Mid-Sem Solution Sketch

① ⓐ Tamojit likes all Indian cricketers. Tamojit likes V.K. Hence, V.K is an Indian.

$\to p$ $\quad \to q \quad \hookrightarrow r$

(Incorrect)

$p \not\to q$

$p \wedge q \not\to r$

$p \to$ like Indian Cricket

$q \to$ doesn't like other crickets

$(p \to q, q)$

$\to p$

✓

(✗) p	✓ q	✓ $p \to q$
0	1	1

(b) $\nexists$ any man who has visited all the countries in the world.

$P(x, i) \rightarrow$ x visited country i

$$\forall x \; \exists i \; \neg P(x, i)$$

$$\forall x \; \neg P(x)$$

(c)

$$\begin{array}{cccc} \boxed{1} & \boxed{2} & \cdots\cdots & \boxed{49} & \boxed{50} \\ x_1 & x_2 & & x_{49} & x_{50} \end{array}$$

$$x_1 + x_3 + \cdots + x_{47} + x_{49} \overset{(?)}{\geq} x_2 + x_4 + \cdots + x_{50}$$

Assume Yes

$S \to x_1, x_3, \ldots\ldots, x_{49}$

$A \to x_2, x_4, \ldots, x_{50}$

(d) Fallacy in Induction Proof:

- $n = 1$ (base case) doesn't hold
  - $n = 0$ is the base case
- $k=0 \to k=1$ (this doesn't hold)

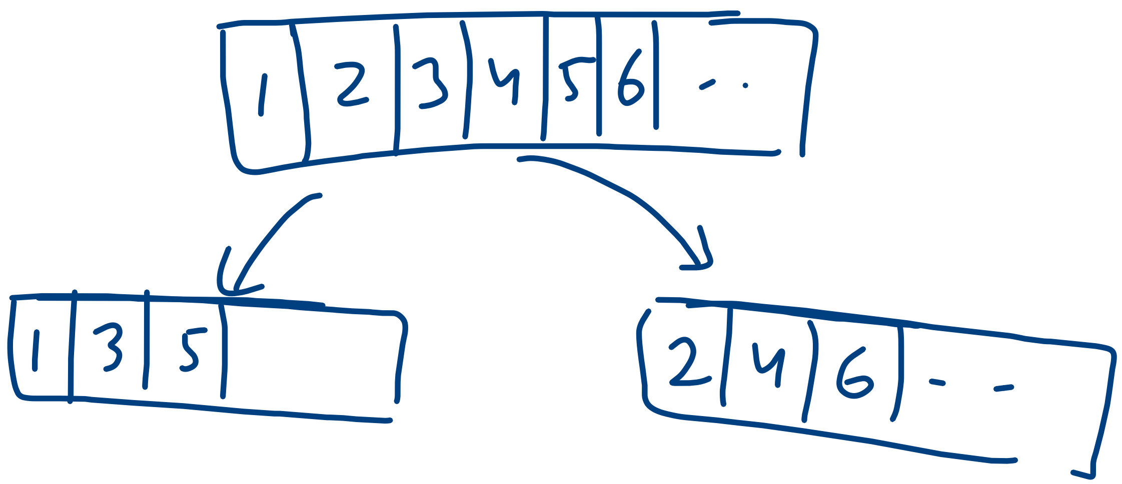
$$\underline{\underline{\frac{d}{dx}(x)}} = x \cdot \frac{d}{dx}(x^0) + \frac{d}{dx}(x)$$

0

1.
2.
3.
4.
5.
6.

wrong

② (a)



— $(1\ 20\ 61\ 78\ n)$

$\downarrow$

— $(1\ 20\ 61\ 78)$

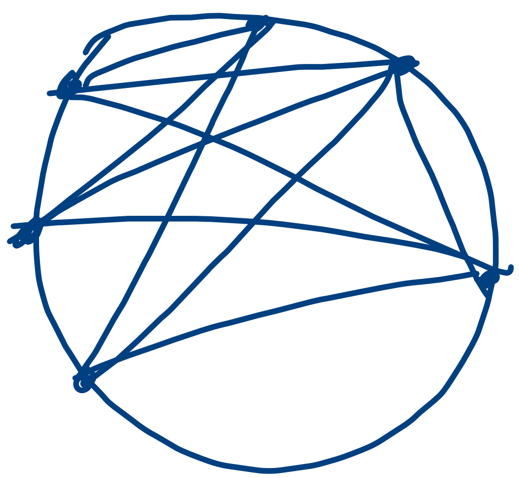
(b) bijection between last person sitting at n^{th} seat & last person sitting at 1^{st} Seat.

