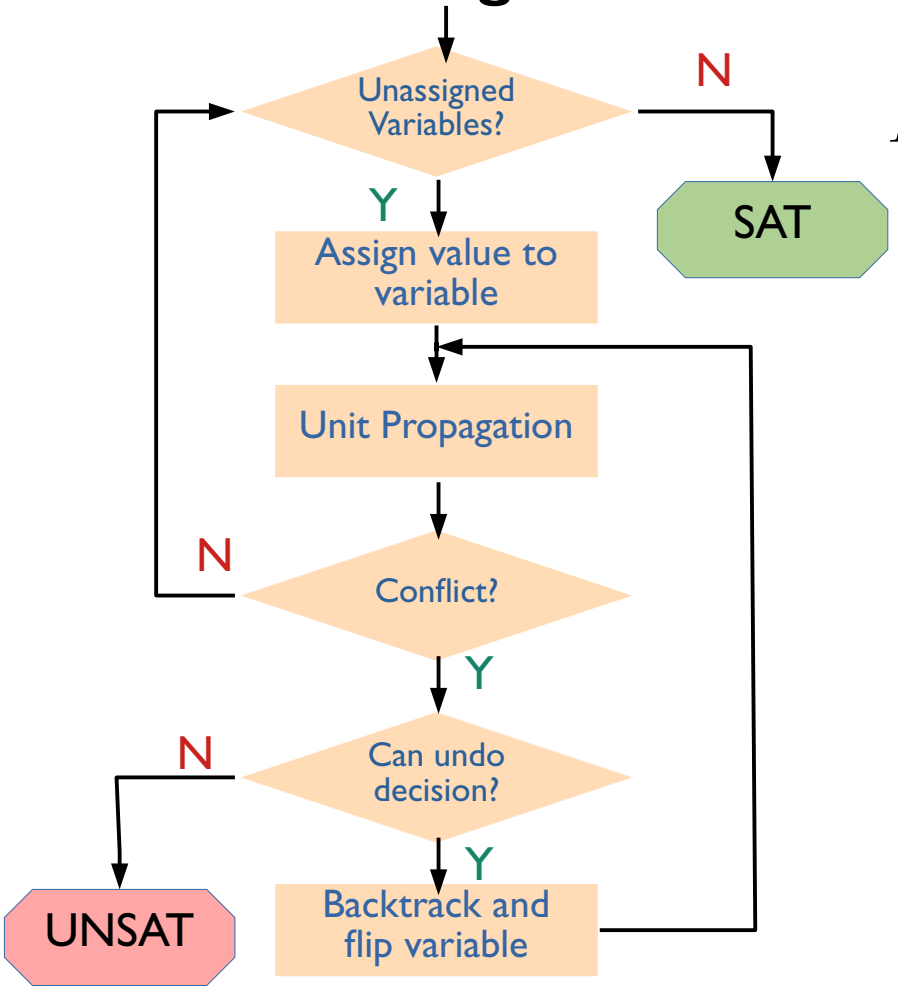

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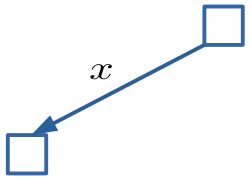

Level	Dec.	Unit Prop.
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1	x	
2	y	
3	a	

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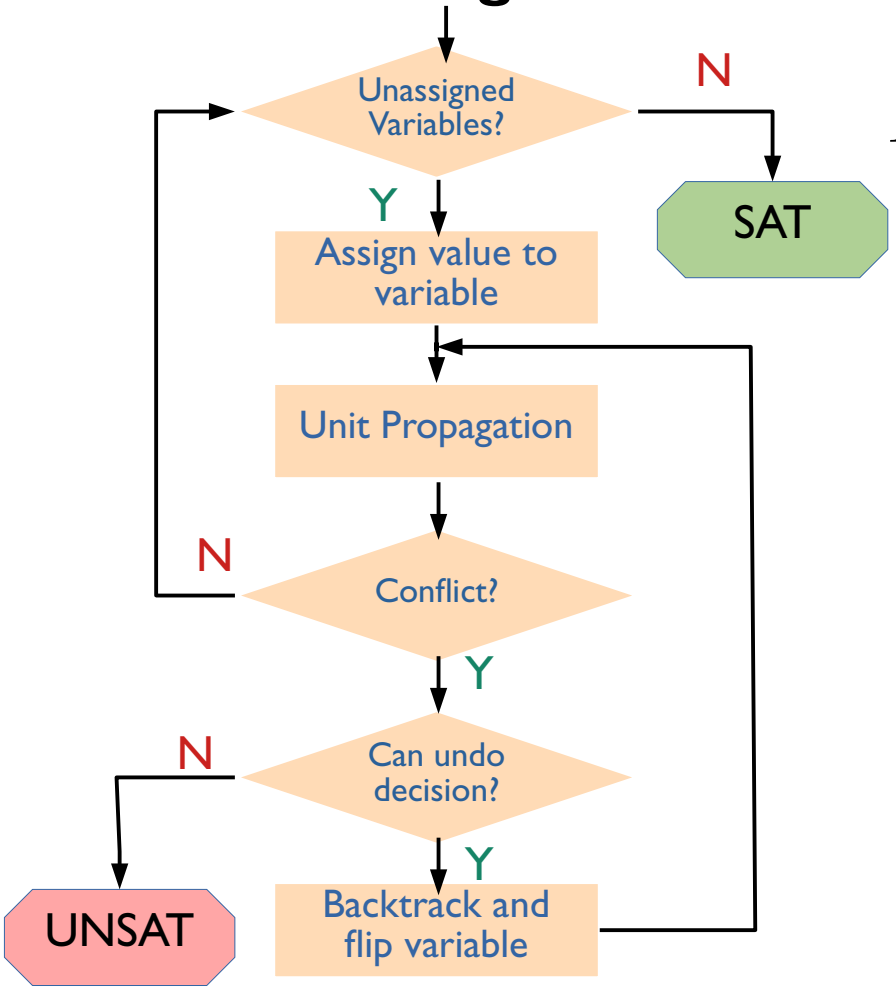

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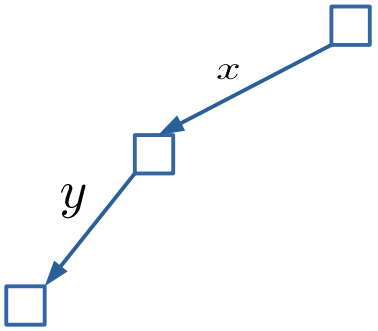

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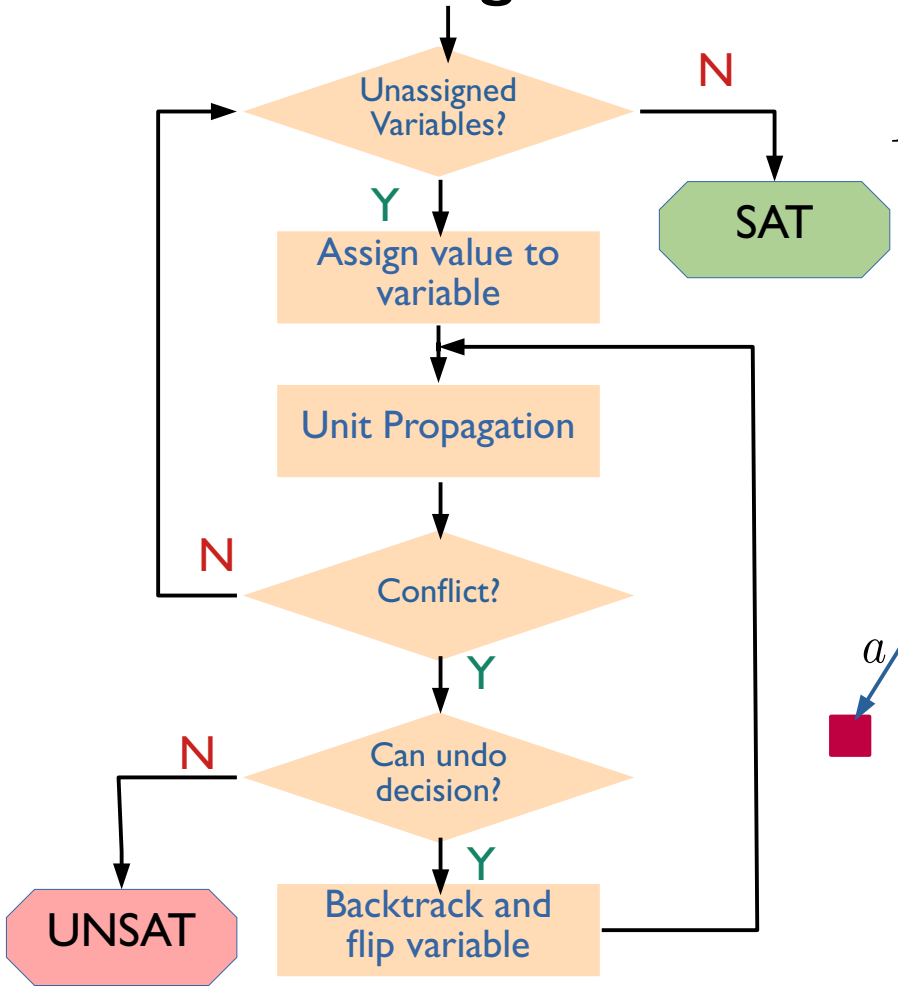

