Practical Attacks on a Class of Secret Image Sharing Schemes: Few Open Issues

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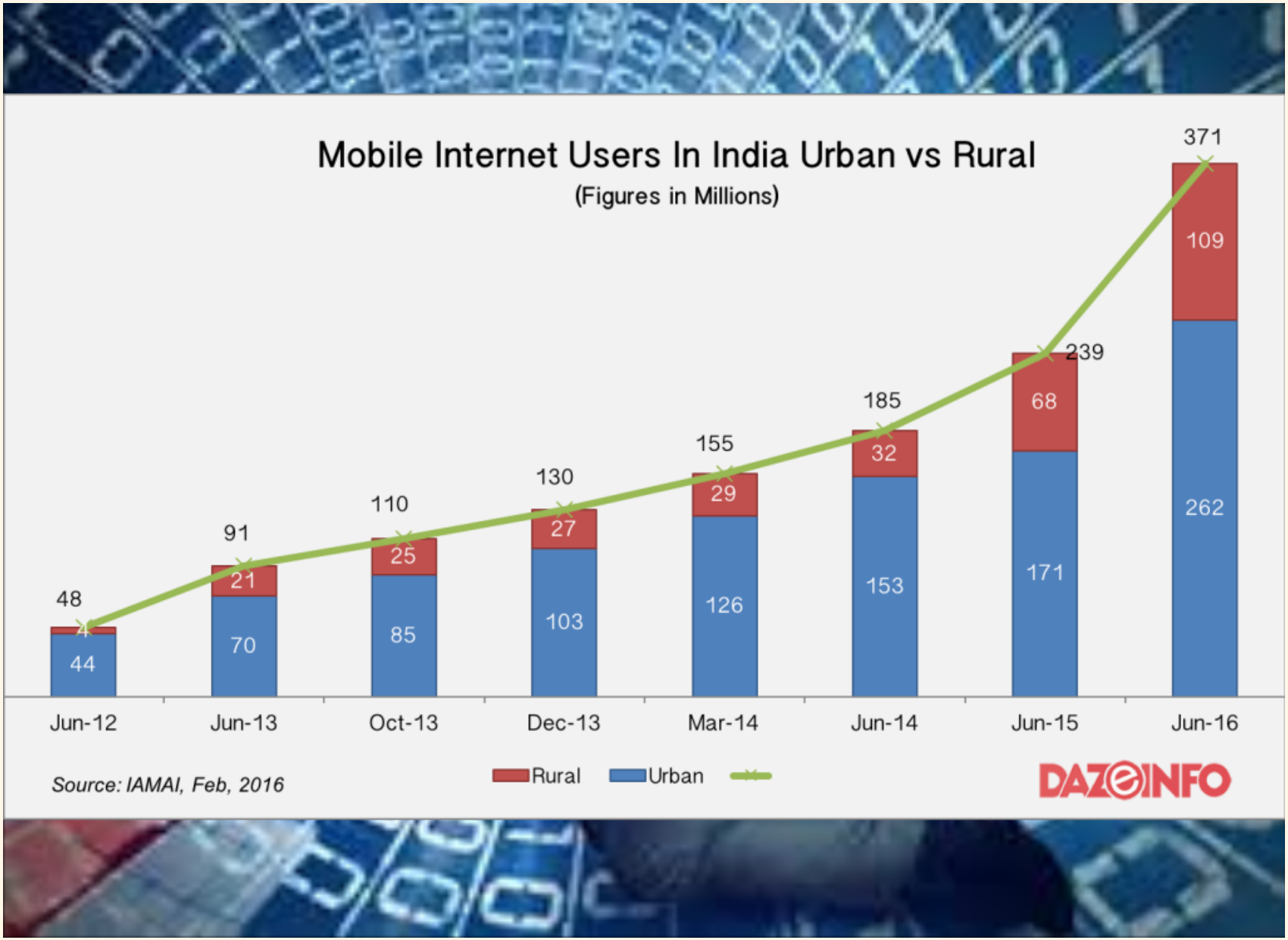
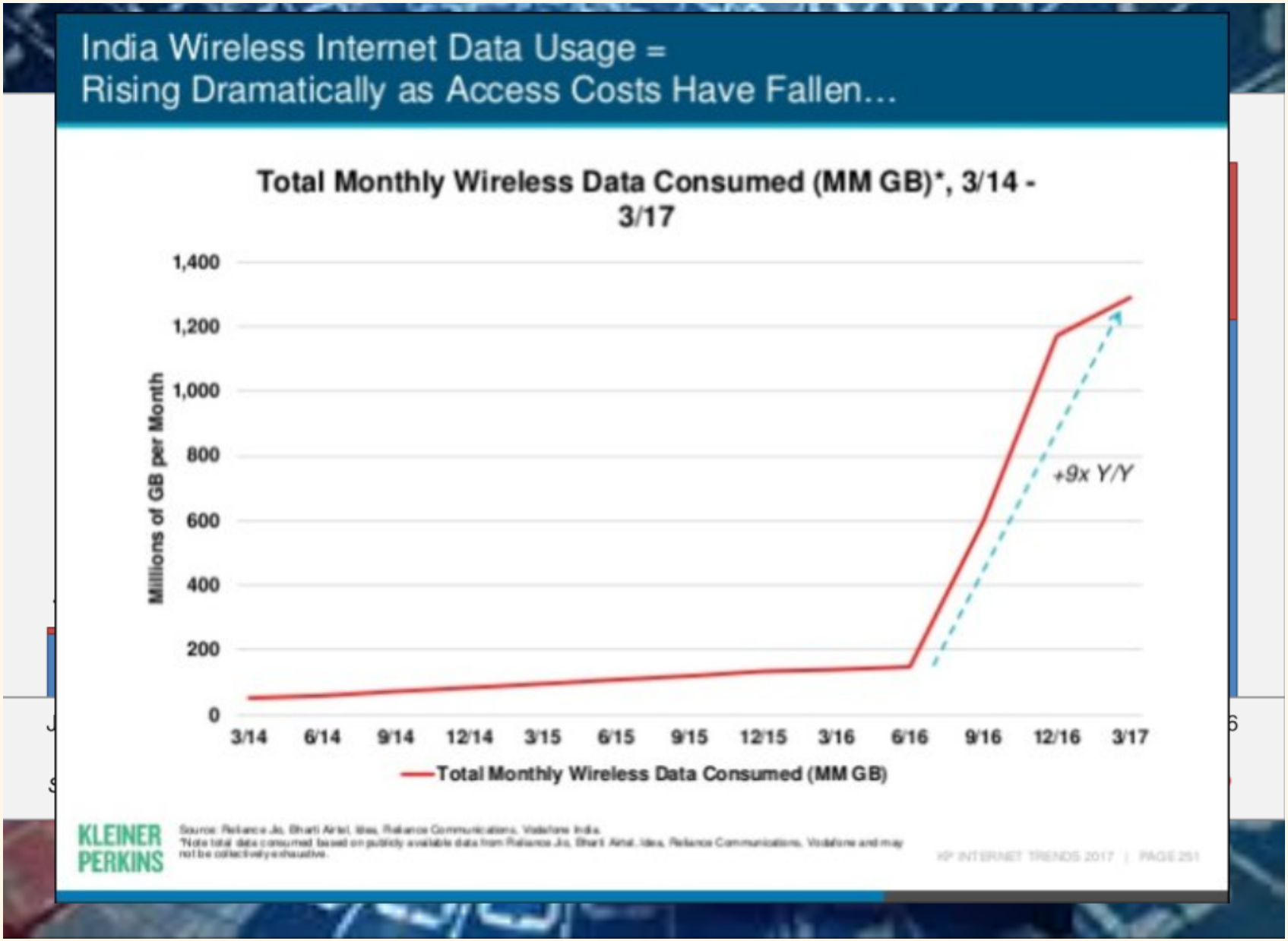
Subarsha Banerjee, Sandip Kumar Mondal
Chandan Goswami and Dr Prakash Dey

Internet in Day-to-Day Life



Internet in Day-to-Day Life



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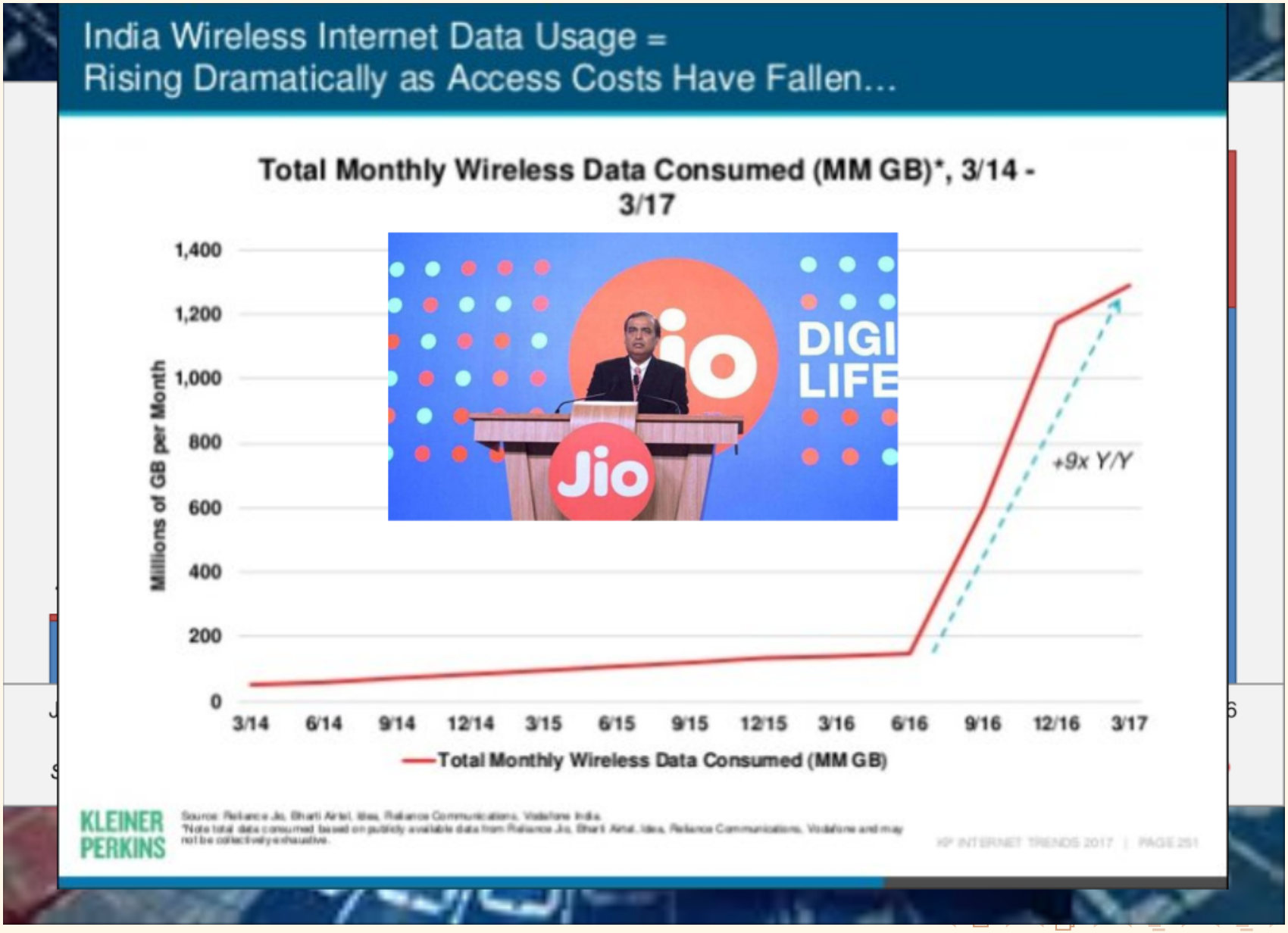

Big Data Usage: 2019

Google processes over 5 exabytes of data per day.
1 Exabytes = $1,000,000,000 = 10^9$ **GB**.
Google handles 1.2 trillion searches every year.
1 trillion=10^{12}.

Facebook generates 4 petabytes of new data per day, where **1 petabytes =** 10^6 **GB.**
350 million photos are uploaded per day. Users generate 4 million likes every minute.

Whatsapp has nearly 500 Million Users. It processes, more than 70 Million Messages a Second.

Nicely said!

Application of Secret Sharing in Digital World

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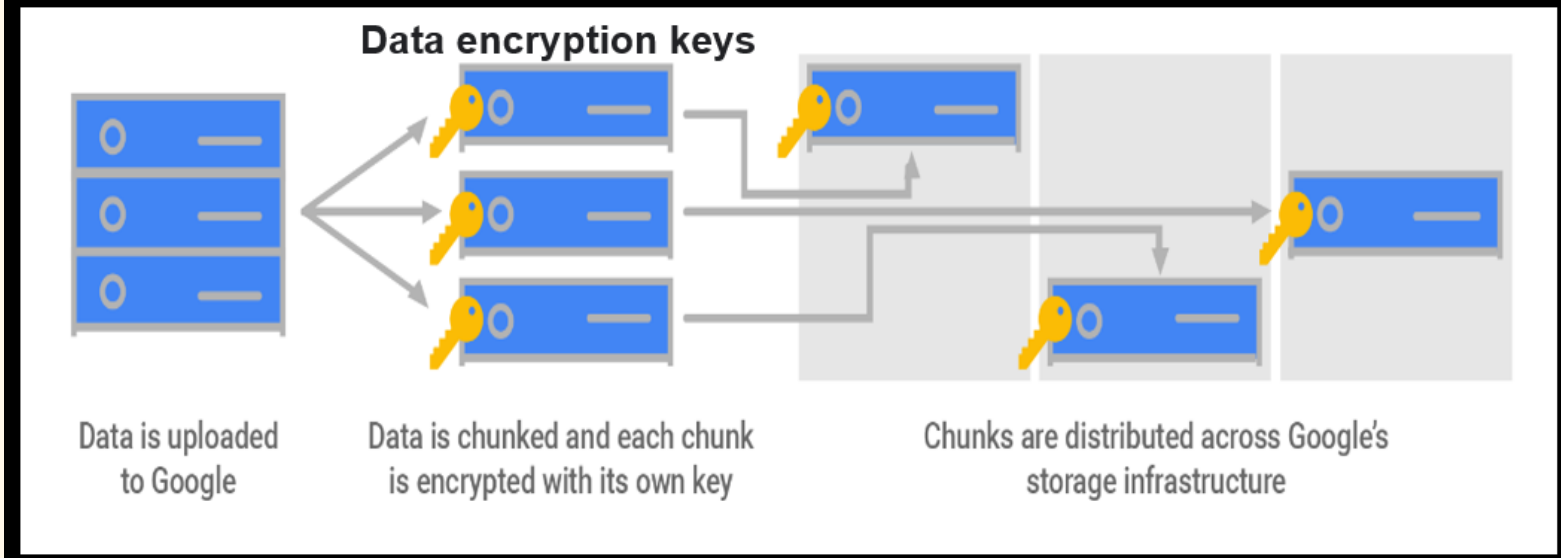
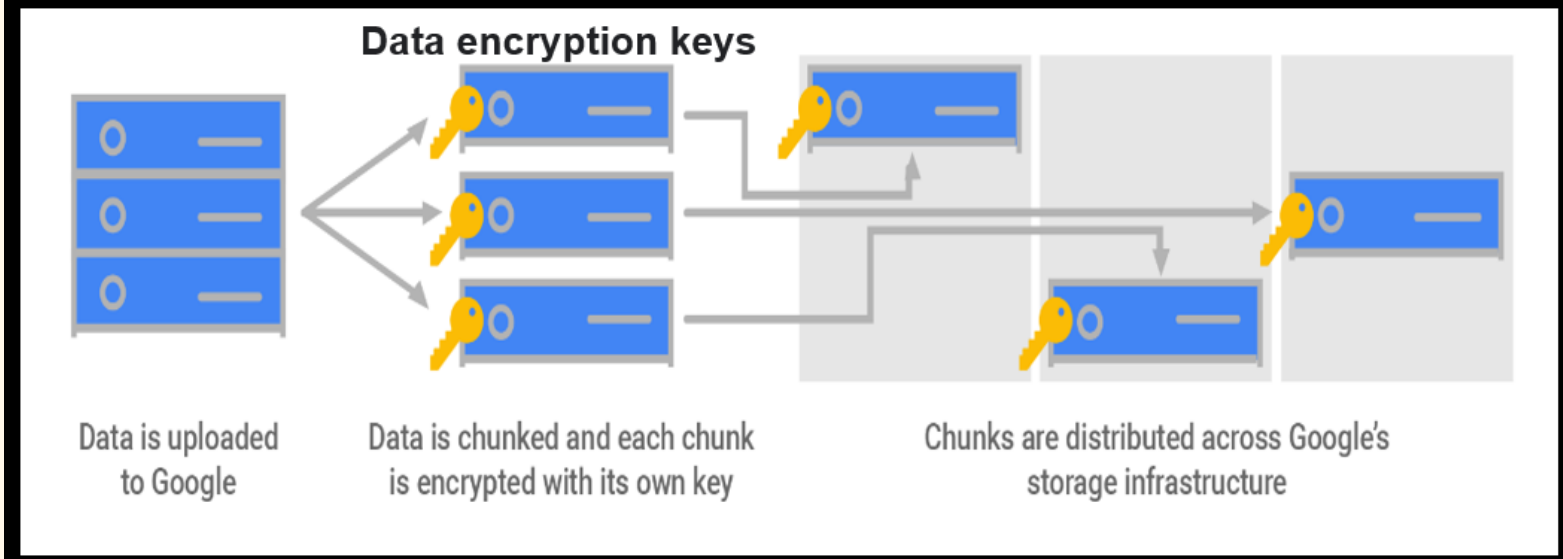
How Google Cloud Stores Data at Rest