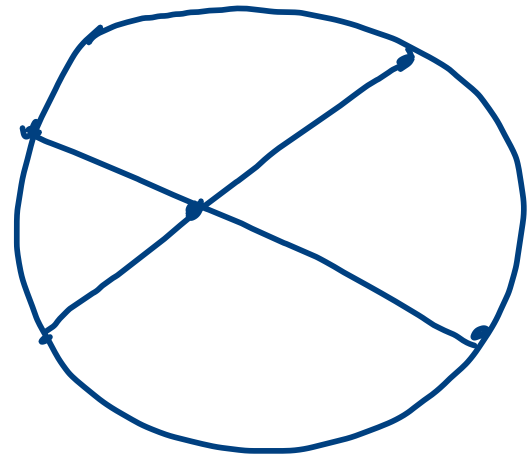
$$\begin{pmatrix} 1 & i_1 & \cdots & i_k & n \\ i_1 & i_2 & \cdots & n & 1 \end{pmatrix} \to \text{last person takes first Seat.}$$

$$\|$$

$$\begin{pmatrix} 1 & i_1 & \cdots & i_k \\ i_1 & i_2 & \cdots & 1 \end{pmatrix} \to \text{last person takes last Seat}$$

(c)

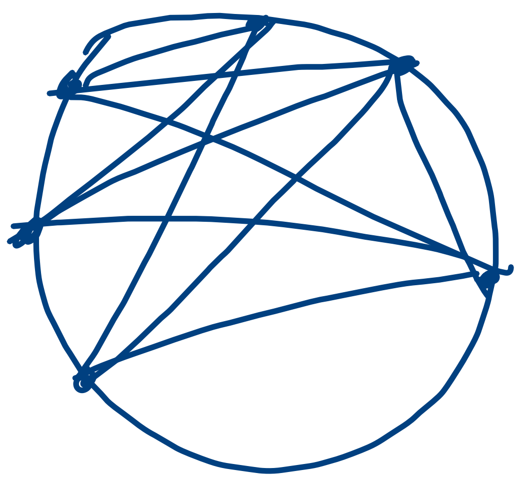


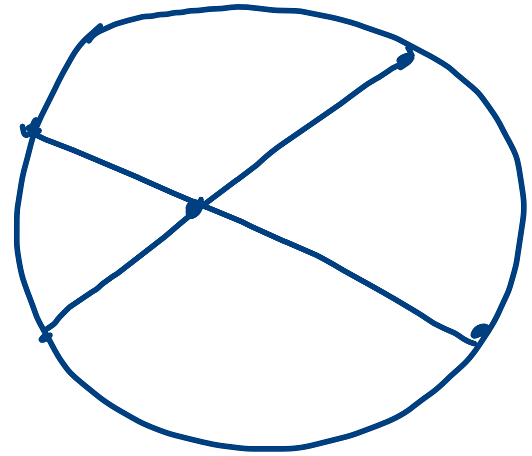


$\binom{n}{4}$

- Take any 4 point, you get exactly 1 intersection inside the circle
- Can you say this is unique? (given cond)

(c)





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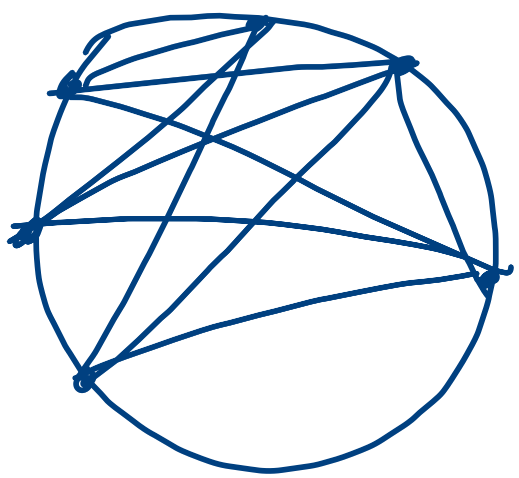