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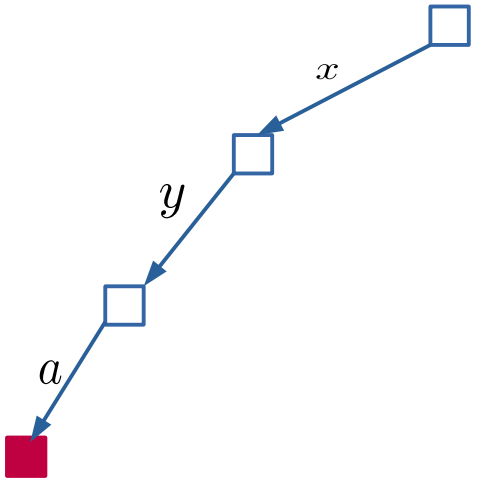

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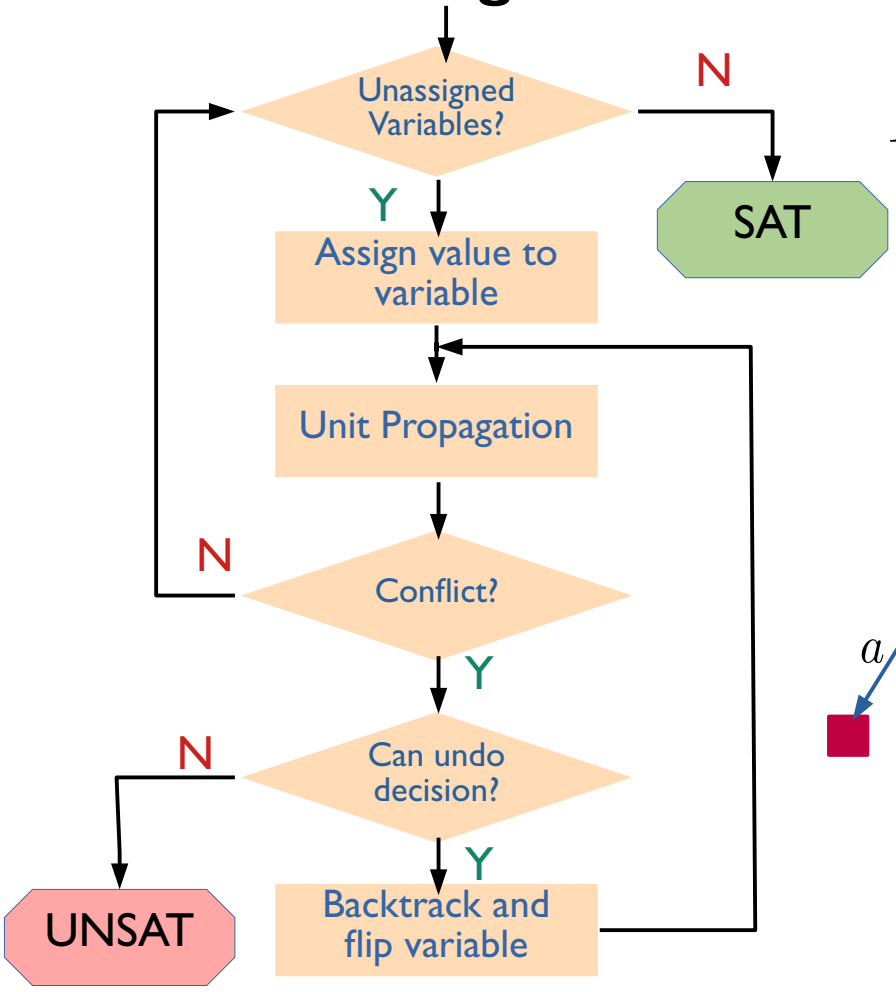

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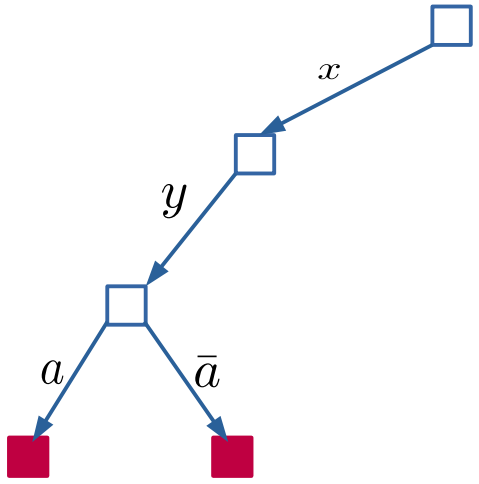

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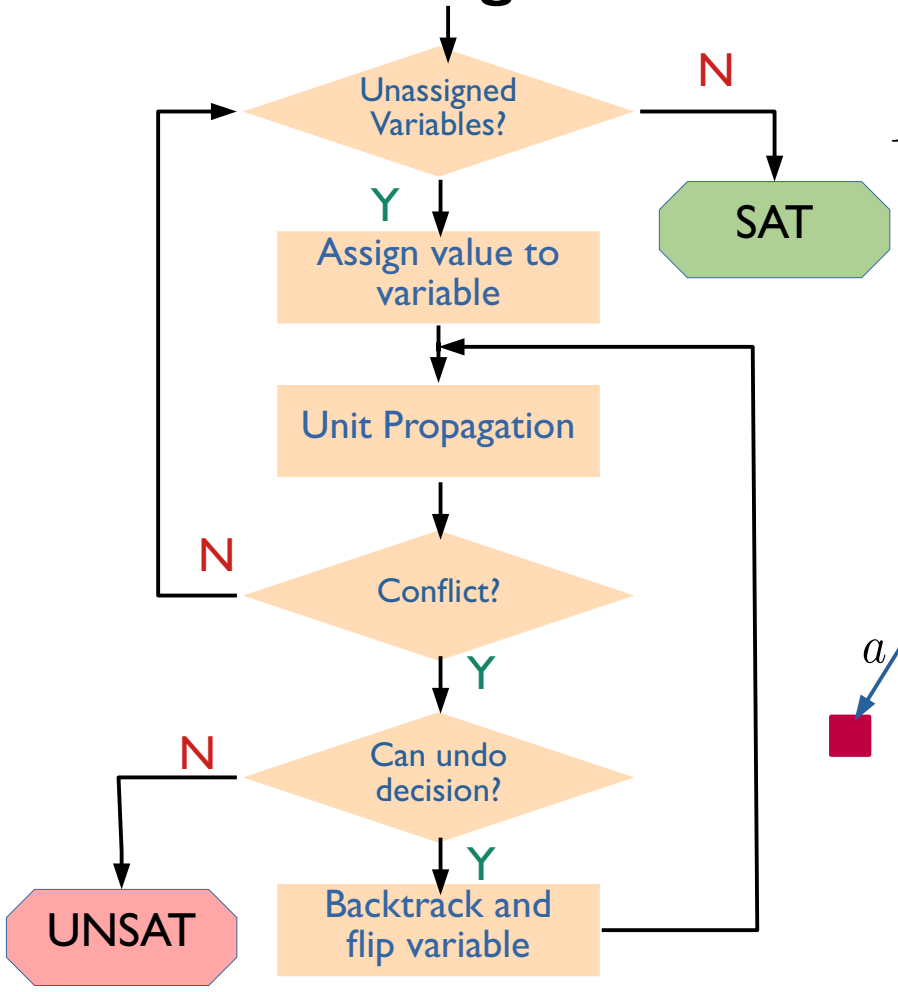