Each chunk is encrypted at the storage level with an individual encryption key: two chunks will not have the same encryption key, even if they are part of the same Cloud Storage object, owned by the same customer, or stored on the same machine.

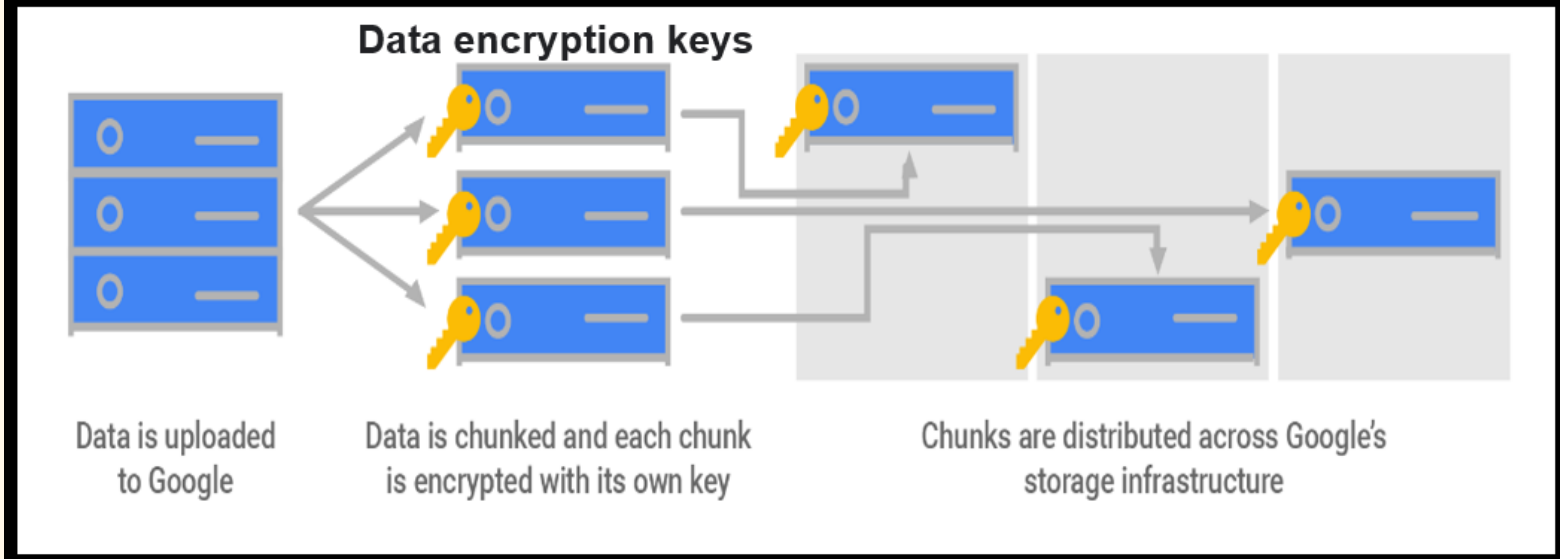
Application of Secret Sharing in Digital World



The DEKs are encrypted with (or "wrapped" by) a *key encryption key (KEK)*. One or more KEKs exist for each Google Cloud service.

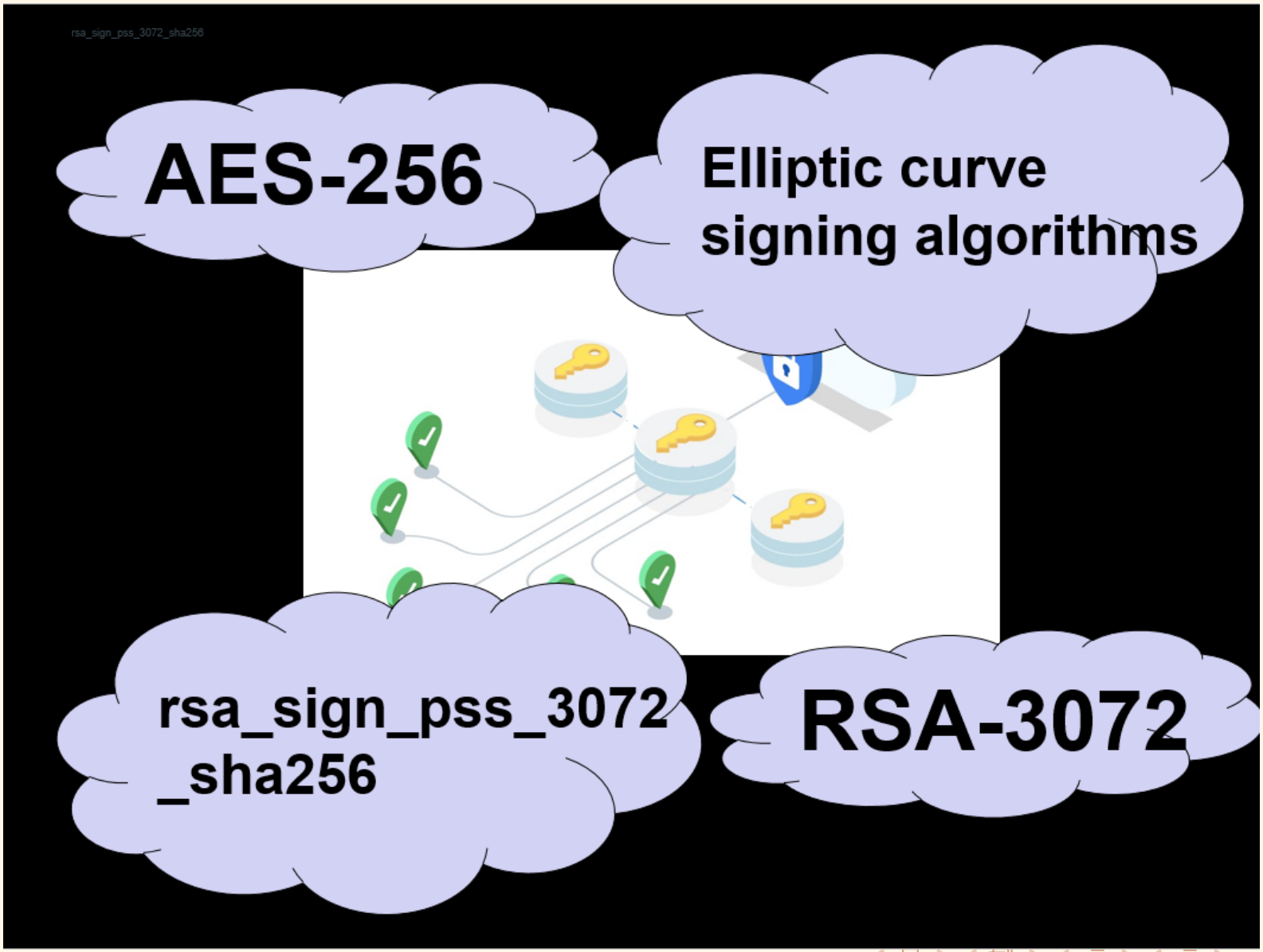
These KEKs are stored centrally in Google's *Key Management Service (KMS)*, a repository built specifically for storing keys.

How Google Cloud Stores Data at Rest



When a storage system needs to retrieve encrypted data, it retrieves the wrapped DEK and passes it to KMS. KMS then verifies that this service is authorized to use the KEK, and if so, unwraps and returns the plaintext DEK to the service. The service then uses the DEK to decrypt the data chunk into plaintext and verify its integrity.

Key Management Services: Cryptographic Tools Used



Prime Numbers in our day to day life (308 digits)

1230231922161117176931558813276752514640713895736833
7157661180291600588006146729487753600678385934595824
2964925405180490851288418089823682358508248206534833
1234959350355845017413023320111360666922624728239756
8804164344783156936750134130907572086903767932966588
1066294182449348845172650530371291600534674790862370
2673480919353936813105736620402352744776903840477883
6511003224093019834883638029305404824879097634840982
5394072868513204440886373475427121259247177864394948
6688511721051561970432780747454823776808464180697103
0838618121843485655227401957966826222055118455120805
5201031005025580158934964592800113374547422071501368
3413907542779063759833876101354235184245096670042160
7206294115815023712480084304471848420986103205804179
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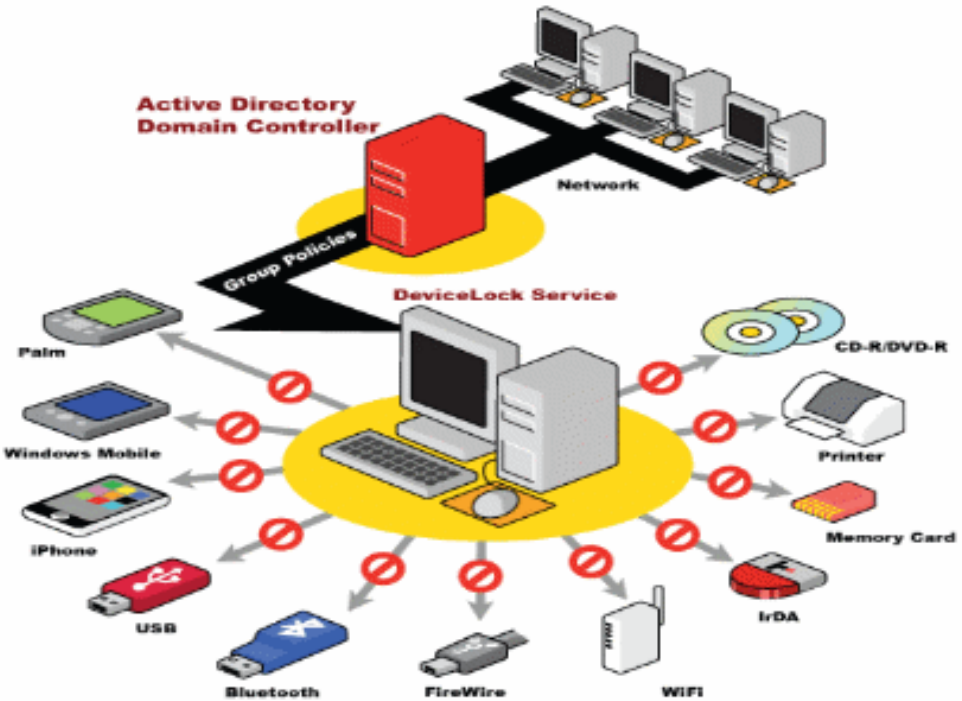
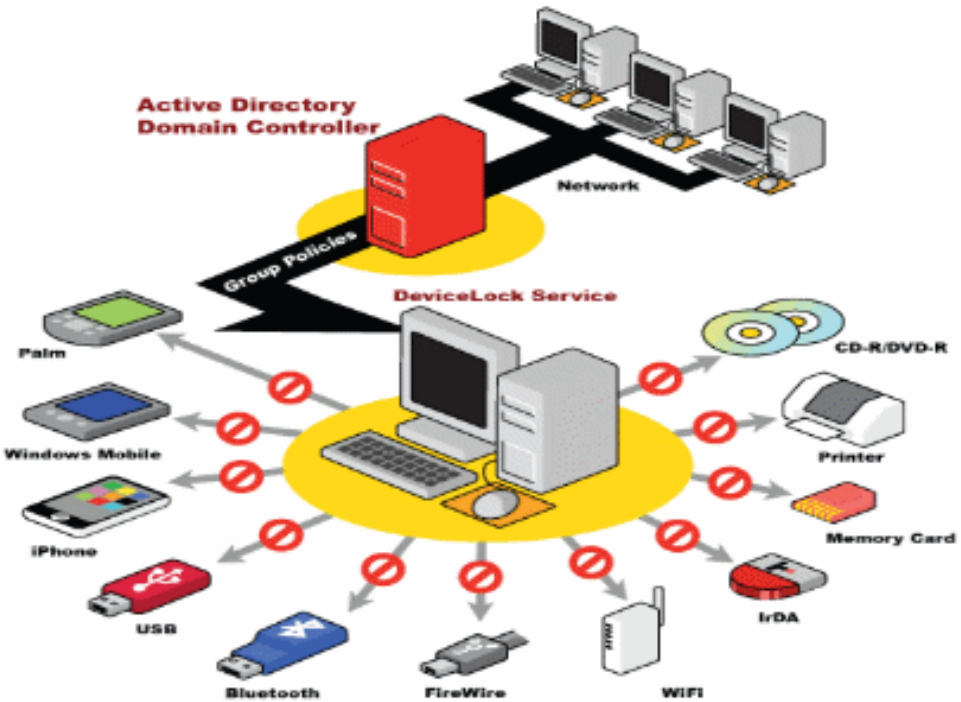
All Have Secrets in Life

Storing Secrets in Different Places



Storing Secrets in Different Places



What is Secret Sharing?