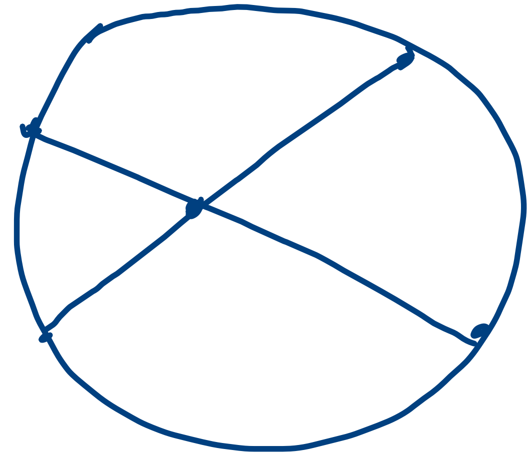

$\binom{n}{4}$

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3 (a) $3 \times \binom{4}{2} \times 2^7$

(b) 10 students — (60 marks)

Two disjoint groups with same marks-sum.

- How many marks-sum? 601 $(0, 1, \ldots, 600)$
- How many subsets? (groups) 2^{10}
- Pigeon-hole principle, there are two subsets with equal marks-sum

$\{x_1, x_2, x_3, x_4\}$ $\{x_3\ x_5\ x_7\}$

- Exclude the intersection elements

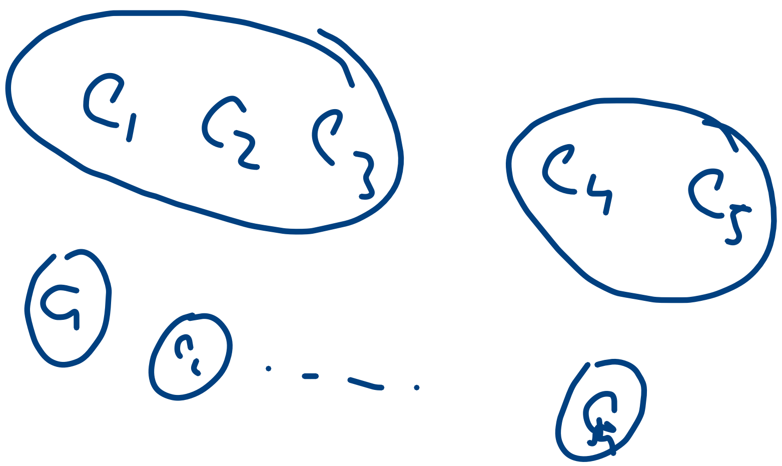
$\{x_1, x_2, x_4\}$ $\{x_5\ x_7\}$

(c) Criminals in prison

- A doesn't beat A
- If A doesn't beat B, B doesn't beat C, A doesn't beat C
- If A doesn't beat B, B can still beat A.