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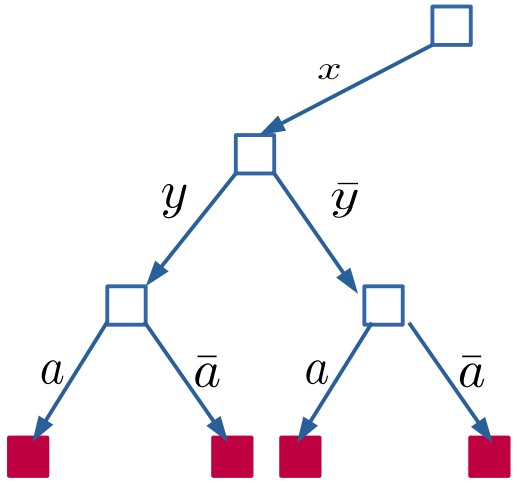

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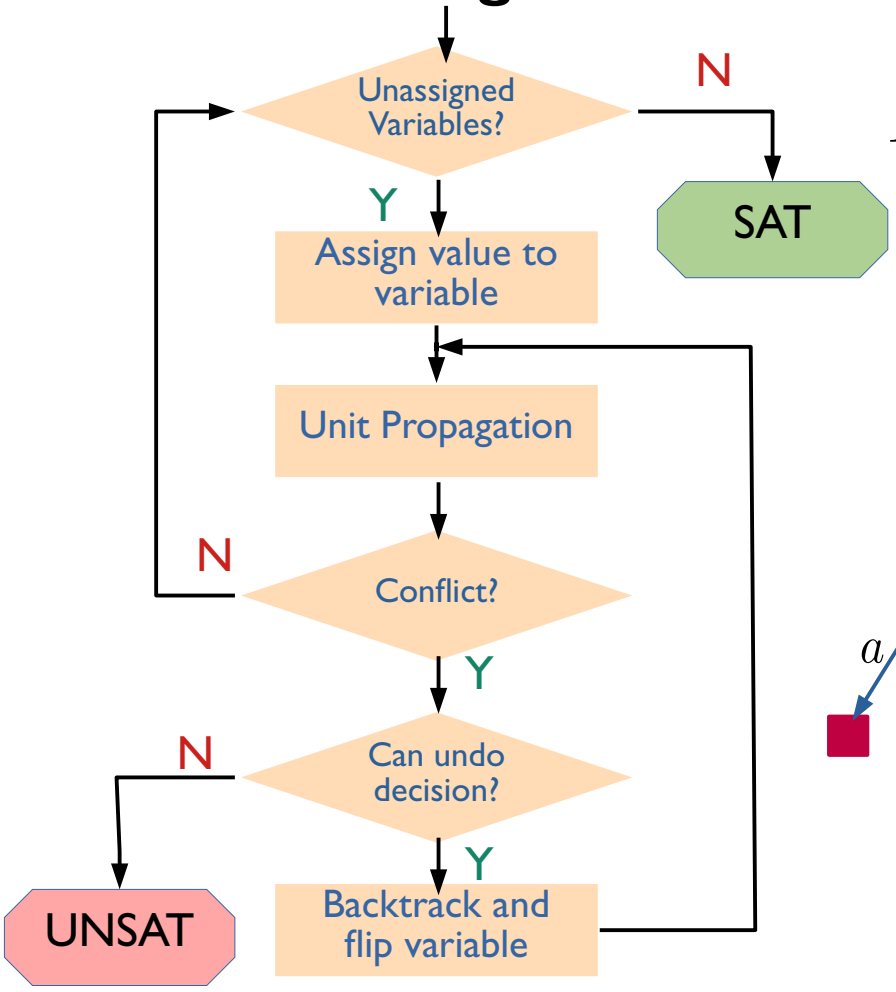

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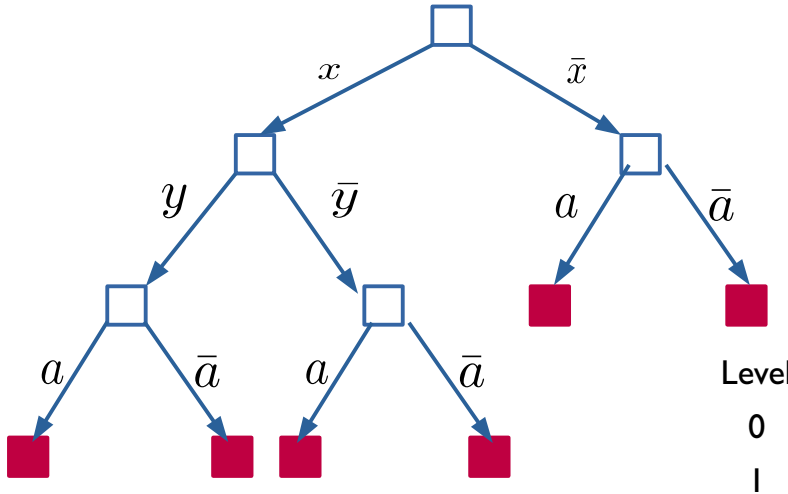

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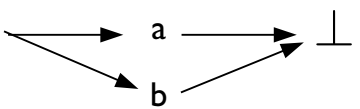

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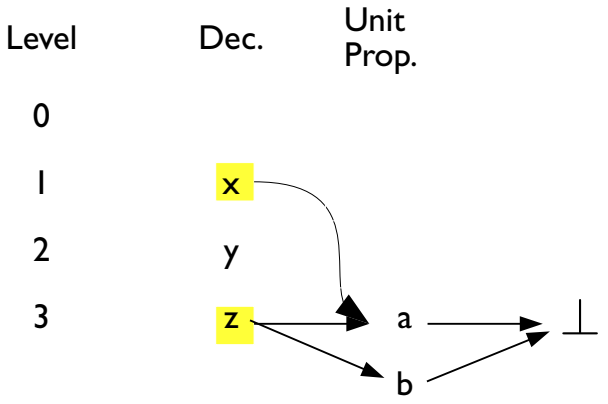

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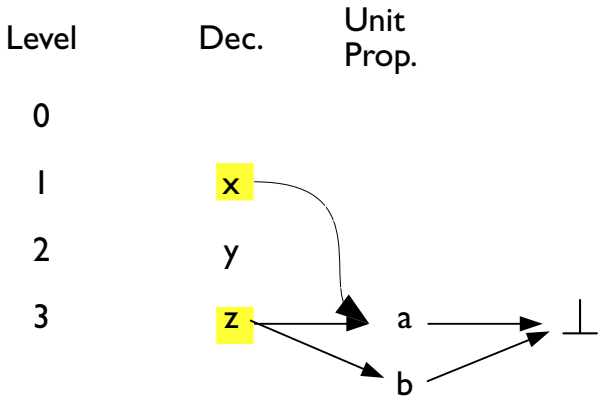

Analyze Conflict

[MSS96]

- Create a new clause $(\bar{x} \vee \bar{z})$

Conflict Driven Clause Learning (CDCL)

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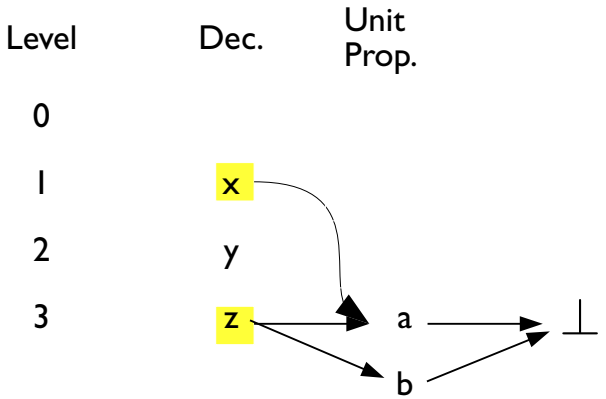

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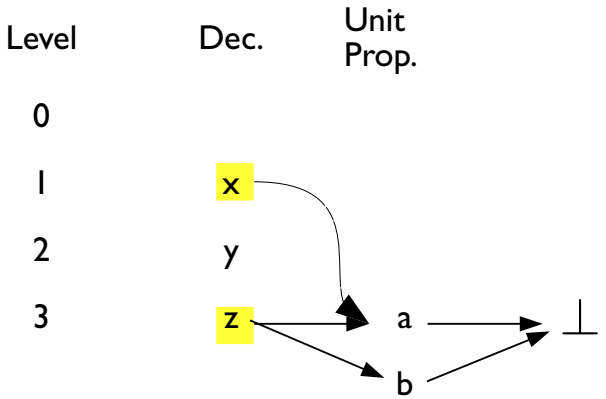