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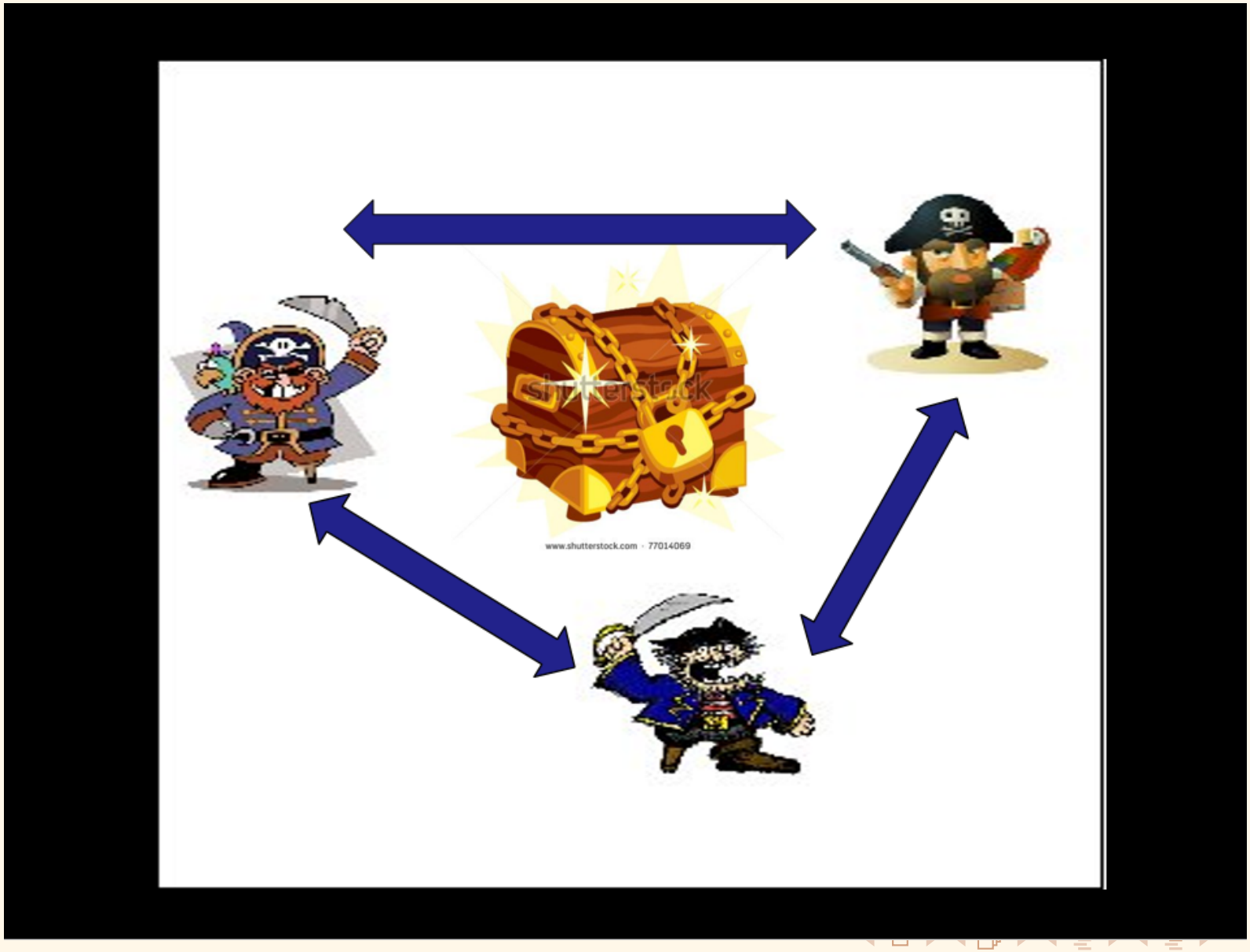

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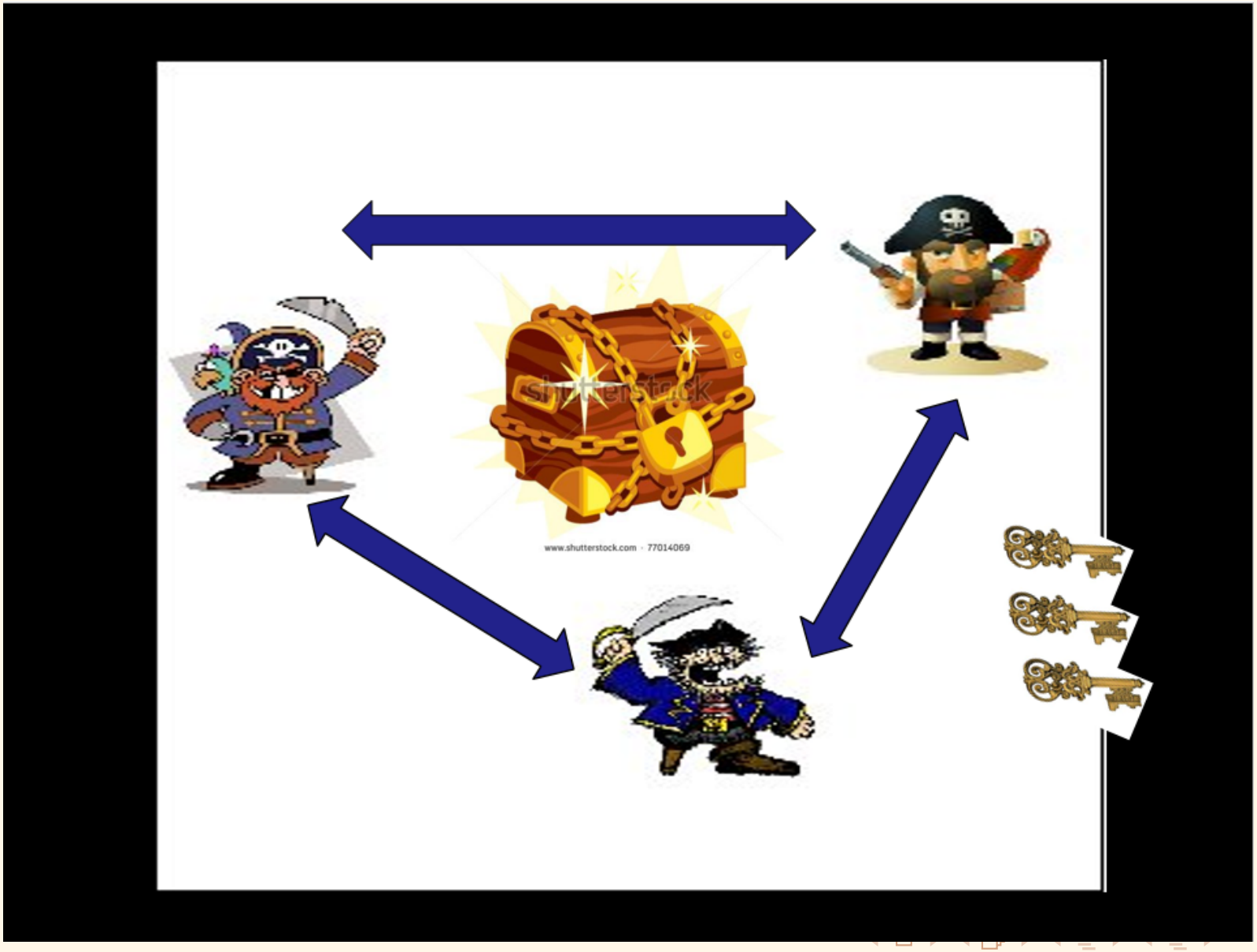

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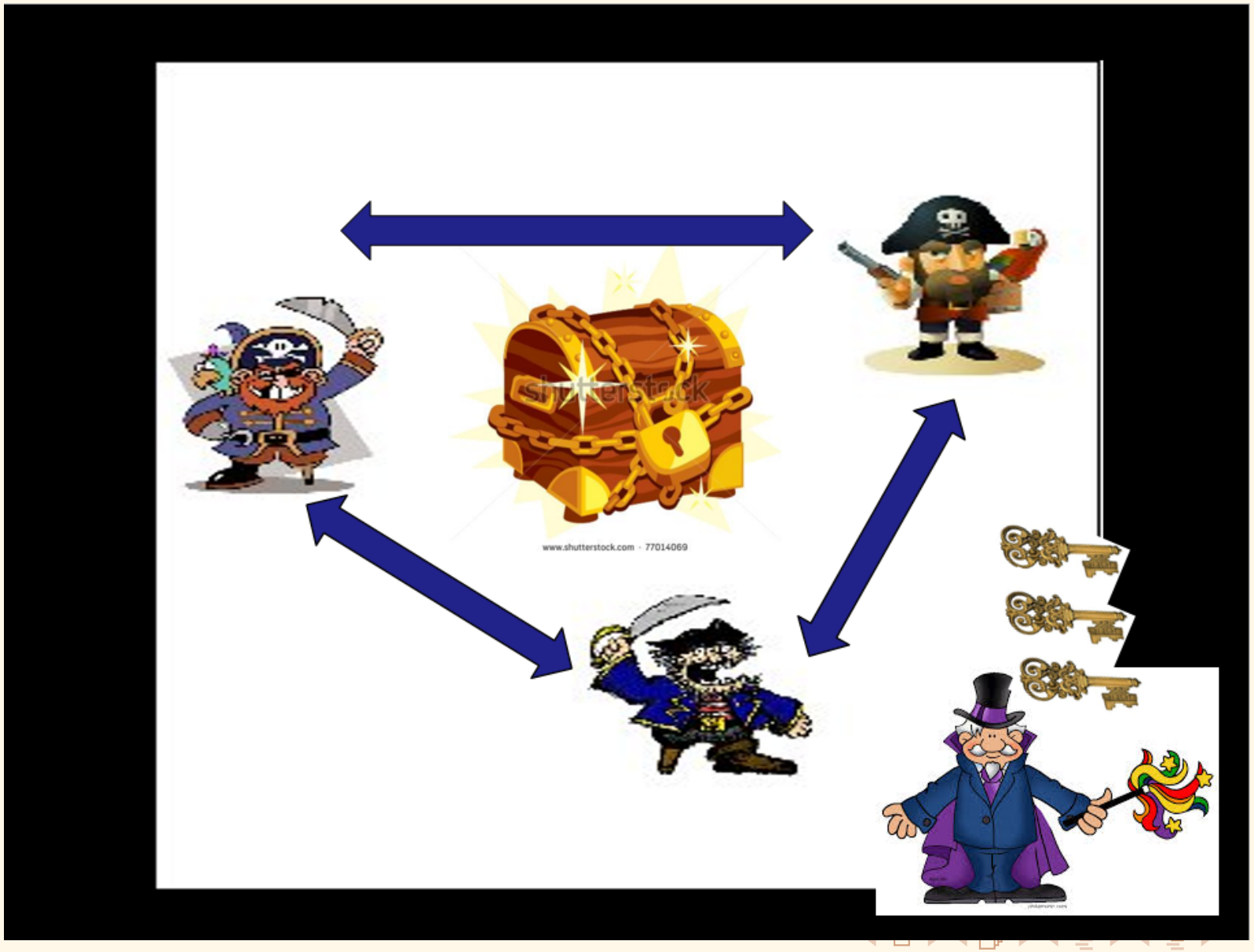

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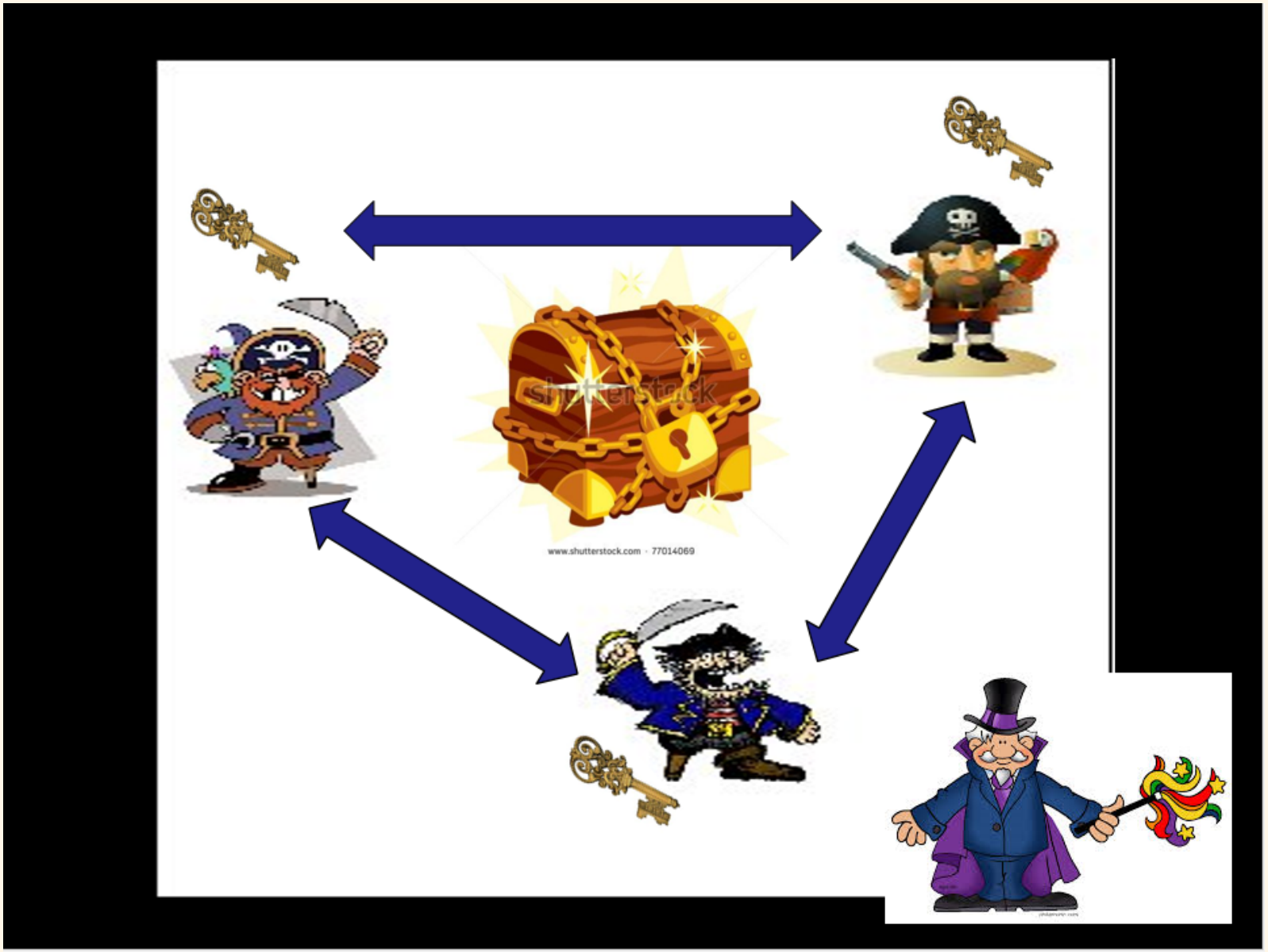