(*) If cond 3

$A \sim B$ iff A doesn't beat B



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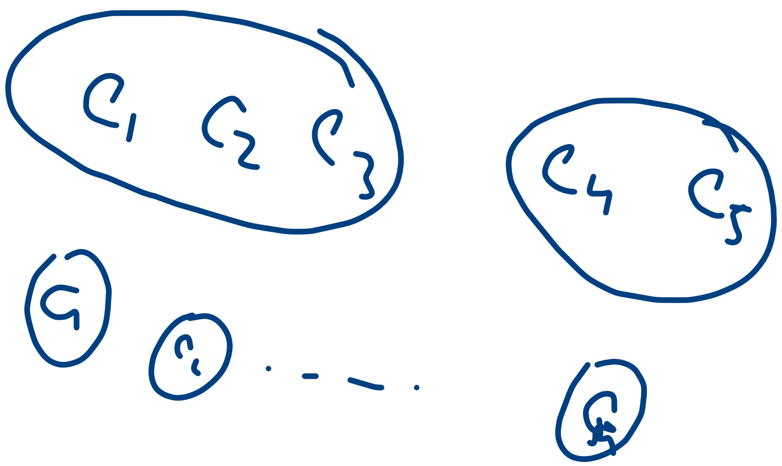
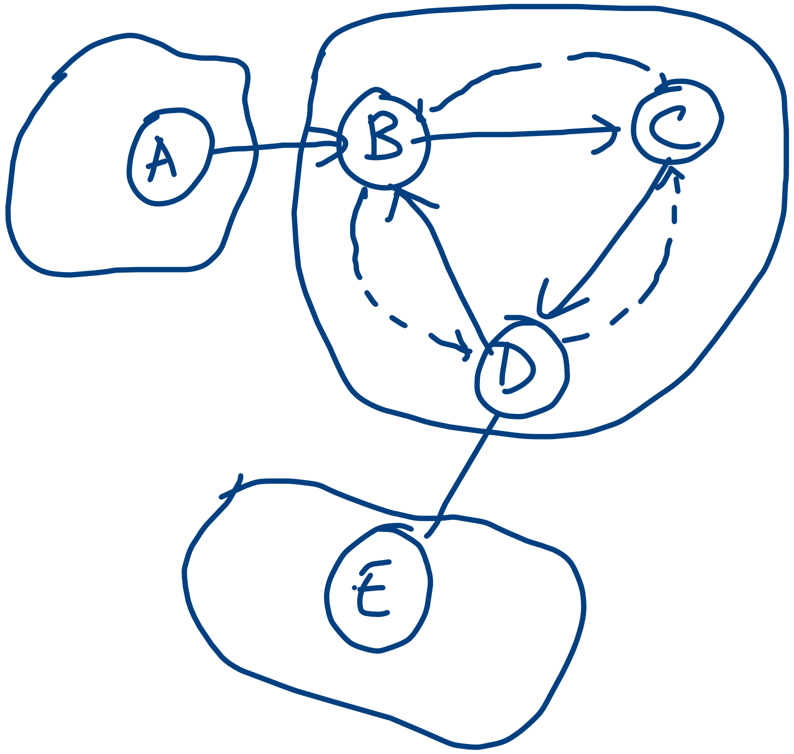
(*) If cond 3

A $\sim$ B iff A doesn't beat B and B doesn't beat A.