Clause learning can be related with resolution

$$\frac{(\bar{a} \vee \bar{b}) \qquad (\bar{z} \vee b) \qquad (\bar{x} \vee \bar{z} \vee a)}{(\bar{x} \vee \bar{z})}$$

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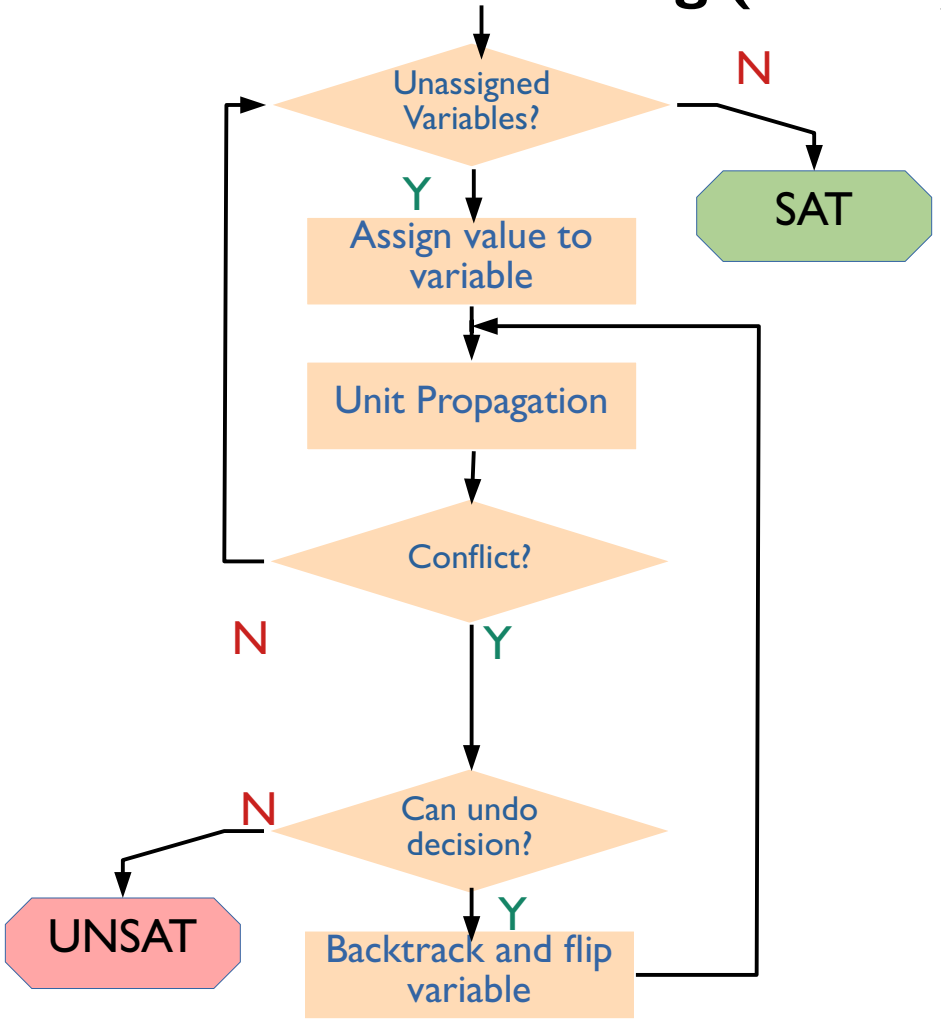

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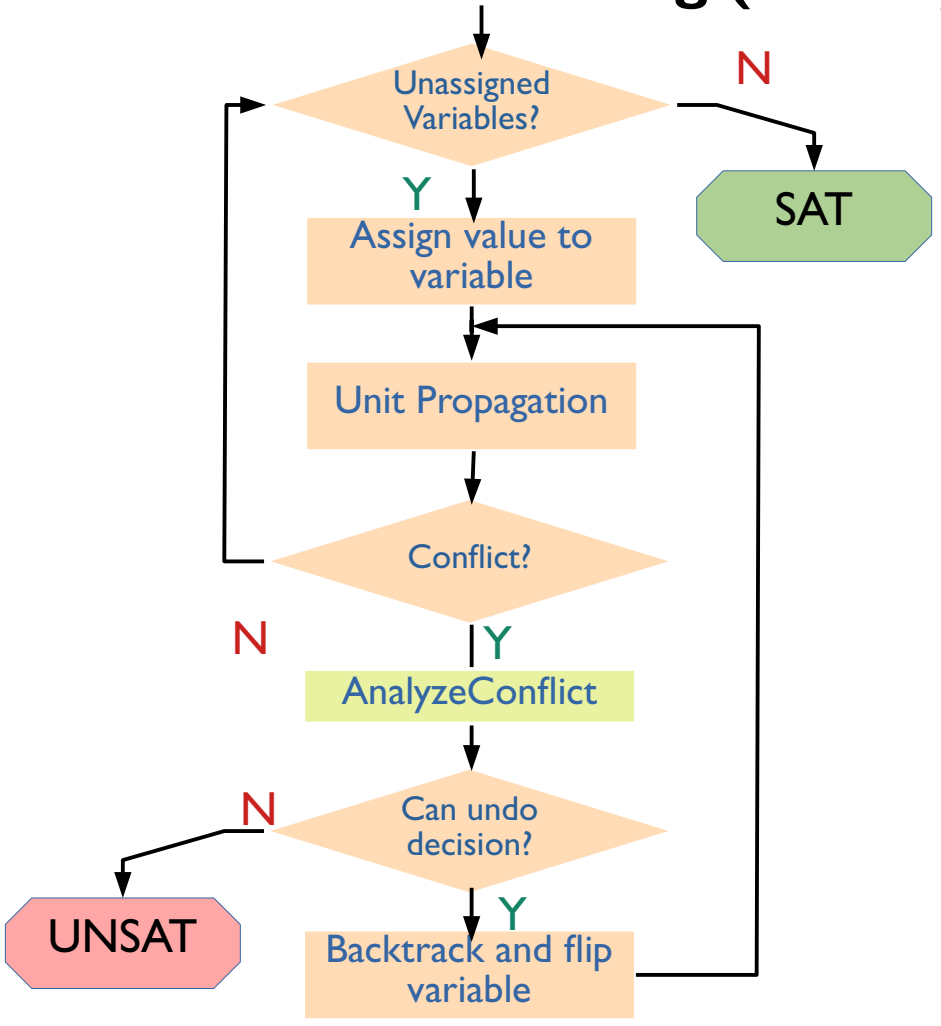