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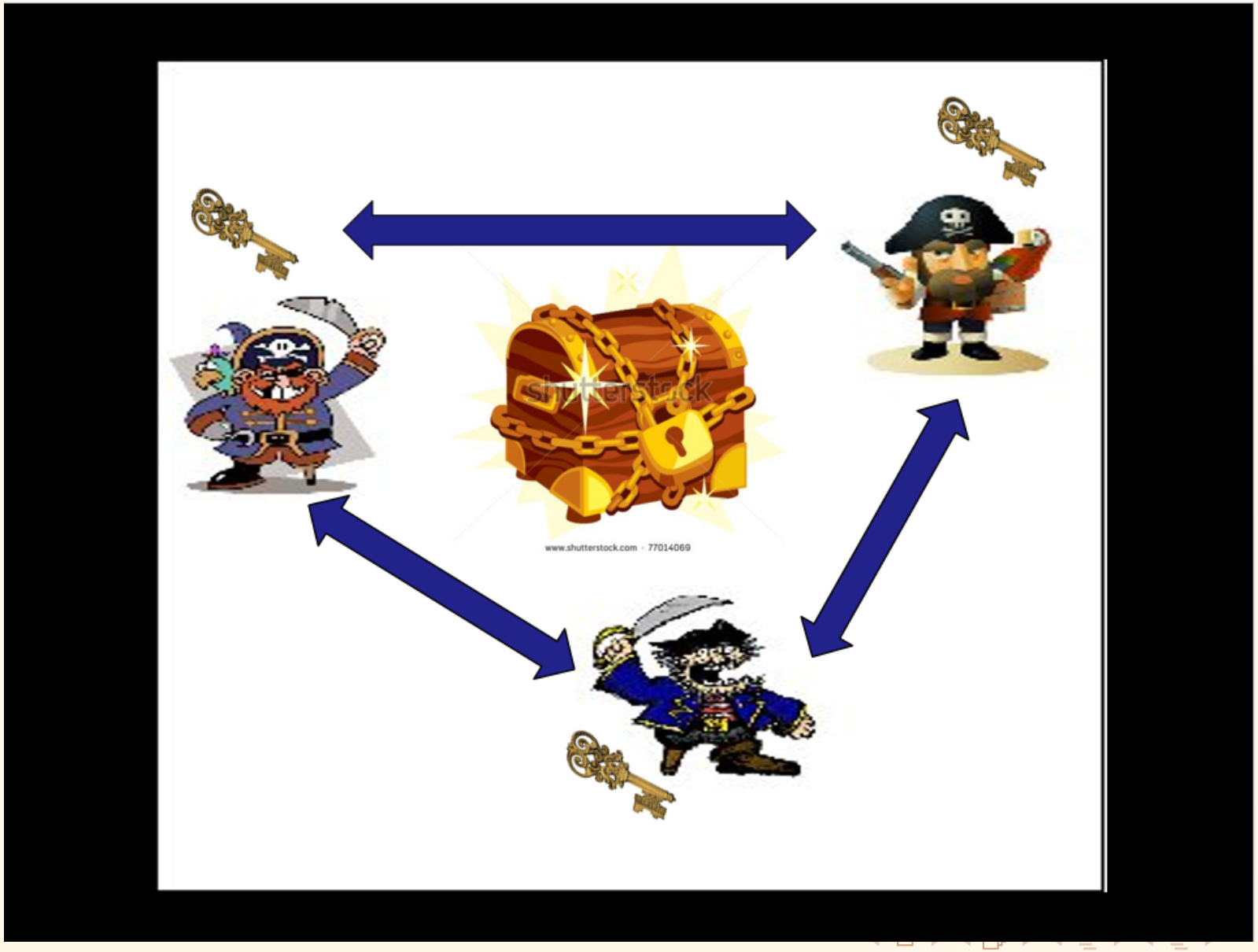

(t, w) threshold scheme

Let t and w be two positive integers, such that $t \leq w$. A (t, w) *threshold scheme* is a method of sharing a scheme key k among a set of w participants in such a way that any t participants can compute the value of k, but no group of $(t-1)$ participants can do so.

Real Life Example of Secret Sharing

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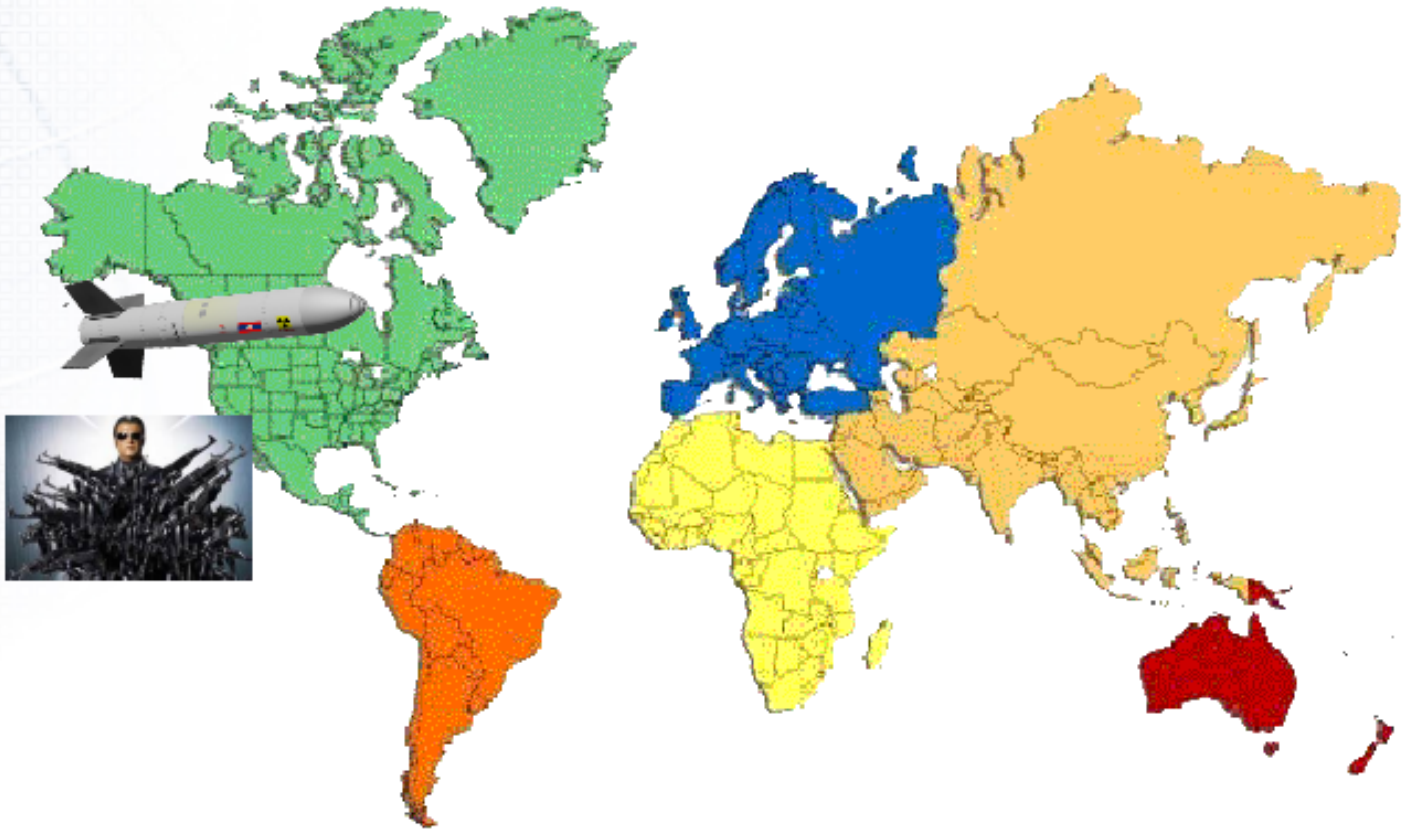

Credit Card and its Pin

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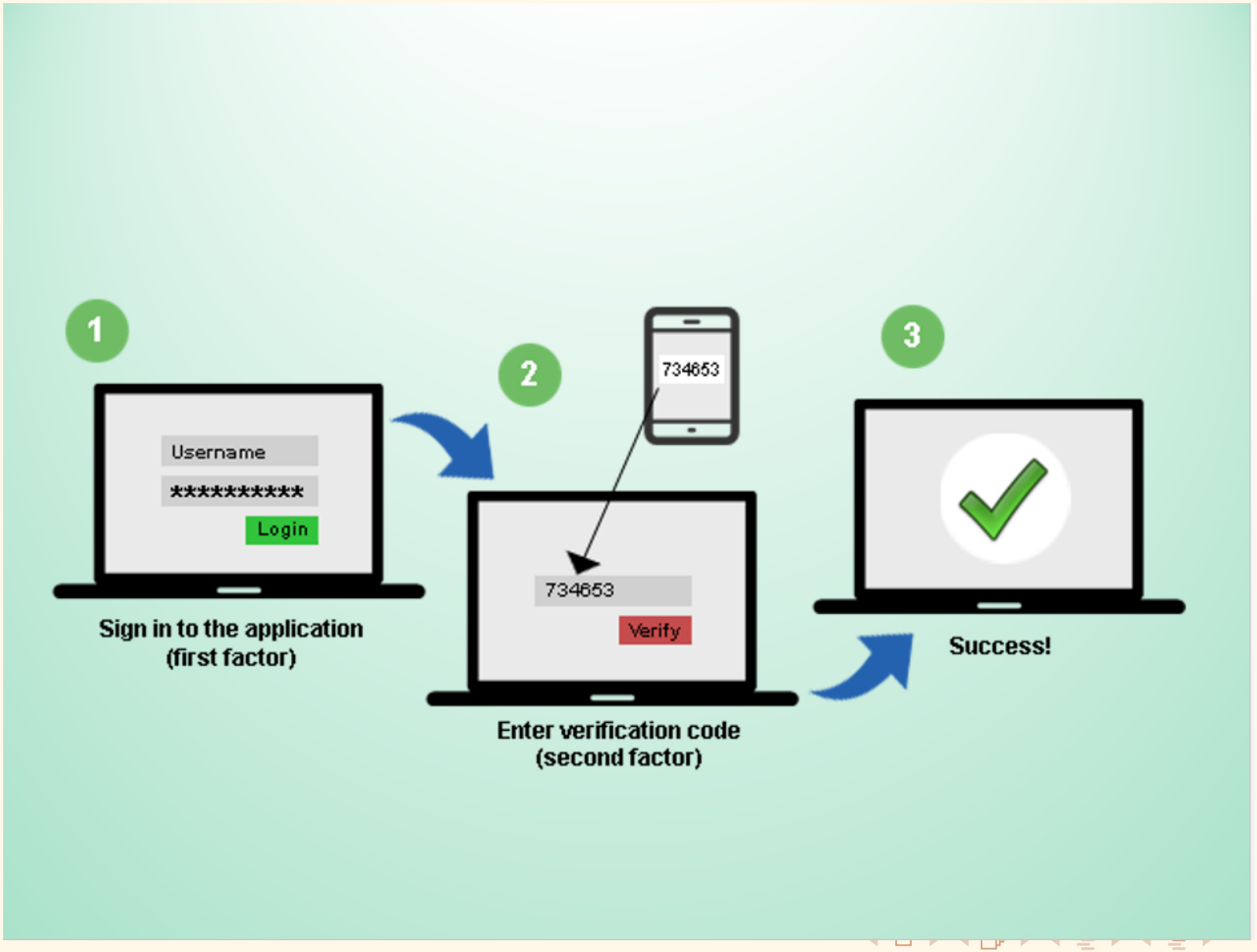
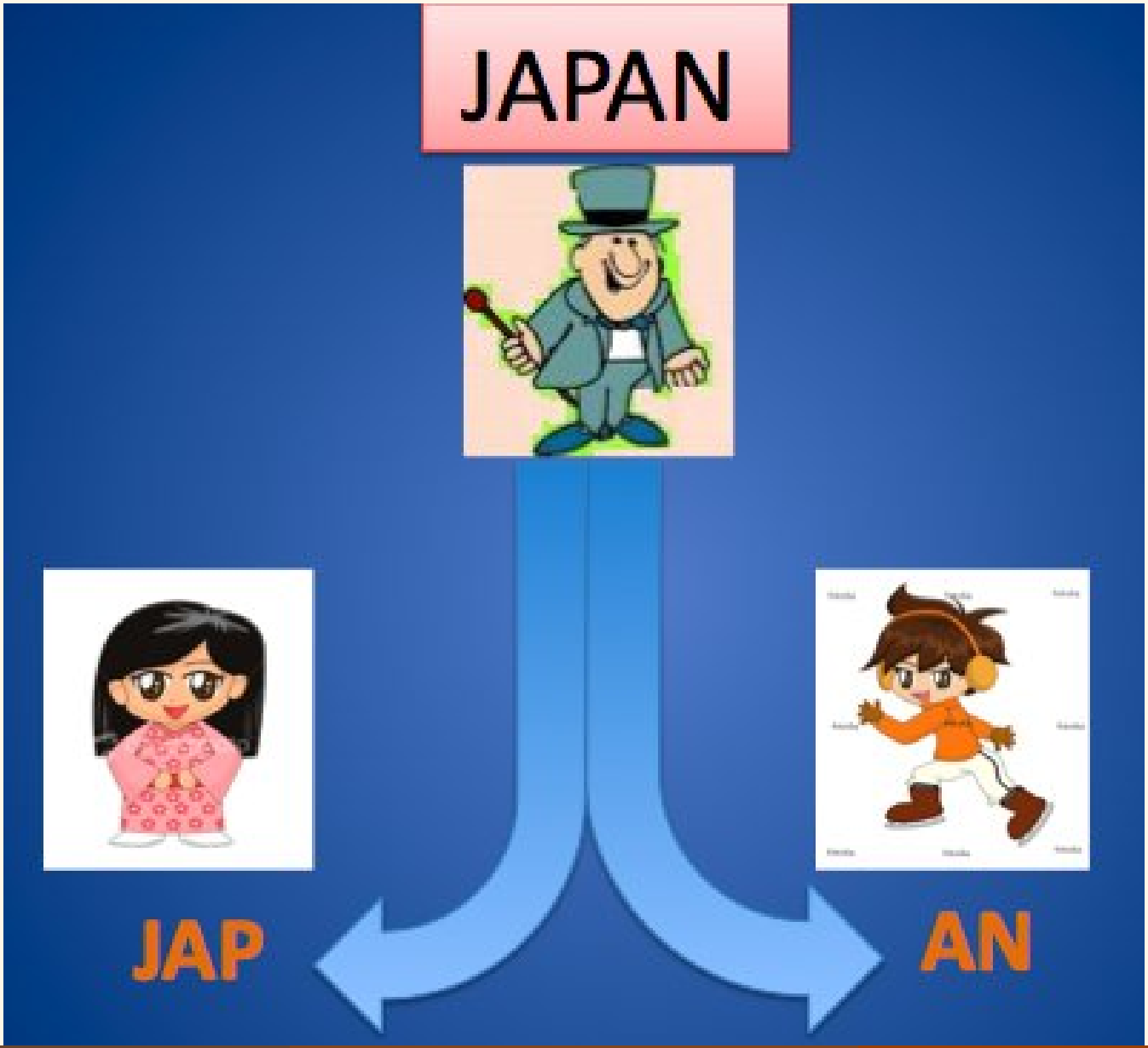
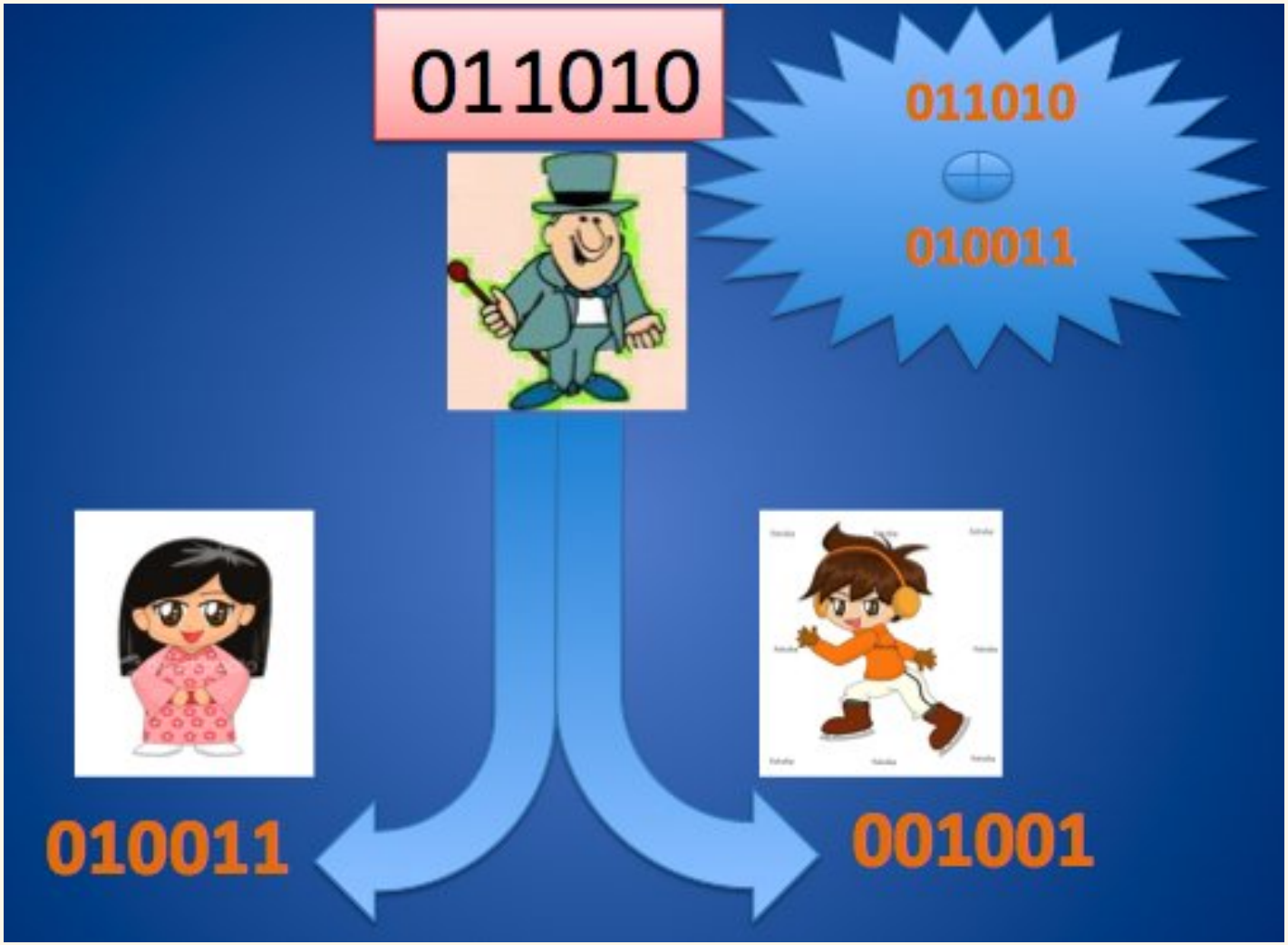
Multi-Level Authentications