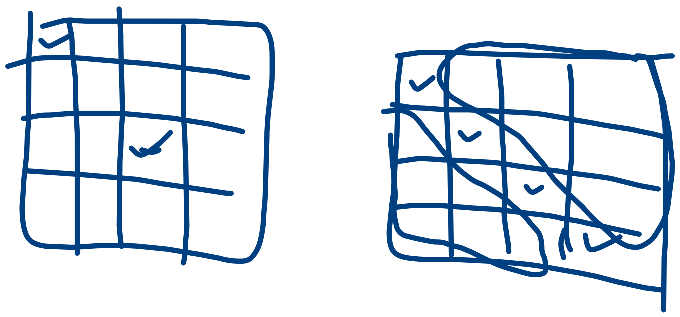


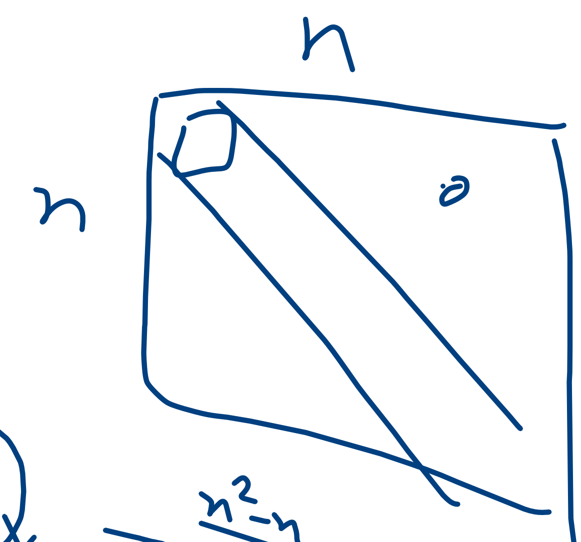
# of Strongly-Connected Components

④
one-directional → # of anti-symmetric relations

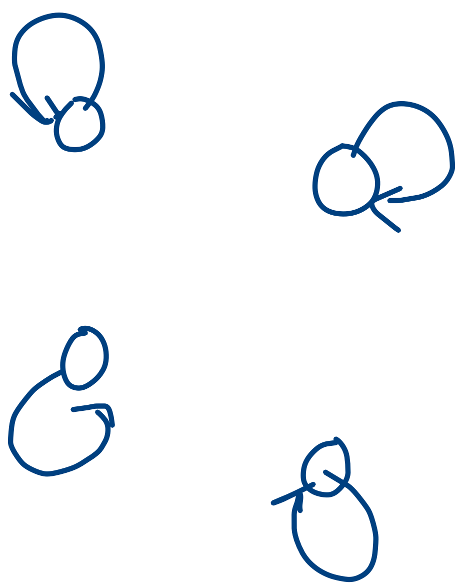
bi-directional → # symmetric relations

both → 2^n

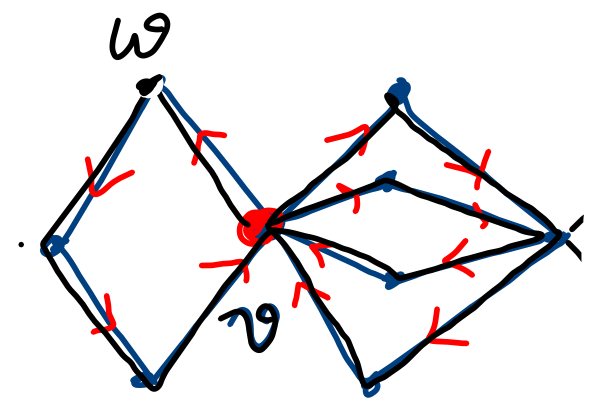


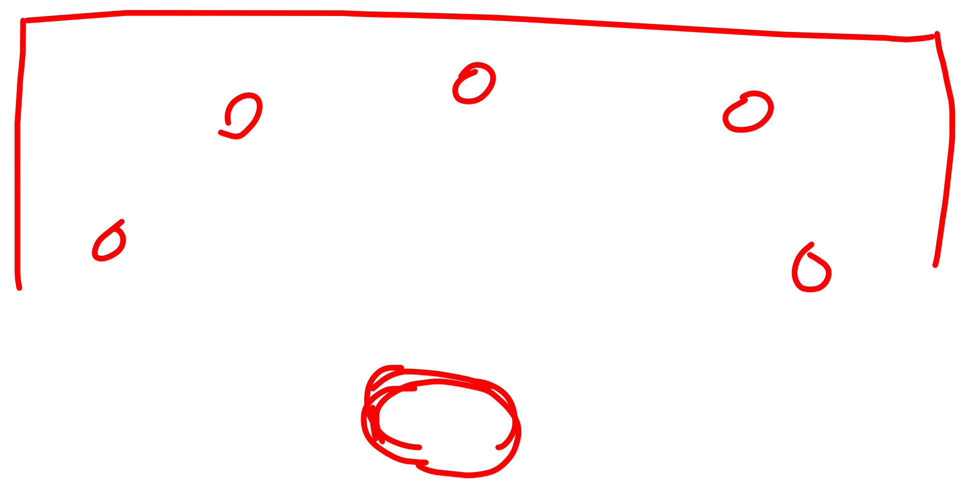


$2^n \times 3^{\frac{n^2-n}{2}}$

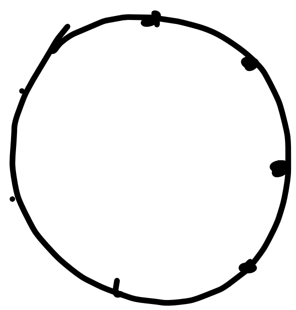


(b) Forced Eulerian Graph from v.





Iff every cycle of G contains v



→ All vertices forced Eulerian.