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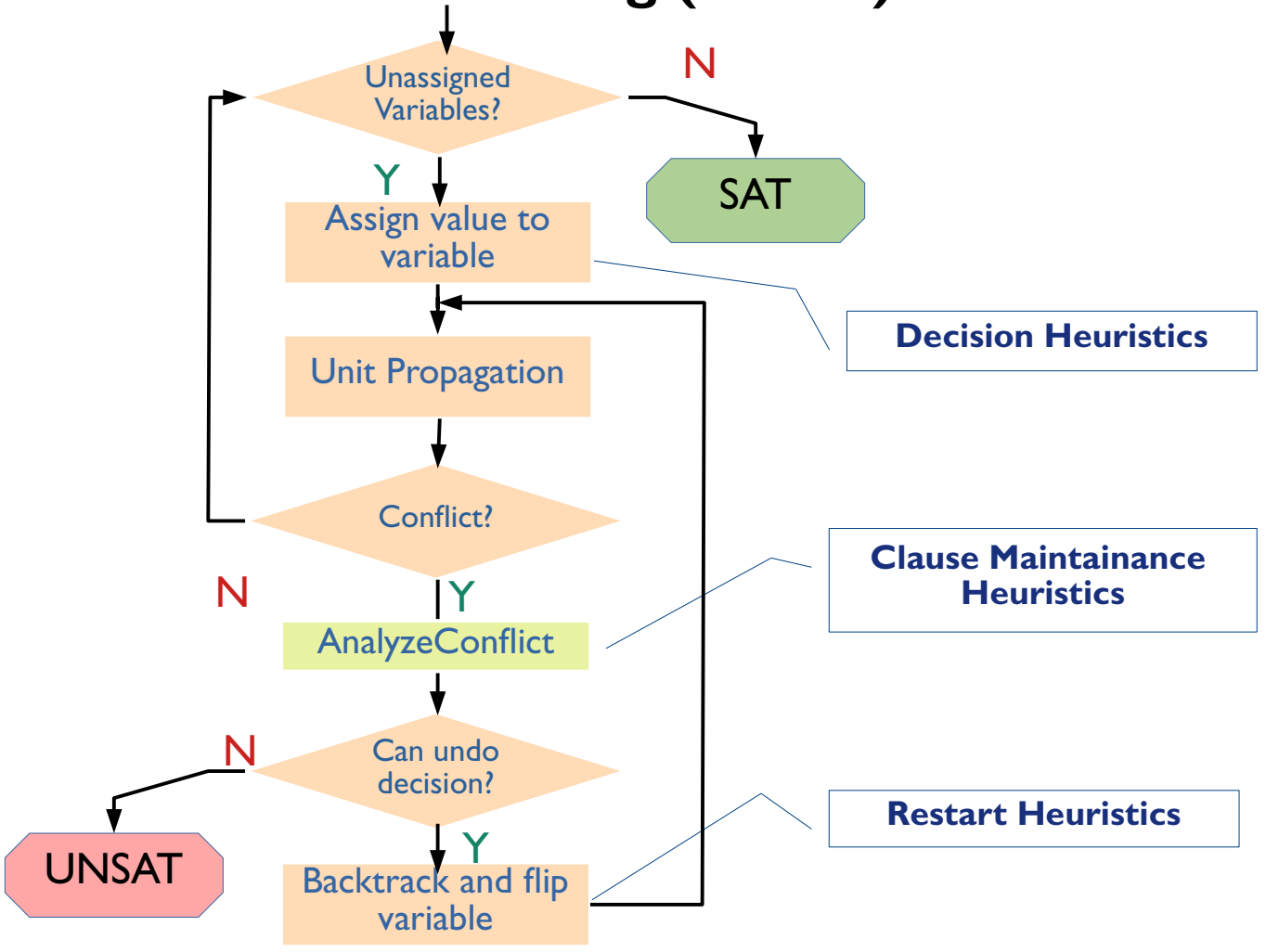

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- **Popular heuristics** include
  - Delete **larger** clauses
  - Delete **less used** clauses
  - Delete clauses based on **Literal block distance**

Data-Driven Design of SAT solver

- View SAT solvers as composition of prediction engines
  - Branching
  - Clause learning
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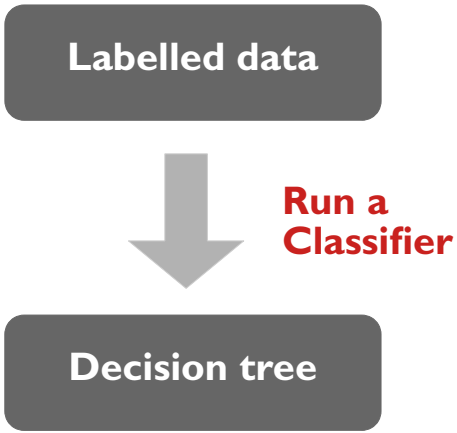

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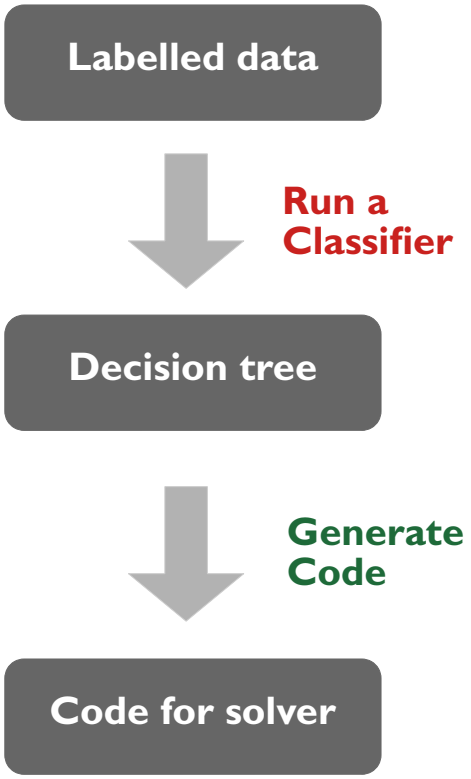

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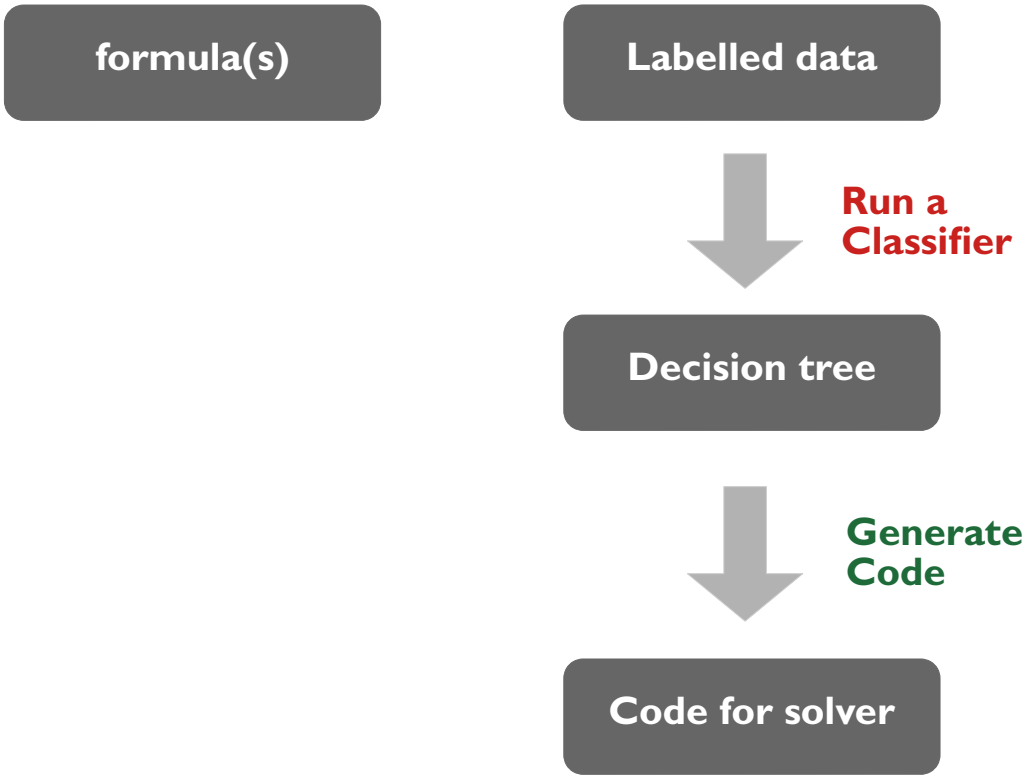

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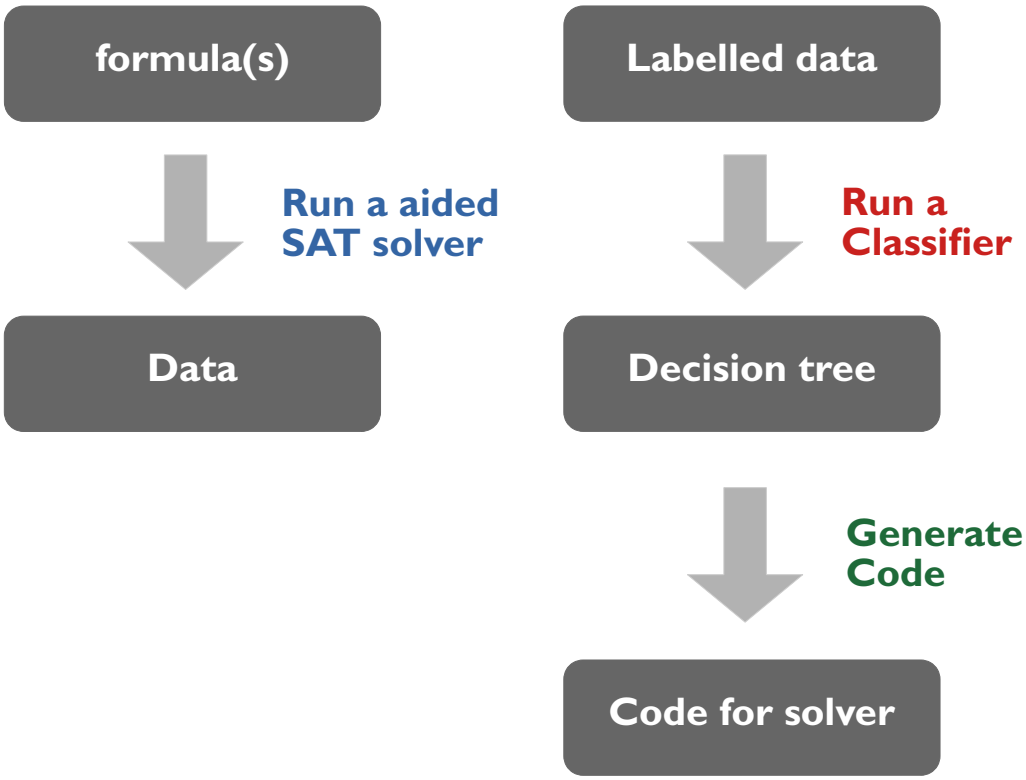