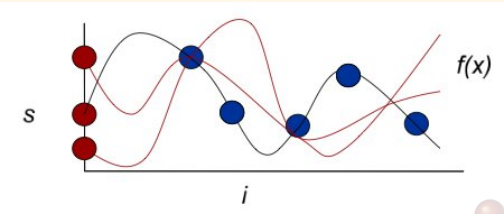
Simple Way!



Perfectly Secure (2,2)-SSS



Shamir's (k, n)-Secret Sharing Scheme



- It takes two points to define a straight line, three points to fully define a quadratic, four points to define a cubic, and so on.
- One can fit a unique polynomial of degree $(k-1)$ to any set of k points that lie on the polynomial.

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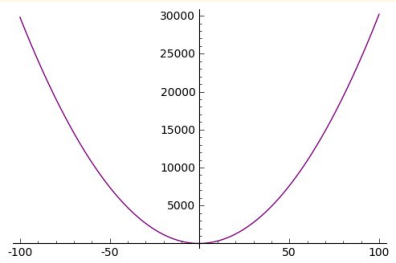

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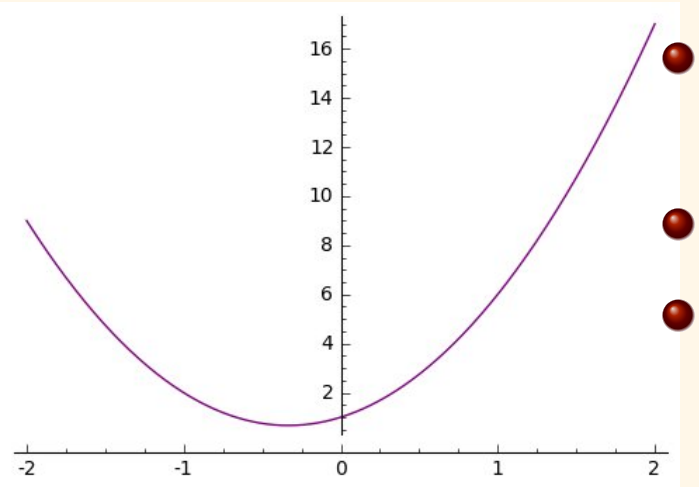
Shamir's (3, 4) threshold scheme



- Let $\mathcal{P}=\{P_1, P_2, P_3, P_4\}$ be a set of 4 participants.
- The key set, $\mathcal{K}=\mathbb{Z}_p$, where $p = 5$ is a prime & $p > n$. Let the secret be 1.
- The set of all possible shares, $\mathcal{S}=\mathbb{Z}_5$.
- The dealer constructs a random polynomial $f(x) \in \mathbb{Z}_5[x]$ of degree $t - 1 = 3 - 1 = 2$, in which the constant term is the secret $K = 1$.

$$f(x) = 1 + 2x + 3x^2$$

Shamir's (3, 4) threshold scheme

- Every participant P_i obtains a point (x_i, y_i) on this polynomial, where $y_i = f(x_i)$ and distinct $x_i \in \mathbb{Z}_p$.
- P_1 gets (1,a(1)=6=1), (P_2) gets (2,2), P_3 gets $(3, 4)$ and P_4 gets (4,2).

Recovery of Secret

- Suppose a subset B of $t = 3$ participants wants to recollect the secret.
- Let the participants P_1, P_2, P_3 want to determine $K = 1$.
- They know that $1 = f(1)$, $2 = f(2)$ and $4 = f(3)$.
- They will assume the form of the secret polynomial as $y = f(x) = a_0 + a_1 x + a_2 x^2$, where a_0, a_1 and a_2 are unknown and belong to $\mathbb{Z}$.
- Thus, these participants can obtain 3 linear equations in the 3 unknowns a_0, a_1, a_2.

Shamir's (t, n) threshold scheme

- $$\begin{bmatrix} 1 & 1 & 1^2 \\ 1 & 2 & 2^2 \\ 1 & 3 & 3^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} f(1) \\ f(2) \\ f(3) \end{bmatrix}$$
  - Now, the coefficient matrix A is the so called Vandermonde's matrix.

$$detA = \prod_{1 \le j < k \le t} (x_{i_k} - x_{i_j}) \mod p = (1-2)(2-3)(3-1) = 4*4*2 = 2 \neq 0$$

Thus multiplying both sides by the inverse of A, we can find the $a_0 = 1$.

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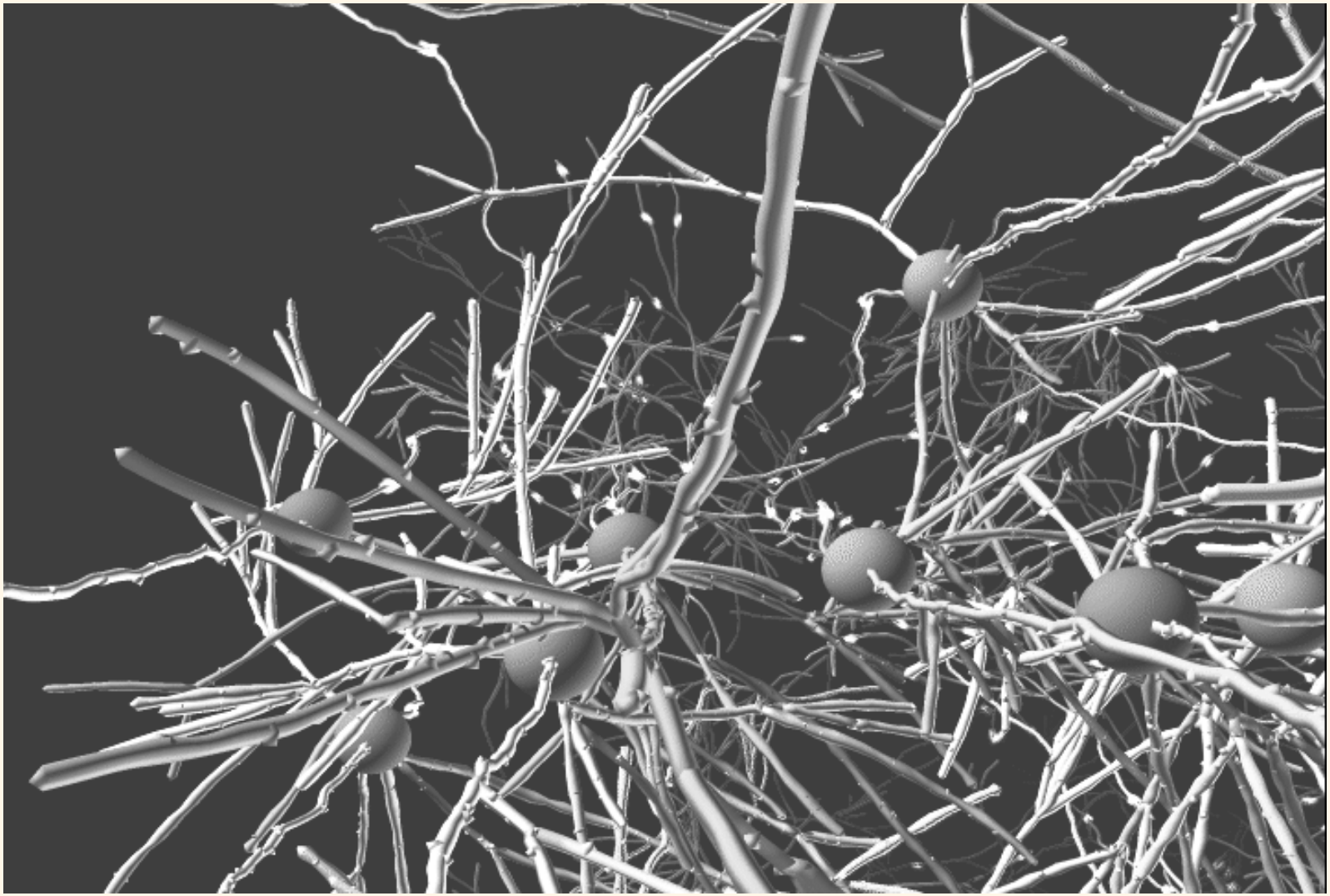
When the Image is a Secret: pgm Extension



Text Version of the Image: pgm Extension

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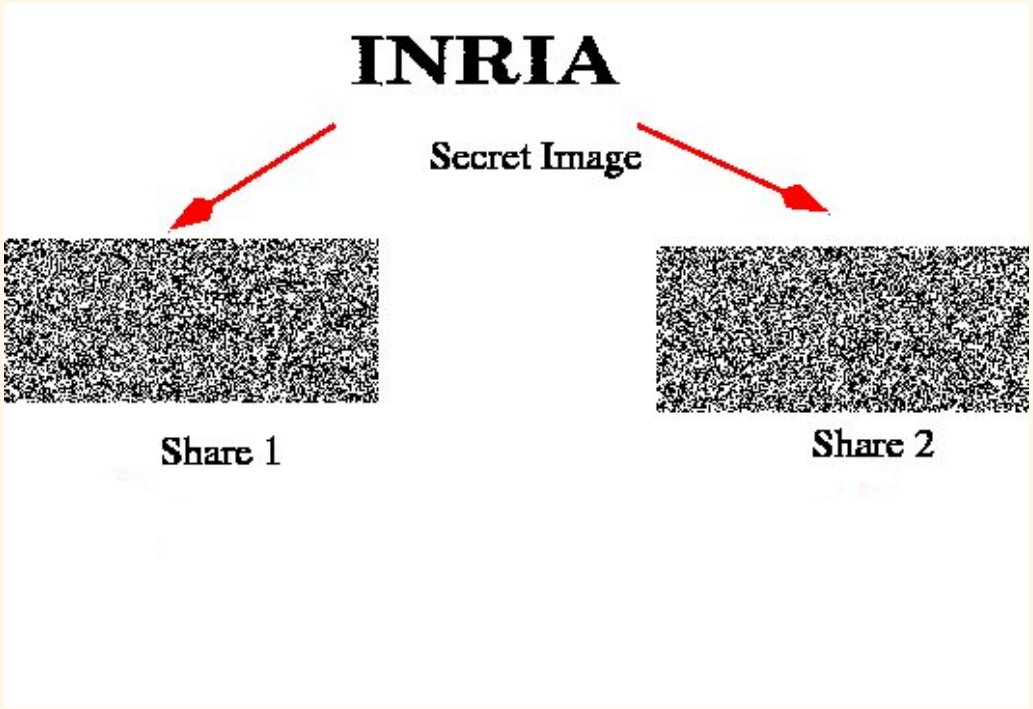
Example of $(2, 2)$-Visual Cryptographic Scheme (VCS)



'x'