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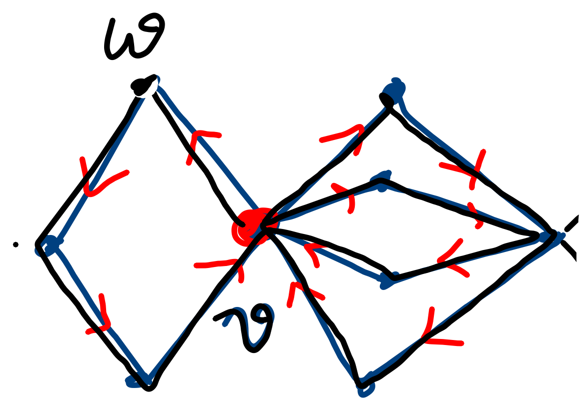

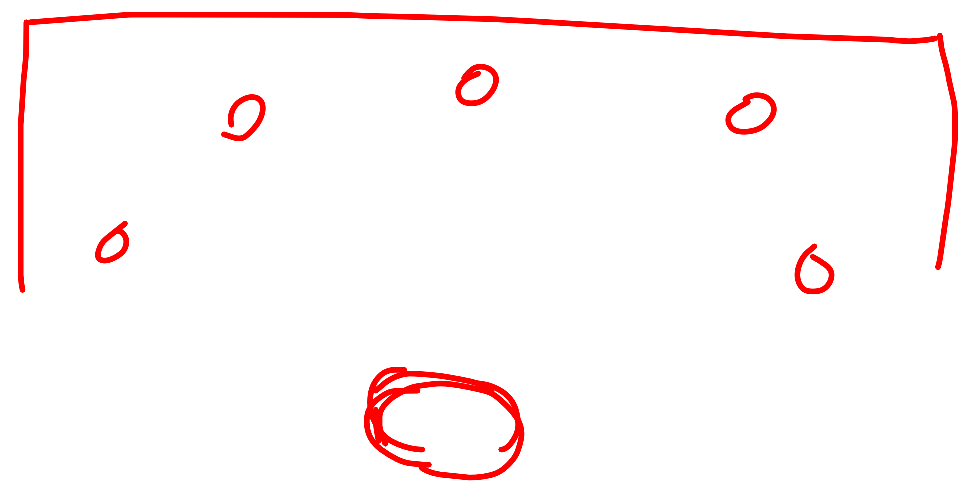

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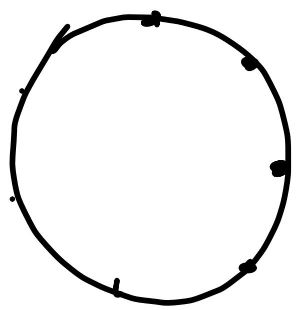

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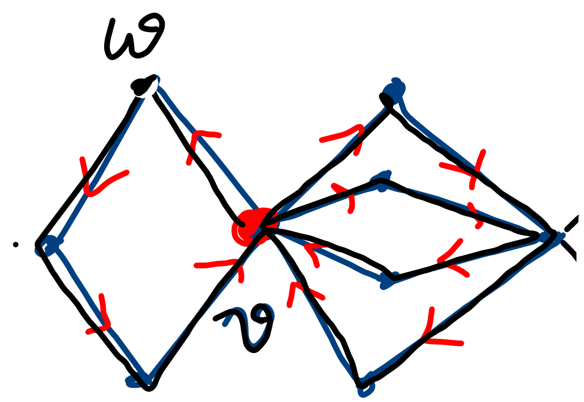