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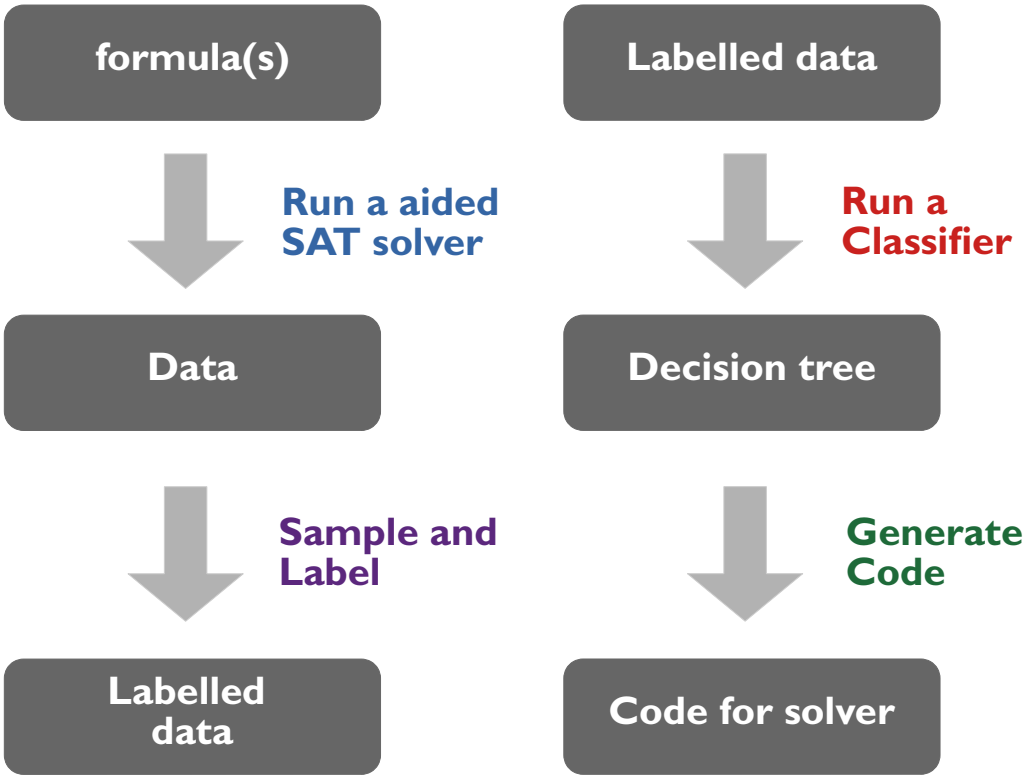

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LBD	size	used_last_10k	activity
10	15	3	top half
7	10	1	bottom half
3	7	0	bottom half

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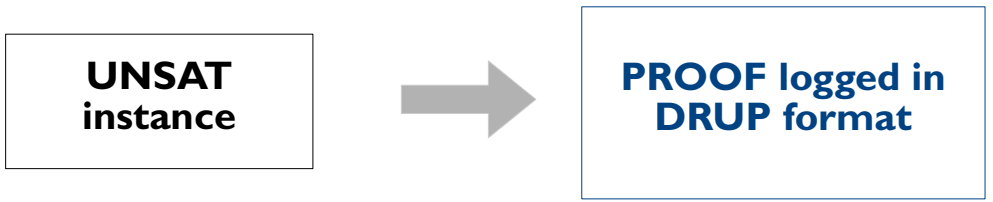

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UNSAT instance

PROOF logged in DRUP format

```
p cnf
-2  3 0
 1  3 0
-1  2 0
-1 -2 0
 1 -2 0
 2 -3 0
 3  6 0
```

Labeling

Task : predict whether the current clause will be useful in future.

UNSAT instance

PROOF logged in DRUP format

```
p cnf
-2  3 0
 1  3 0
-1  2 0
-1 -2 0
 1 -2 0
 2 -3 0
 3  6 0
```

```
1 -2  3 0 0
2  1  3 0 0
3 -1  2 0 0
4 -1 -2 0 0
5  1 -2 0 0
6  2 -3 0 0
7 -2  0 4 5 0
8  3  0 1 2 3 0
9  0  6 7 8 0
```

Labeling

Task : predict whether the current clause will be useful in future.

UNSAT instance

PROOF logged in DRUP format

DRAT - trim

```
p cnf
-2  3 0
 1  3 0
-1  2 0
-1 -2 0
 1 -2 0
 2 -3 0
 3  6 0
```

```
1 -2  3 0 0
2  1  3 0 0
3 -1  2 0 0
4 -1 -2 0 0
5  1 -2 0 0
6  2 -3 0 0
7 -2  0 4 5 0
8  3  0 1 2 3 0
9  0  6 7 8 0
```

Backward pass to construct optimal proof.

Labeling

- Look at DRAT-trim's proof
- Check which learnt clauses are useful
- Label accordingly.

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glue	size	used_last_10k	activity
10	15	3	top half
7	10	1	bottom half
3	7	0	bottom half

Labeling

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glue	size	used_last_10k	activity	label
10	15	3	top half	keep
7	10	1	bottom half	throw
3	7	0	bottom half	throw

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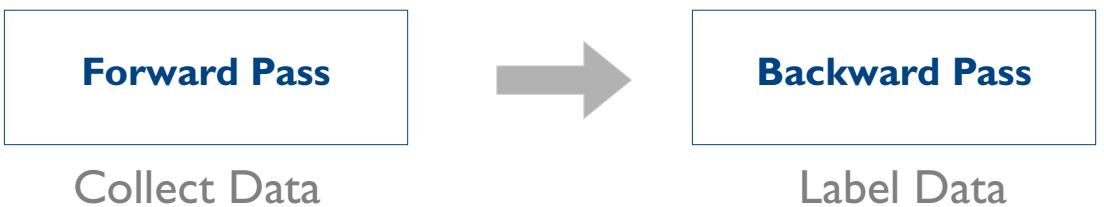
glue	size	used_last_10k	activity	label
10	15	3	top half	keep
7	10	1	bottom half	throw
3	7	0	bottom half	throw



Inference Engine

glue	size	used_last_10k	label
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7	10	1	throw
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Inference Engine