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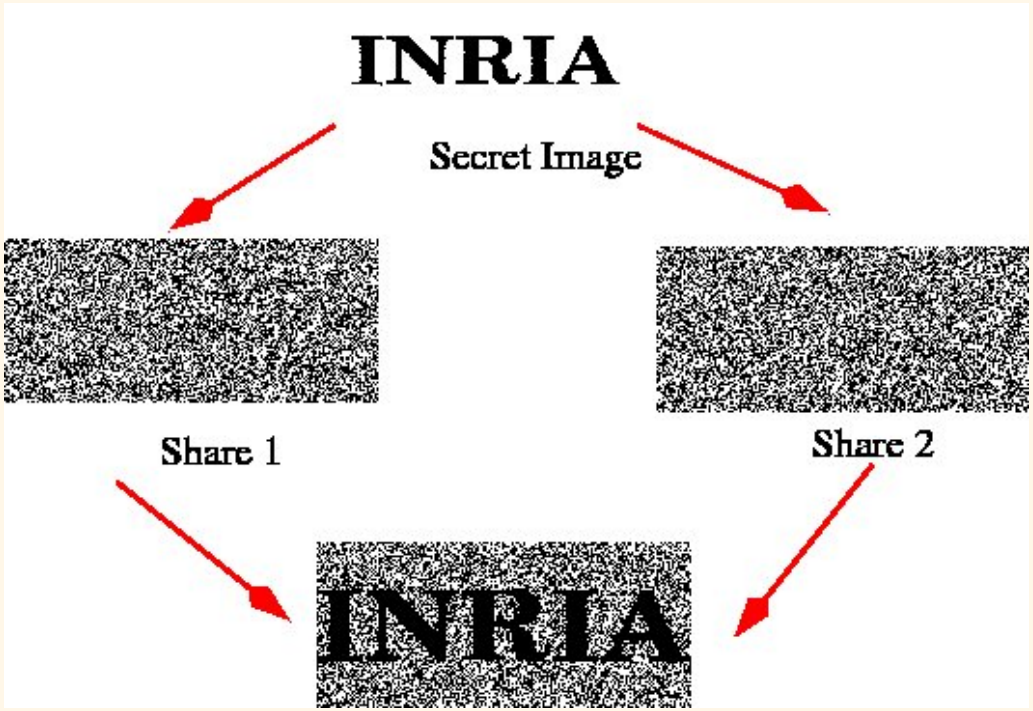
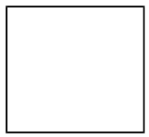


'x'

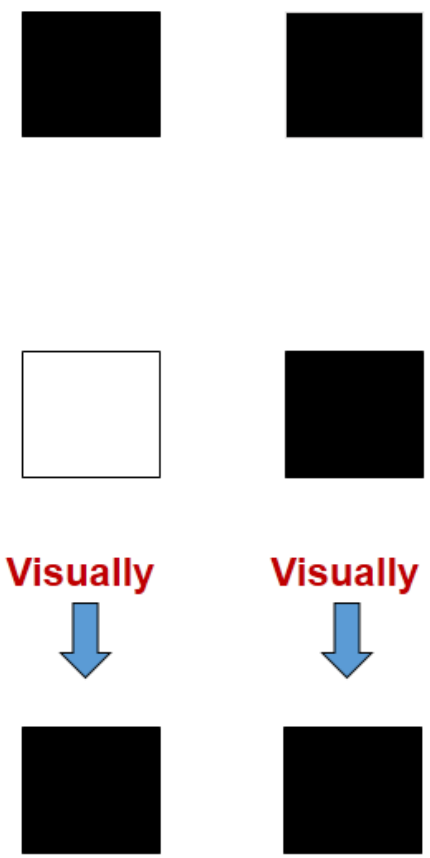
Superimposition of pixels



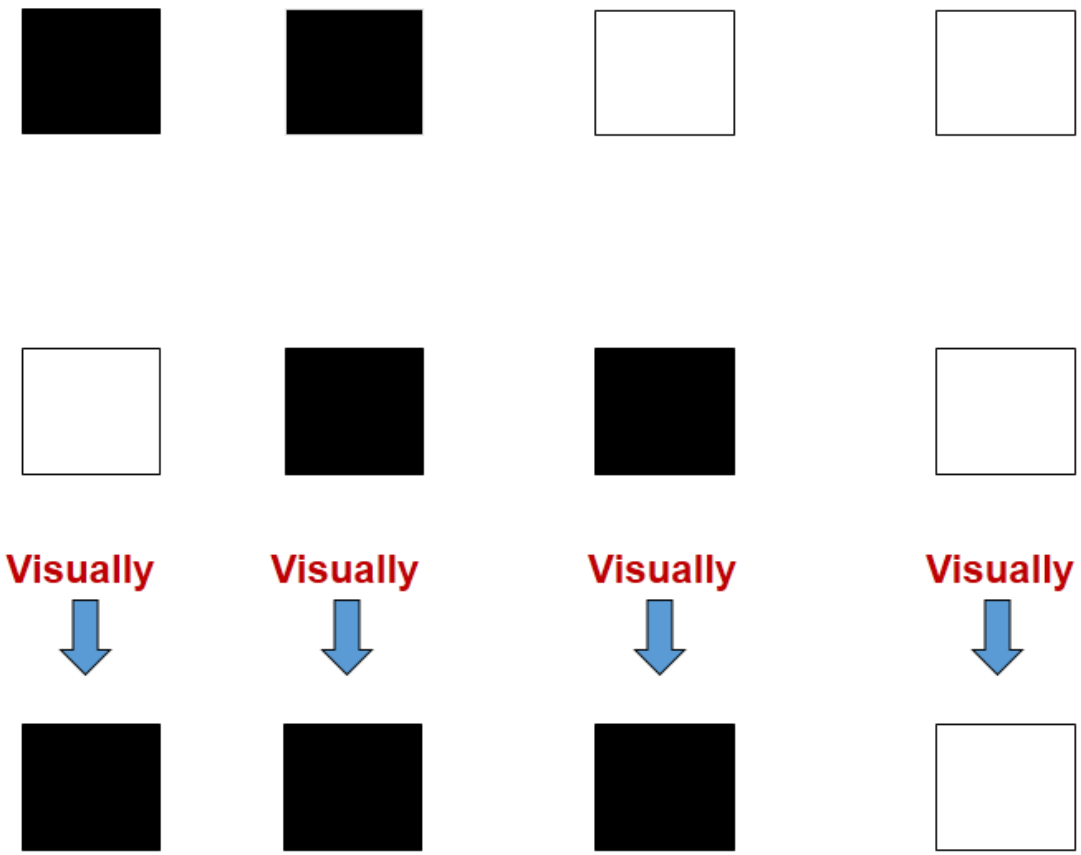
Superimposition of pixels

Visually

Superimposition of pixels



Superimposition of pixels



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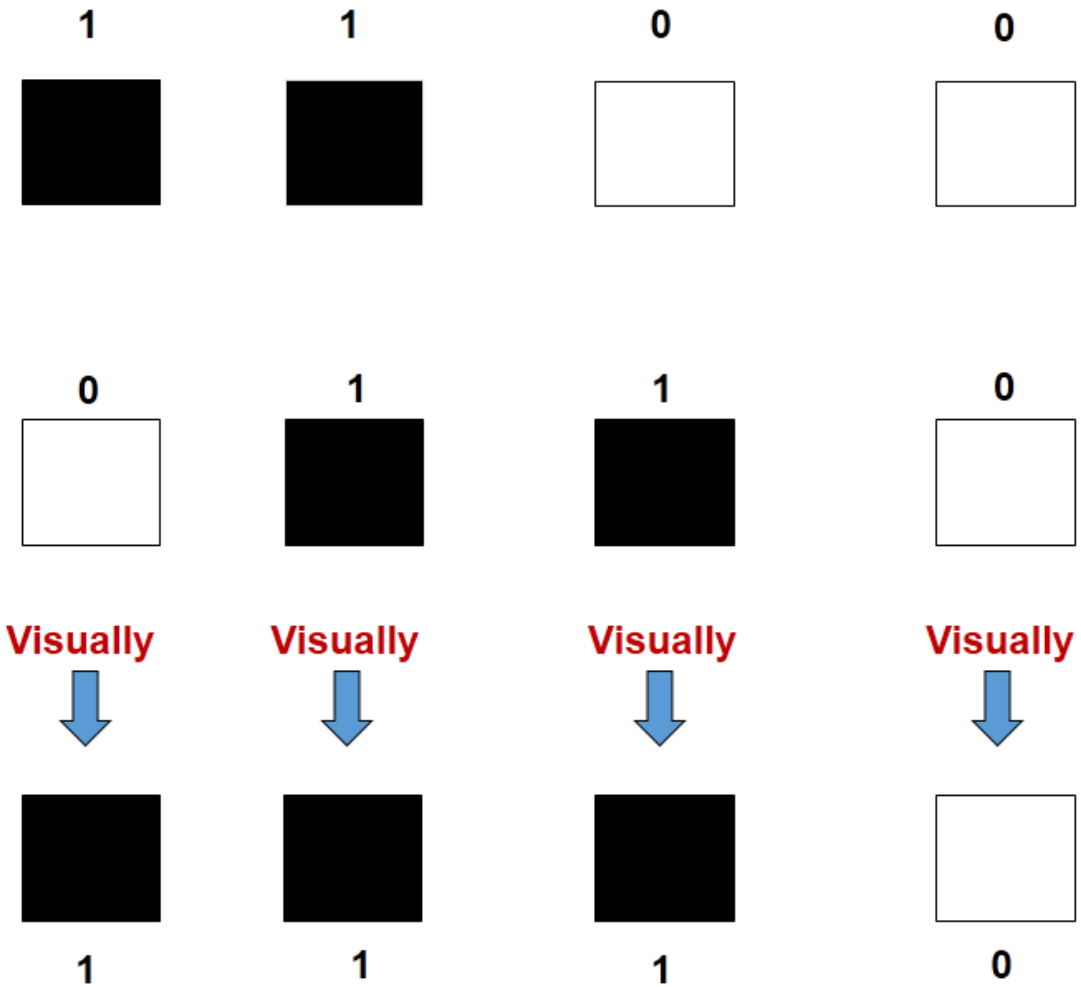

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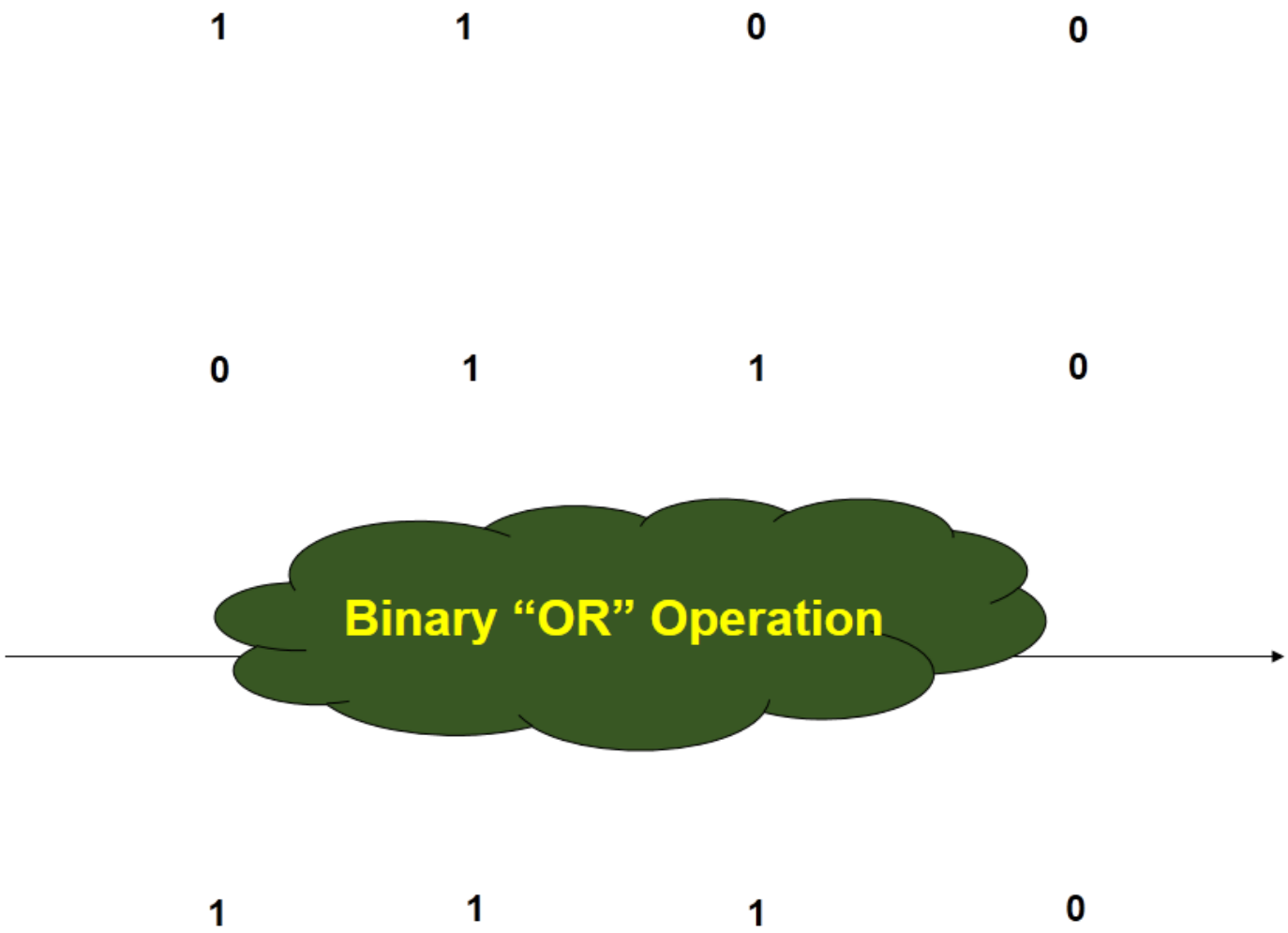

$(2, 2)$-VCS

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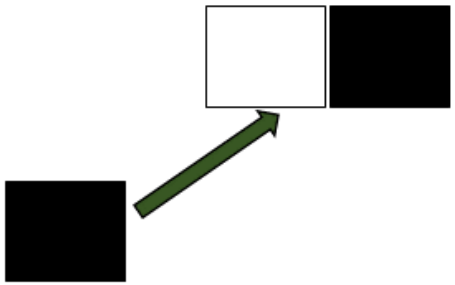

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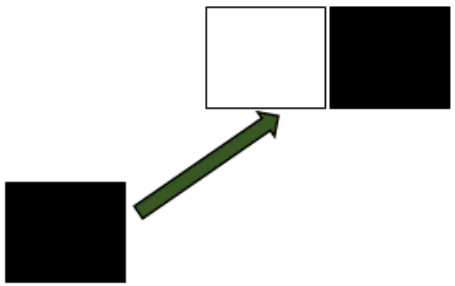

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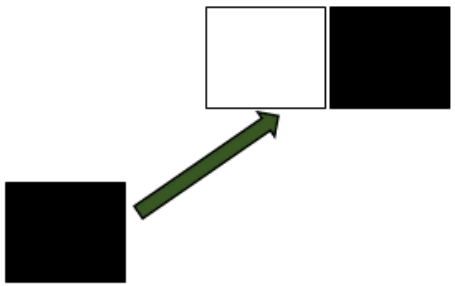

