Assignment 4

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1. A tree T is called an $(1,3)$-tree if all of it's vertices have degree either 1 or 3.

   (a) How many $(1,3)$-trees are there with the degree sequence $(3,3,3,3,\,1,1,1,1)$?

   (b) Draw an $(1,3)$-tree with 19 vertices.

   (c) If T has n leaves, find the number of vertices with degree 3.

   (d) Let T has $n \geq 4$ vertices. Show that there is some internal vertex which is adjacent to exactly two leaves.

   (e) Find the number of $(1,3)$-trees with n leaves.

2. Justify with proper arguments:

   (a) Suppose an undirected graph G with unique positive weights has a Minimum Spanning Tree (MST) T. If we square all the edge weights and compute the MST again, we will get the same tree structure again.

   (b) We can sort the vertices of a di-graph topologically *if and only if* the graph is acyclic. (Hint: In an acyclic digraph, there exist at least one source and at least one sink)

3. The Minimum Bottleneck Spanning Tree (MBST) in an undirected graph is a tree whose most expensive edge is as minimum as possible.

   (a) Prove that every MST is a MBST.

   (b) Does every MBST corresponding to a graph is a MST?

4. Four people need to cross a rickety bridge at night. Unfortunately, they have only one torch and the bridge is too dangerous to cross without one. The bridge is only strong enough to support two people at a time. Not all people take the same time to cross the bridge. Times for each person: 1 min, 2 mins, 7 mins and 10 mins. What is the shortest time needed for all four of them to cross the bridge?