glue	size	used_last_10k	label
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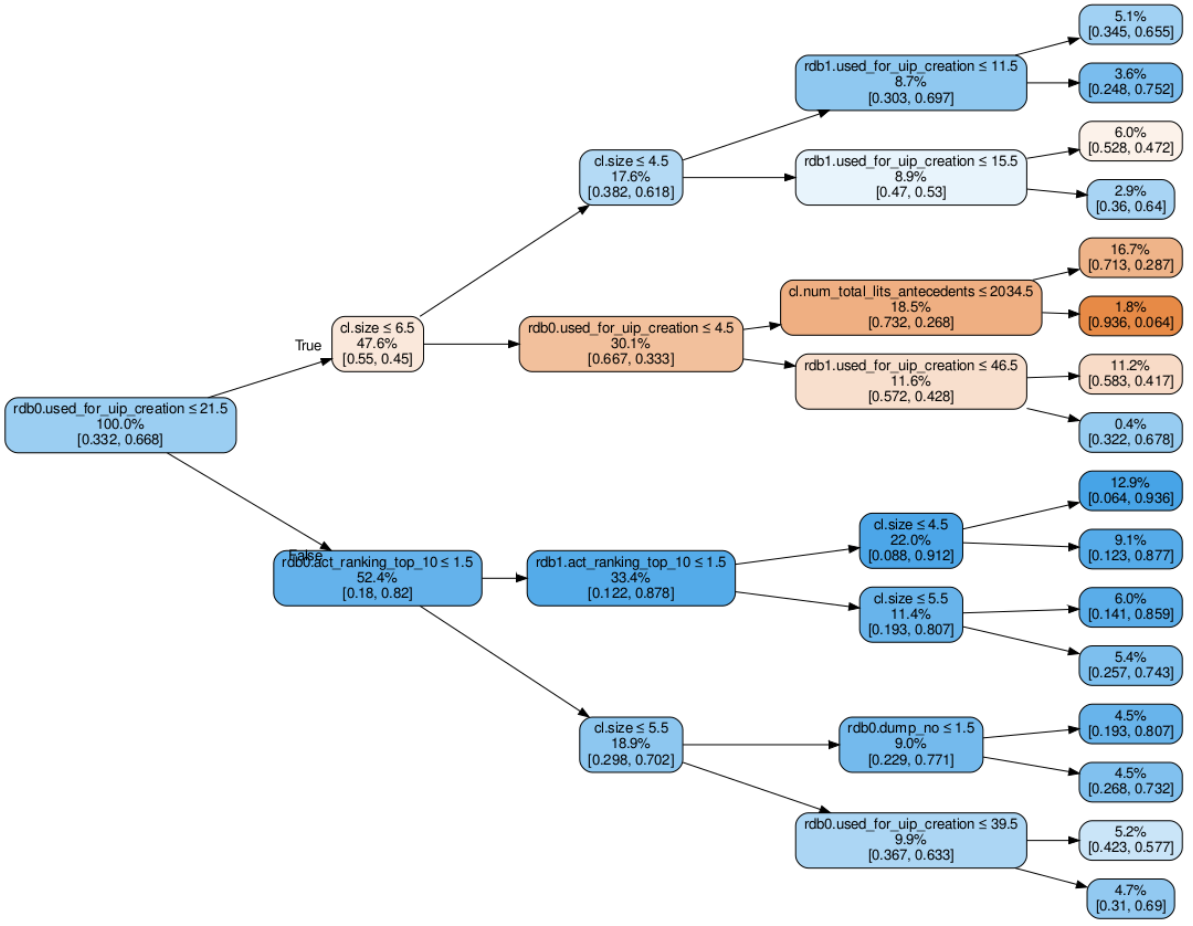

Inference Engine

glue	size	used_last_10k	label
10	15	3	keep
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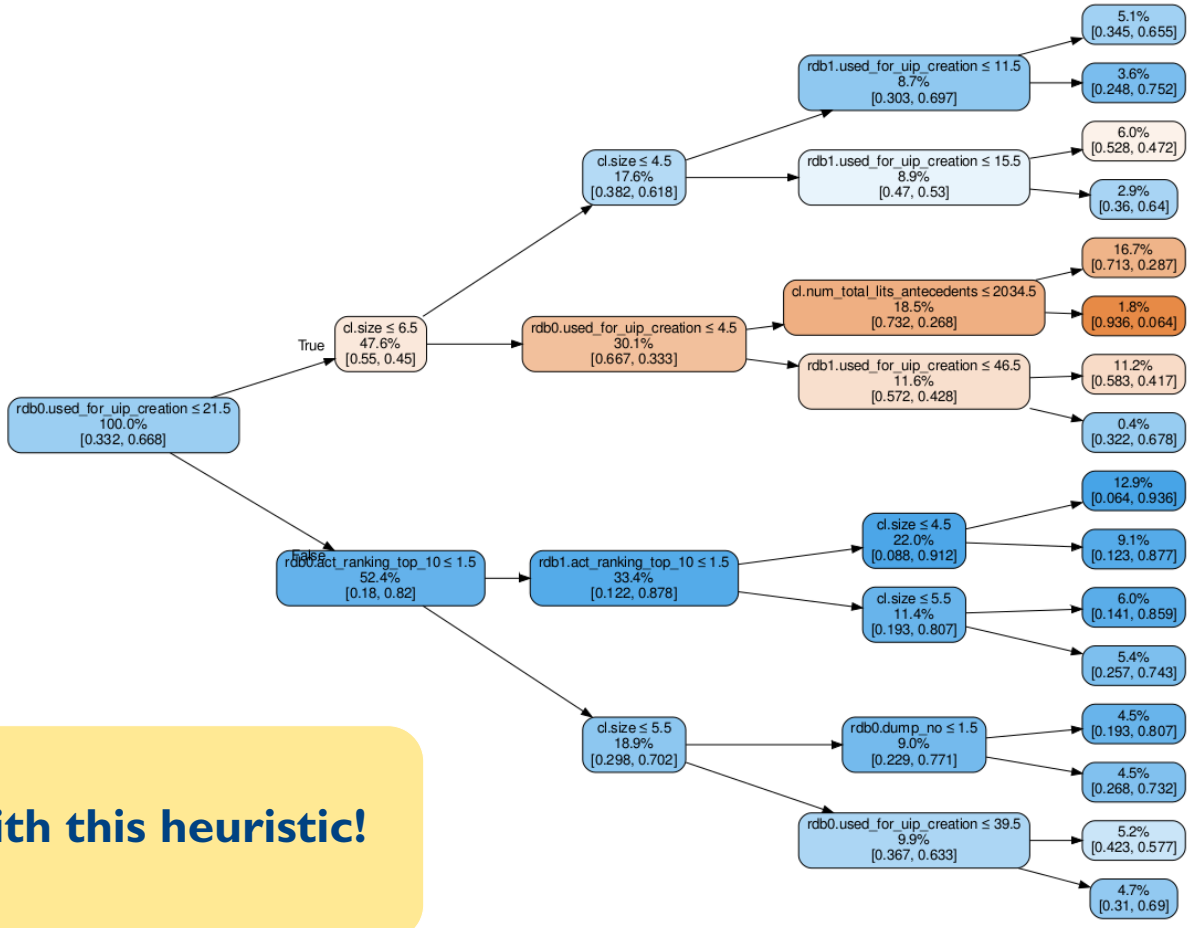

Create new solver with this heuristic!

Experimental Setup

- All the UNSAT instances from SAT 2014-17.
- The number of learnt clauses for different problems varied from few hundreds to millions
- In total, we had 422K data points.
- Tested on SAT competition 2019 benchmarks.

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on 400 SAT '19 instances

solver	# solved	Avg. Runtime*
CryptoMiniSat	291	9939
PredCryptoMiniSat	**299**	**9710**

Domain specific solver

comparing average runtime in seconds

	Benchmark	
Solver trained on	SATCompetition '19	SHA-1
SATCompetition	2440	1263
SHA-1	2805	1165

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	Benchmark
Solver trained on	AES
SATCompetition	1474
AES	1340

	Benchmark
Solver trained on	Grain
SATCompetition	1860
Grain	1973

Domain specific solver

comparing average runtime in seconds

	Benchmark	
Solver trained on	SATCompetition '19	SHA-1
SATCompetition	2440	1263
SHA-1	2805	1165

Rooms for improvement

- Sampling
- Labelling
- Different models to try

...

	Benchmark
Solver trained on	AES
SATCompetition	1474
AES	1340

	Benchmark
Solver trained on	Grain
SATCompetition	1860
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Conclusion

- First time, **white-box access** to SAT solvers.
- Democritizing SAT solver research : **easy to test new features**.
- Extend for **branching and restarts**.

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Code : **meelgroup.github.io/crystalball**