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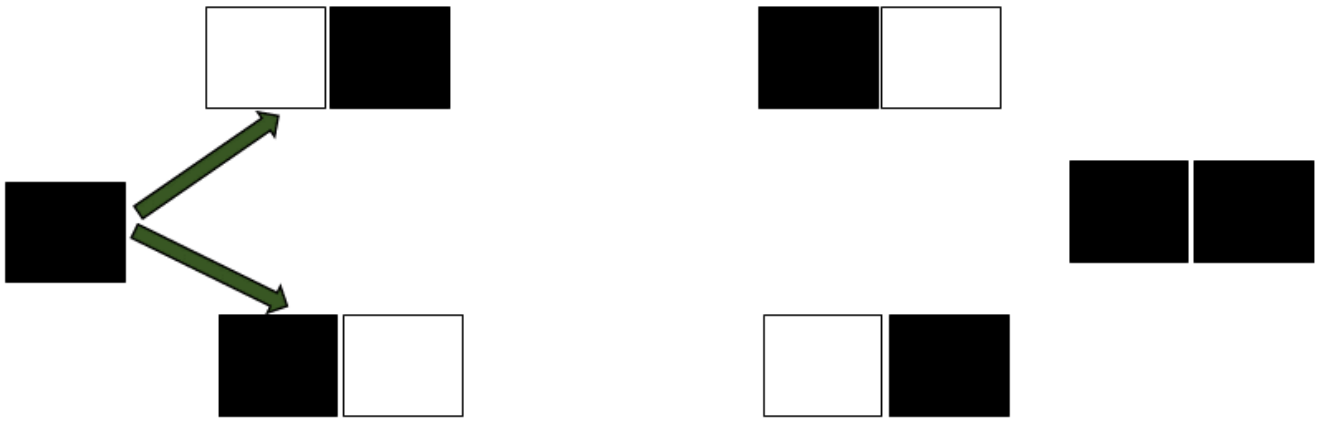

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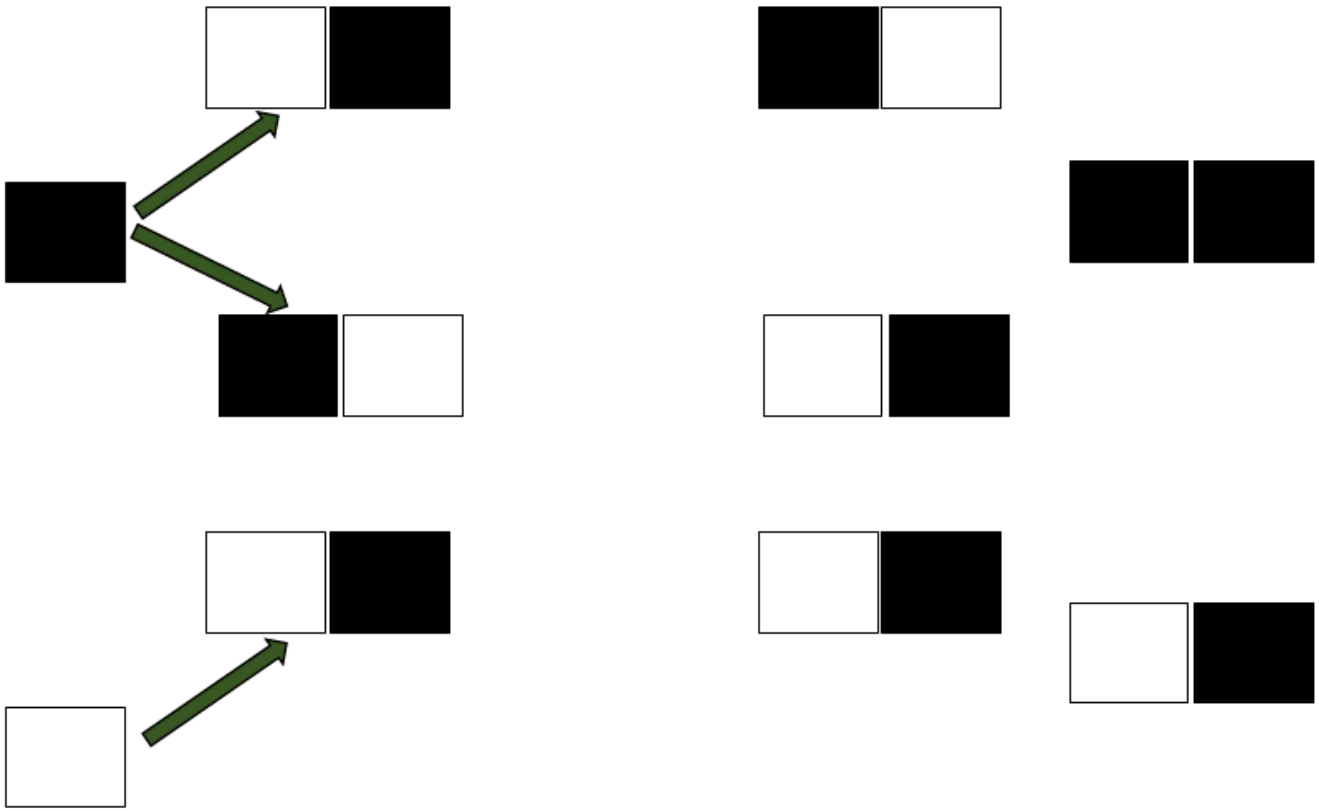
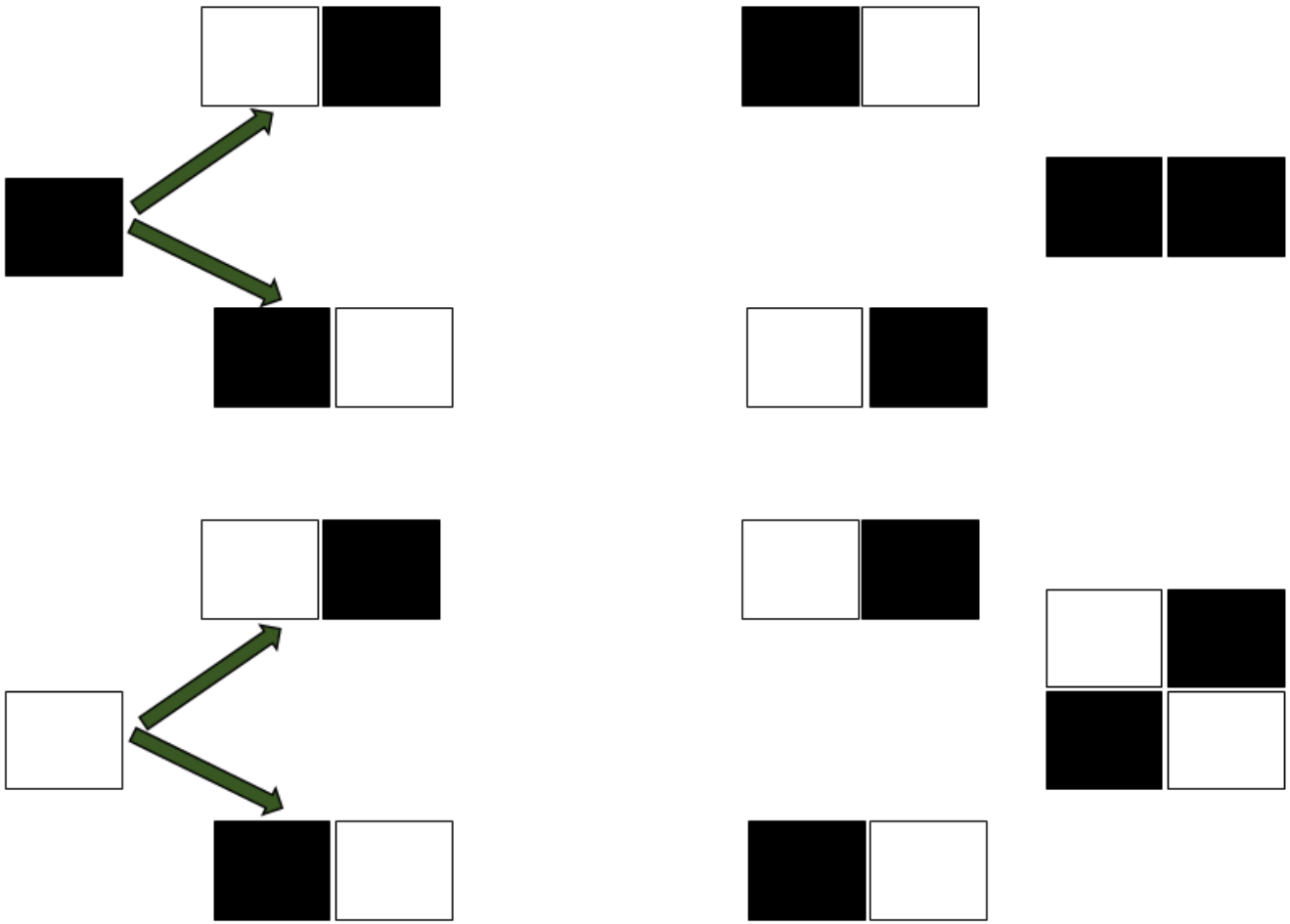
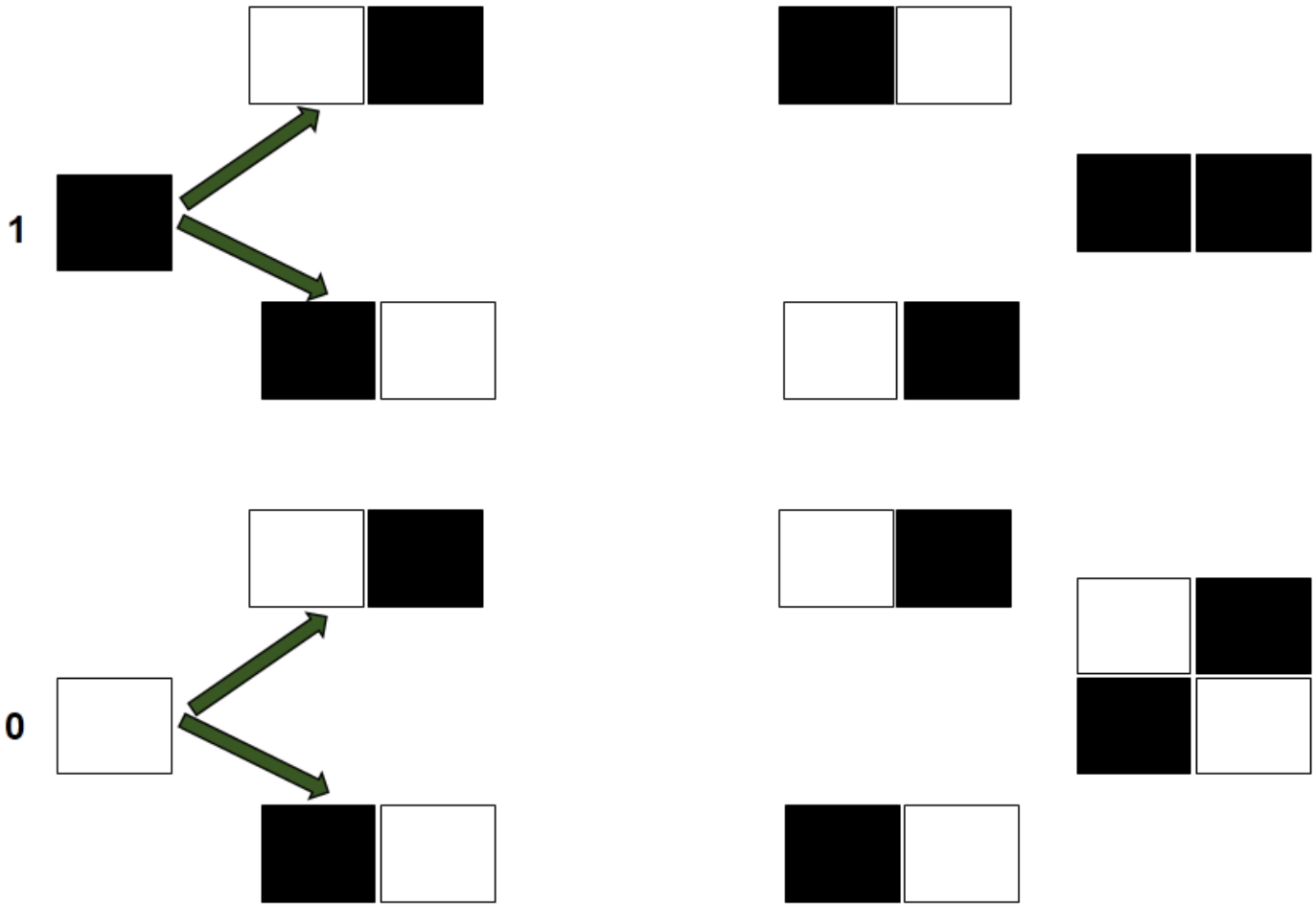