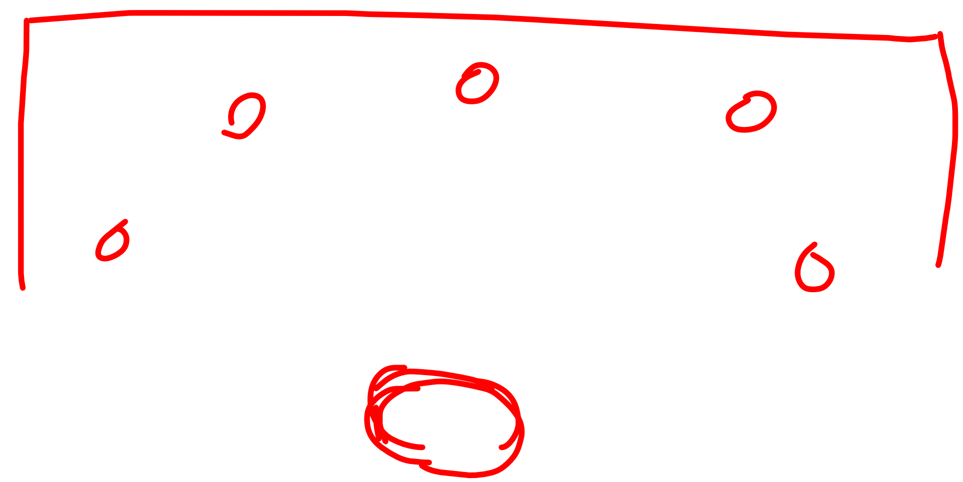

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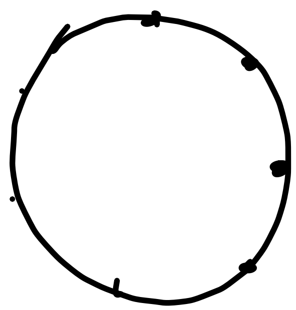

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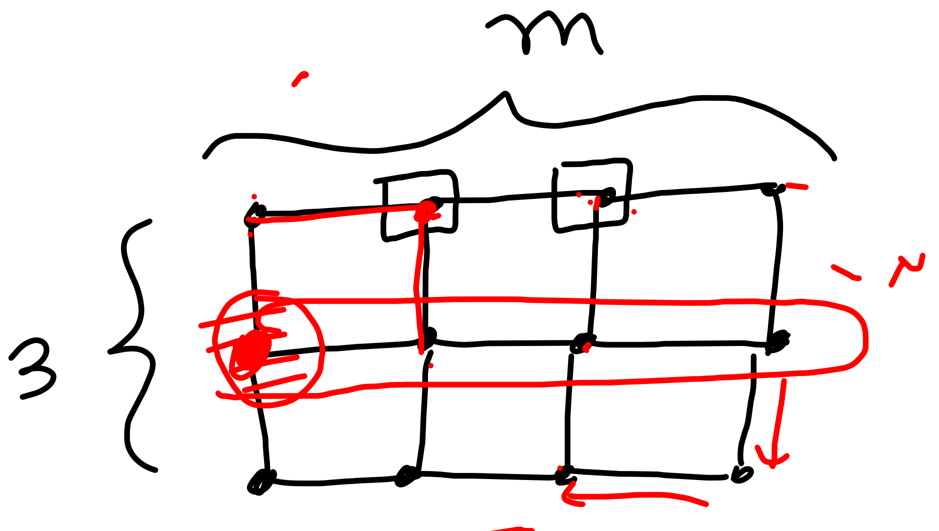


Eulerian → No

Hamiltonian → Even (Intuition)

odd (X)