\end{document}

An approach in solving Cryptographic Problems

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Algebraic Cryptanalysis

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Optimize SIMON Family

An approach in solving Cryptographic Problems

Algebraic Cryptanalysis

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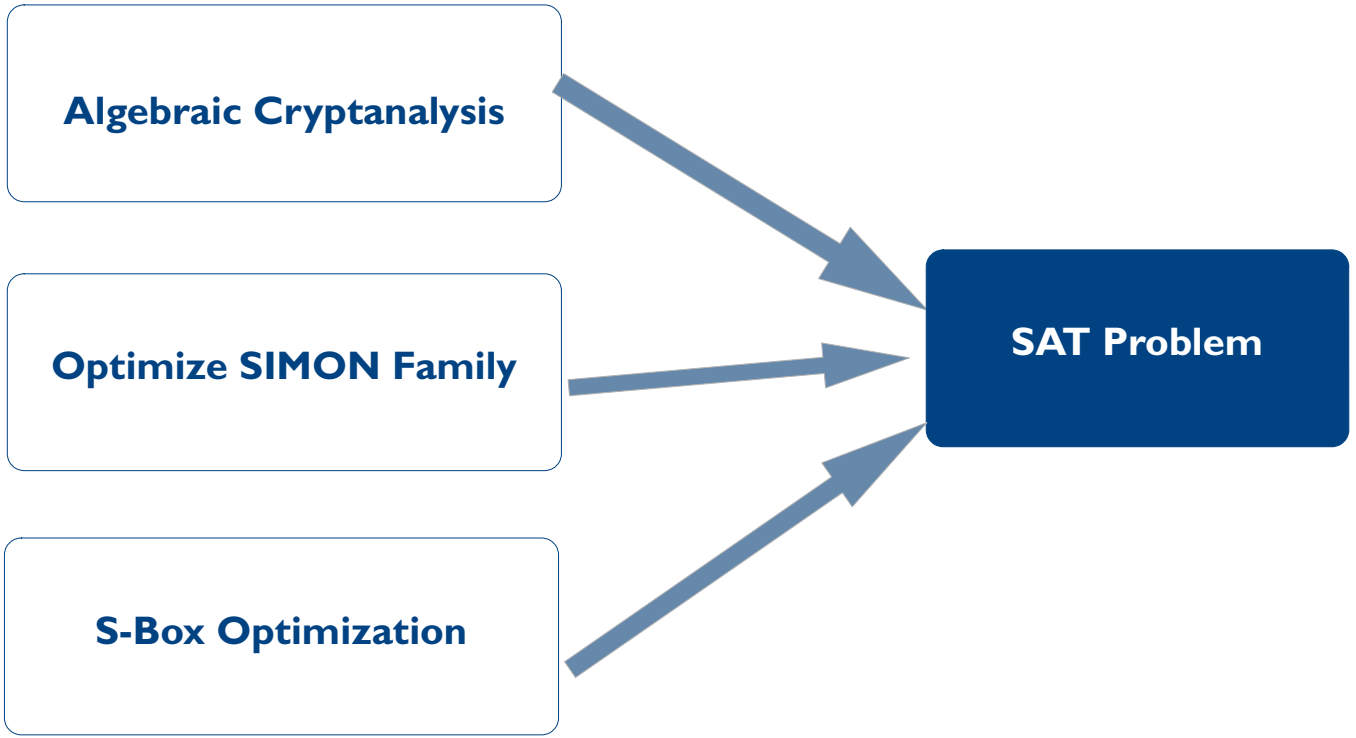
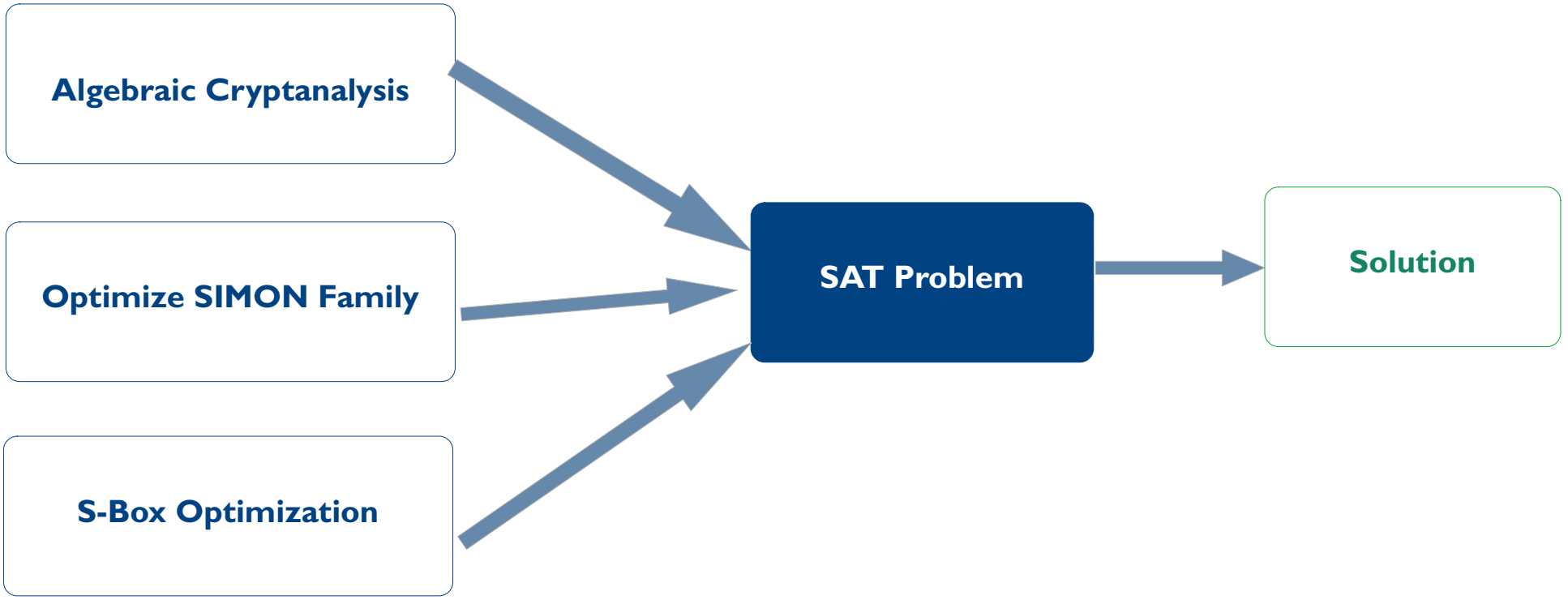
S-Box Optimization

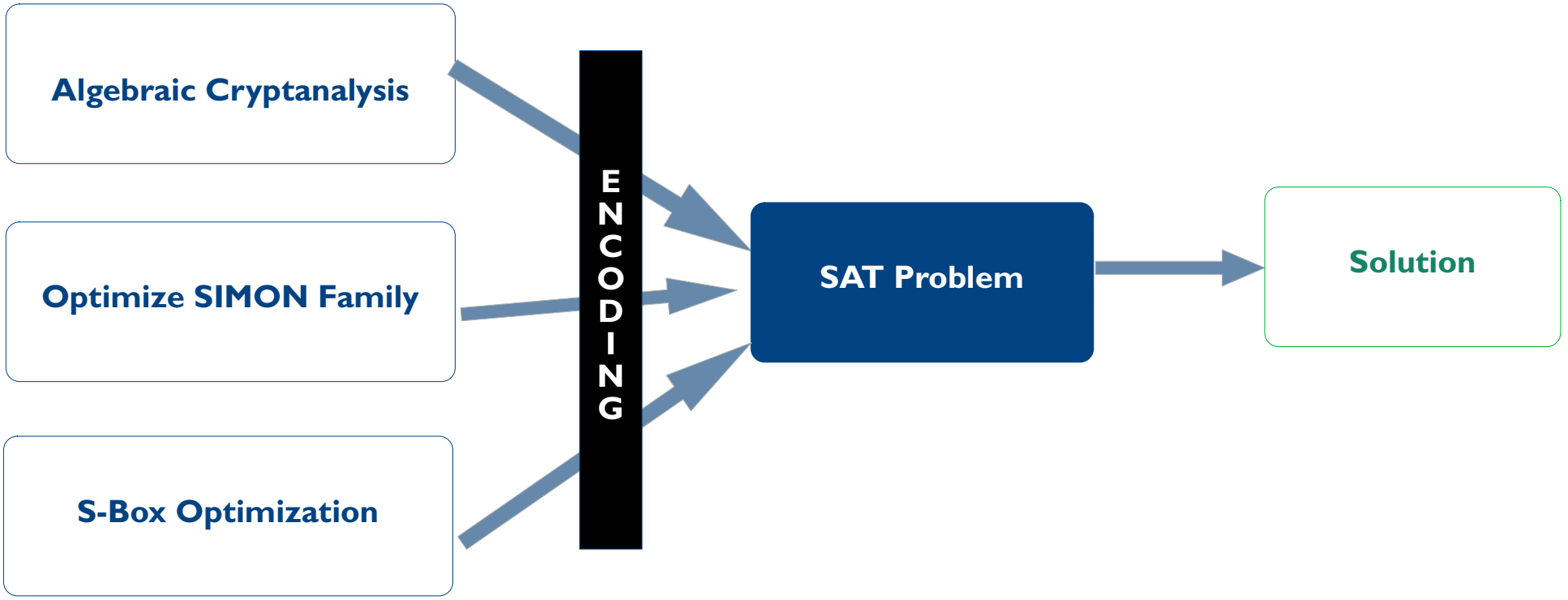
An approach in solving Cryptographic Problems



An approach in solving Cryptographic Problems



An approach in solving Cryptographic Problems



An approach in solving Cryptographic Problems