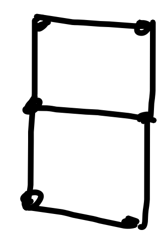
↳ Try to see

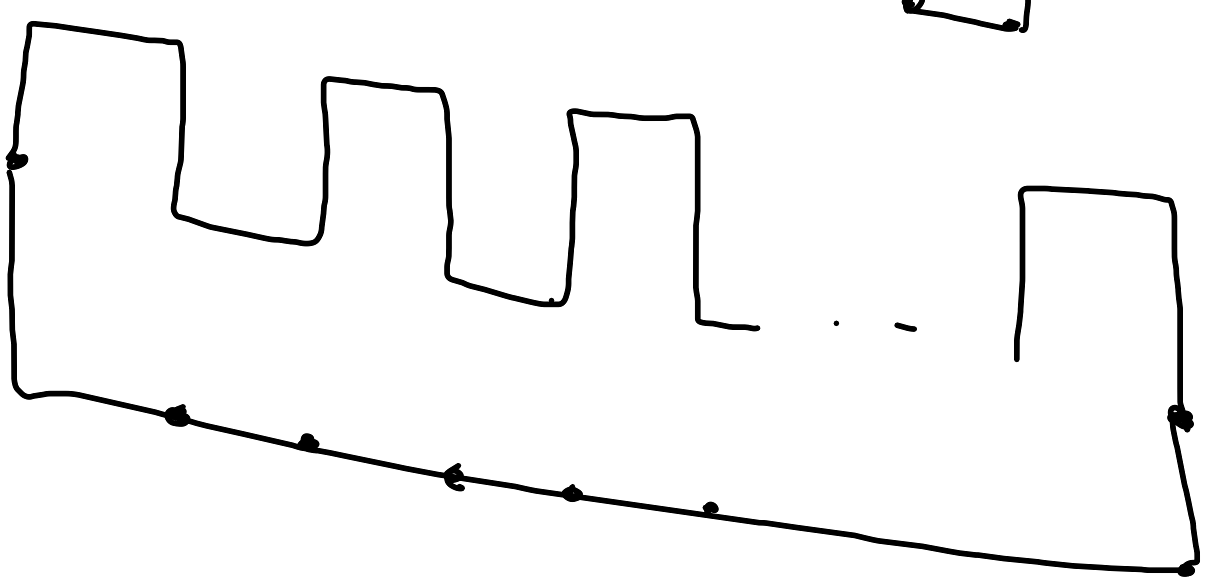
G_m

$m \to$ even

$m=1$?

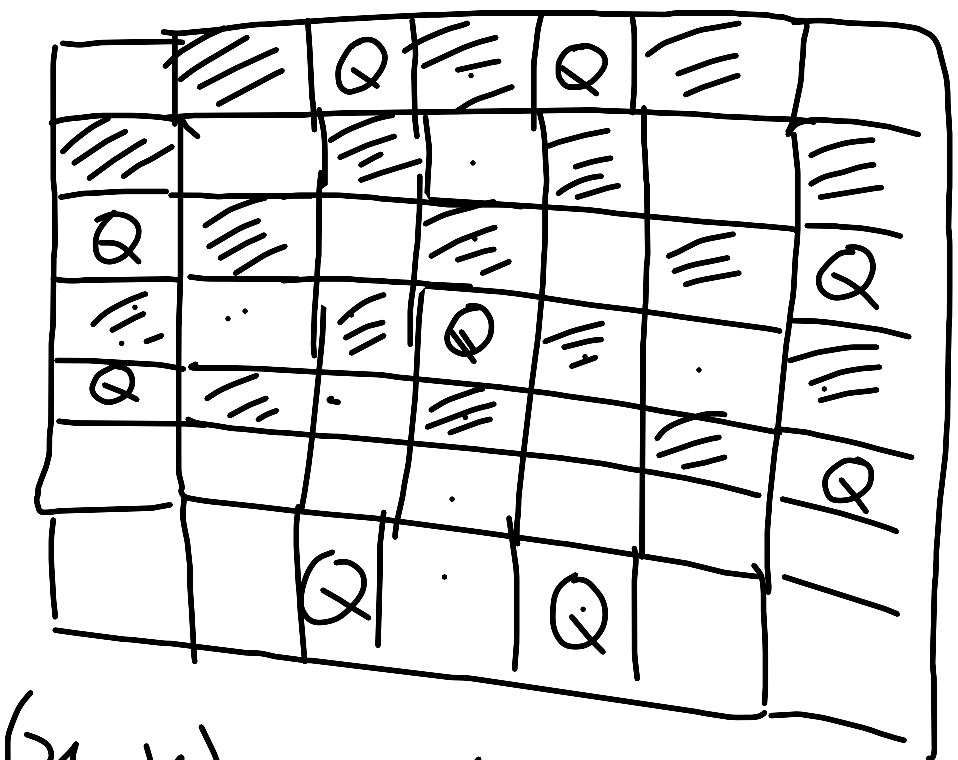
$m=2$?





(b)

Special Knight



$(x, y) \to (x \pm 3, y \pm 1)$

$(x \pm 1, y \pm 3)$

(c) $(3, 3, 3, 3, 3, 3, 3, 3)$

Non Isomorphic