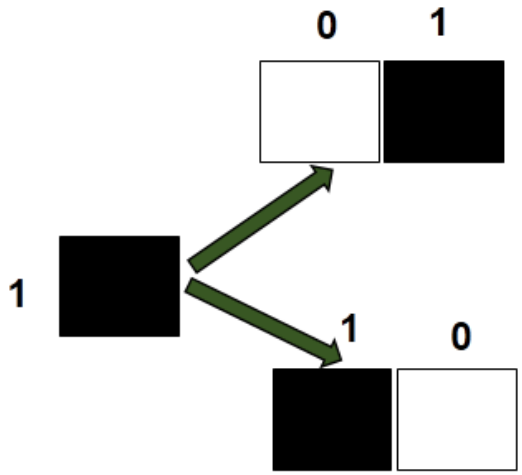
(2, 2)-VCS

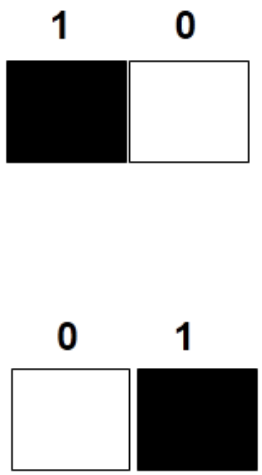


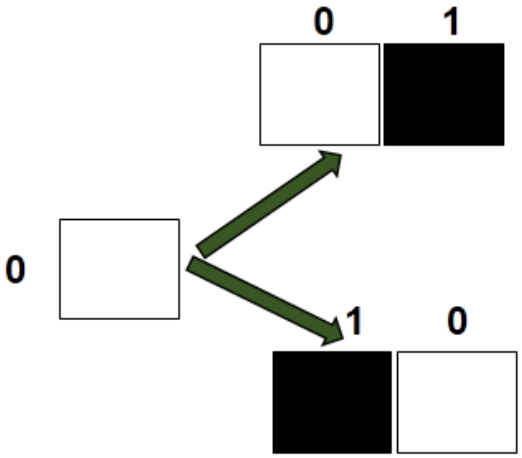
(2, 2)-VCS

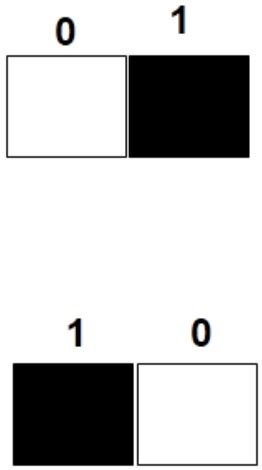


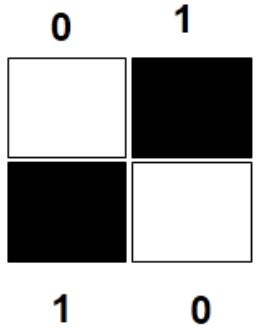
(2, 2)-VCS



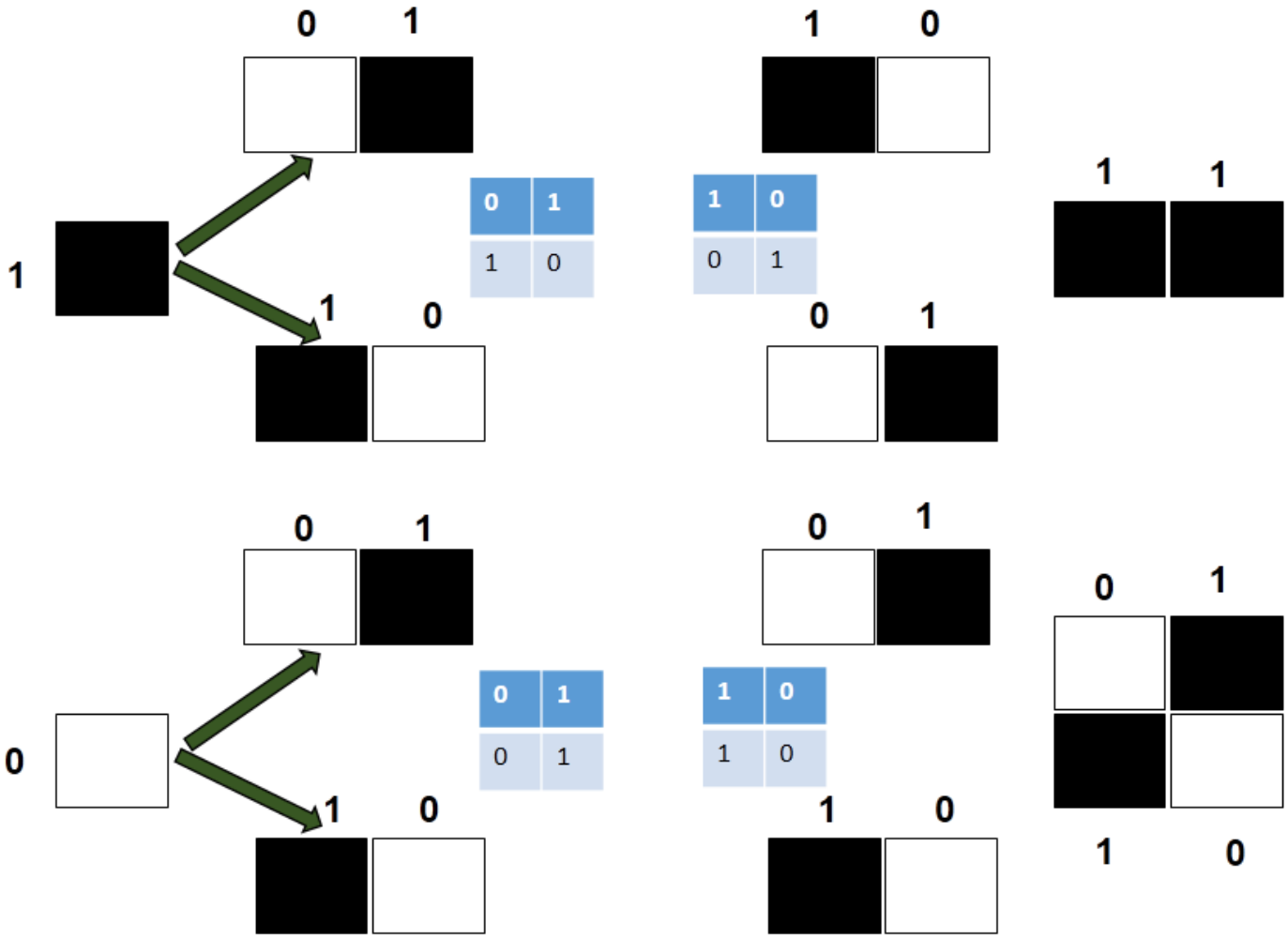




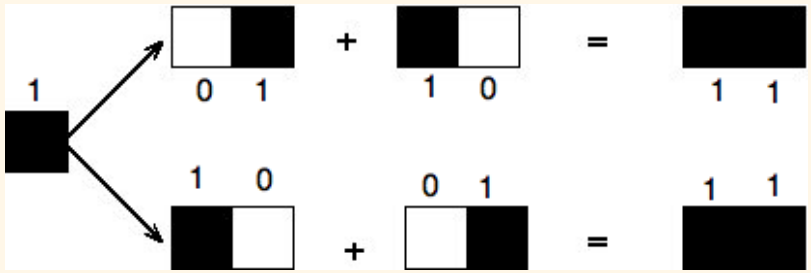




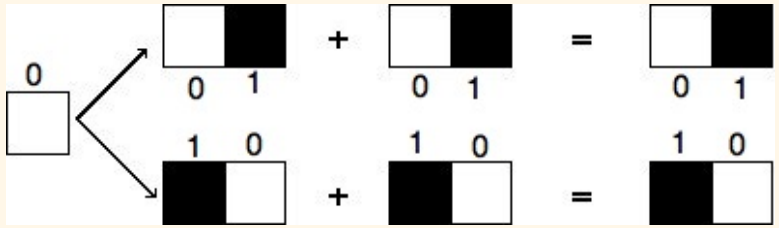
(2, 2)-VCS



(2, 2)-VCS



For Black pixel



For White pixel

$$S^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } S^0 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

Relative contrast

Let us consider a $(2, n)$-VCS on a set $\mathcal{P} = \{1, 2, \ldots, n\}$ of n participants with basis matrices S^0 and S^1 and having *pixel expansion m* which is the number of columns of the basis matrices. Then the *relative contrast* for the participants corresponding to X, $X \subseteq \mathcal{P}$, is denoted by $\alpha_X(m)$ and is defined as

$$\frac{w(S^1_X) - w(S^0_X)}{m}.$$

Few Significant Contributions on VCS

1. Sabyasachi Dutta, Avishek Adhikari, Sushmita Ruj: Maximal contrast color visual secret sharing schemes. **Des. Codes Cryptogr. 87(7): 1699-1711 (2019)**.
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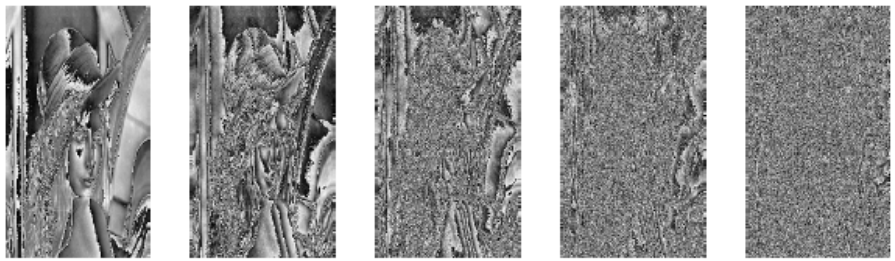
3. Avishek Adhikari: Linear algebraic techniques to construct monochrome visual cryptographic schemes for general access structure and its applications to color images. **Des. Codes Cryptogr.** 73(3): 865-895 (2014).

(k, n)-Image Sharing Scheme (Thien and Lin, '02)

Secret



S_1 S_2 S_3 S_4 S_5

$S_{\{1,2\}}$ $S_{\{1,2,3\}}$ $S_{\{1,2,3,4\}}$

Polynomial Based SIS: Same Share Size

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Polynomial Based SIS: Reduced Share Size

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A New Lossless Secret Color Image Sharing Scheme with Small Shadow Size.
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Sardar, M. K. and Adhikari, A. (2020b).
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Thien, C.-C. and Lin, J.-C. (2002).
Secret Image Sharing.
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Li, P., Liu, Z., and Yang, C. (2018).
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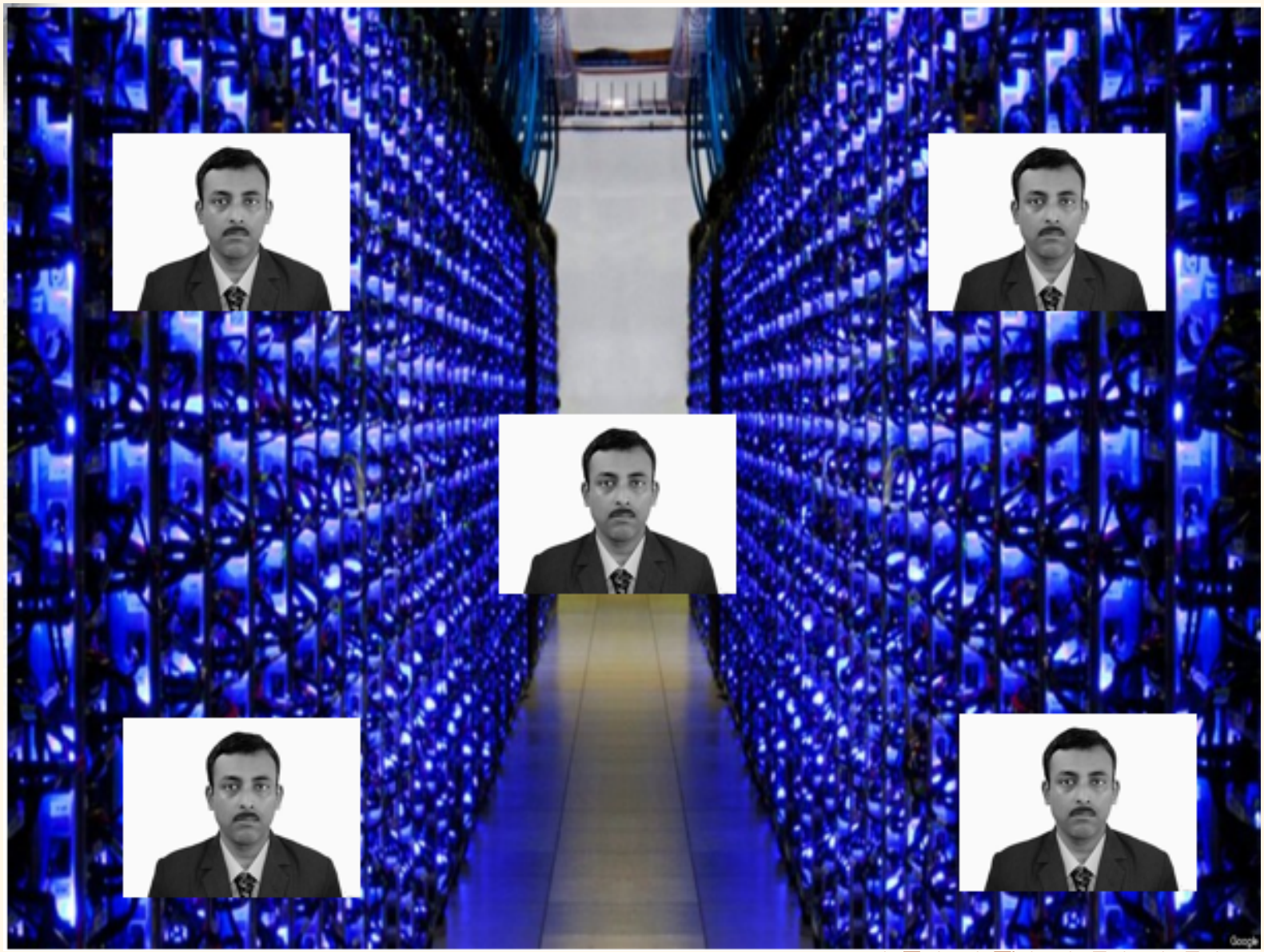
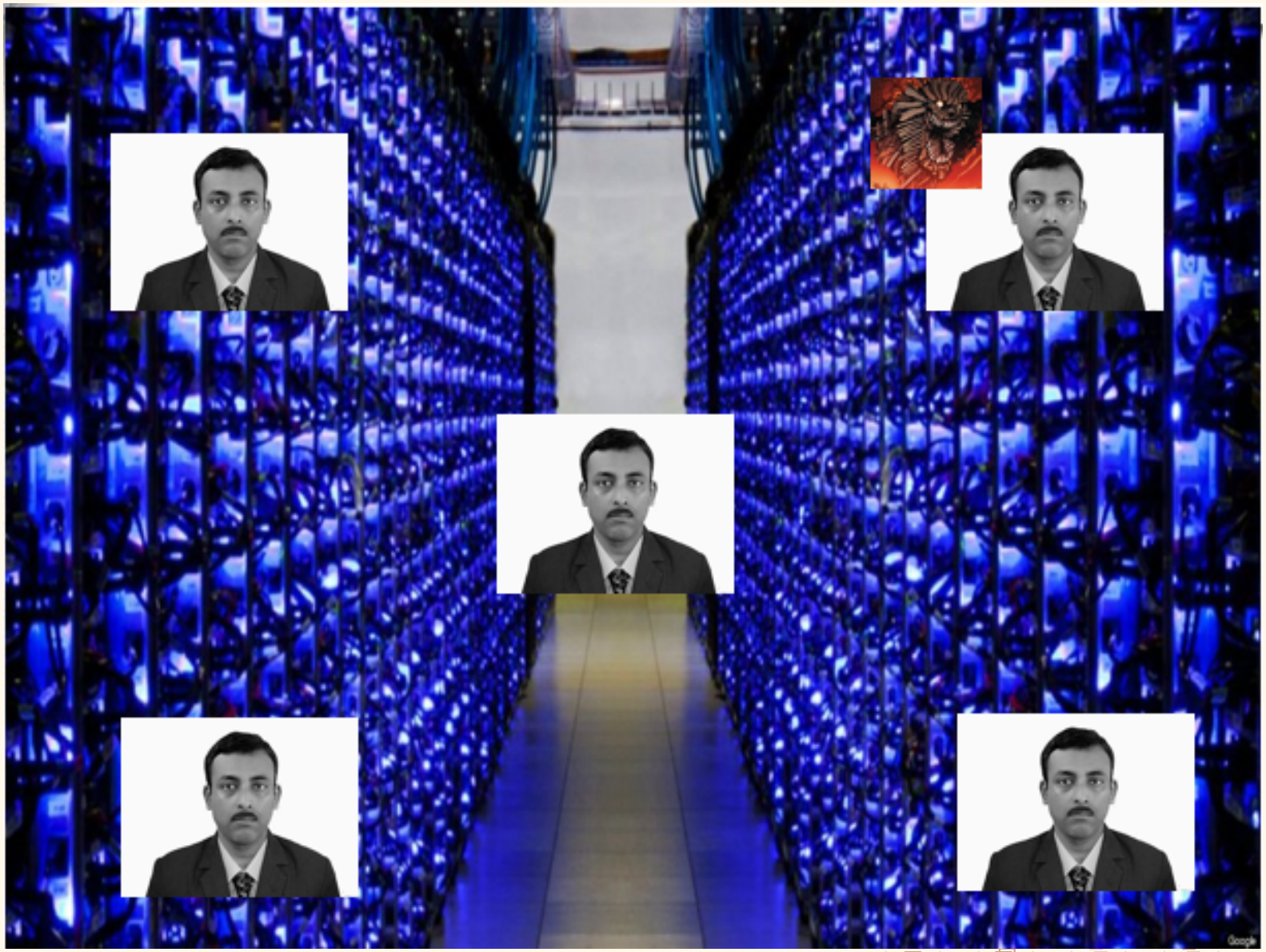
Attacks on (k, n)-SISS

Attacks on (k, n)-SISS

Attacks on (k, n)-SISS



Attacks on (k, n)-SISS



Attacks on (k, n)-SISS

Attacks on (k, n)-SISS

Attacks on (k, n)-SISS

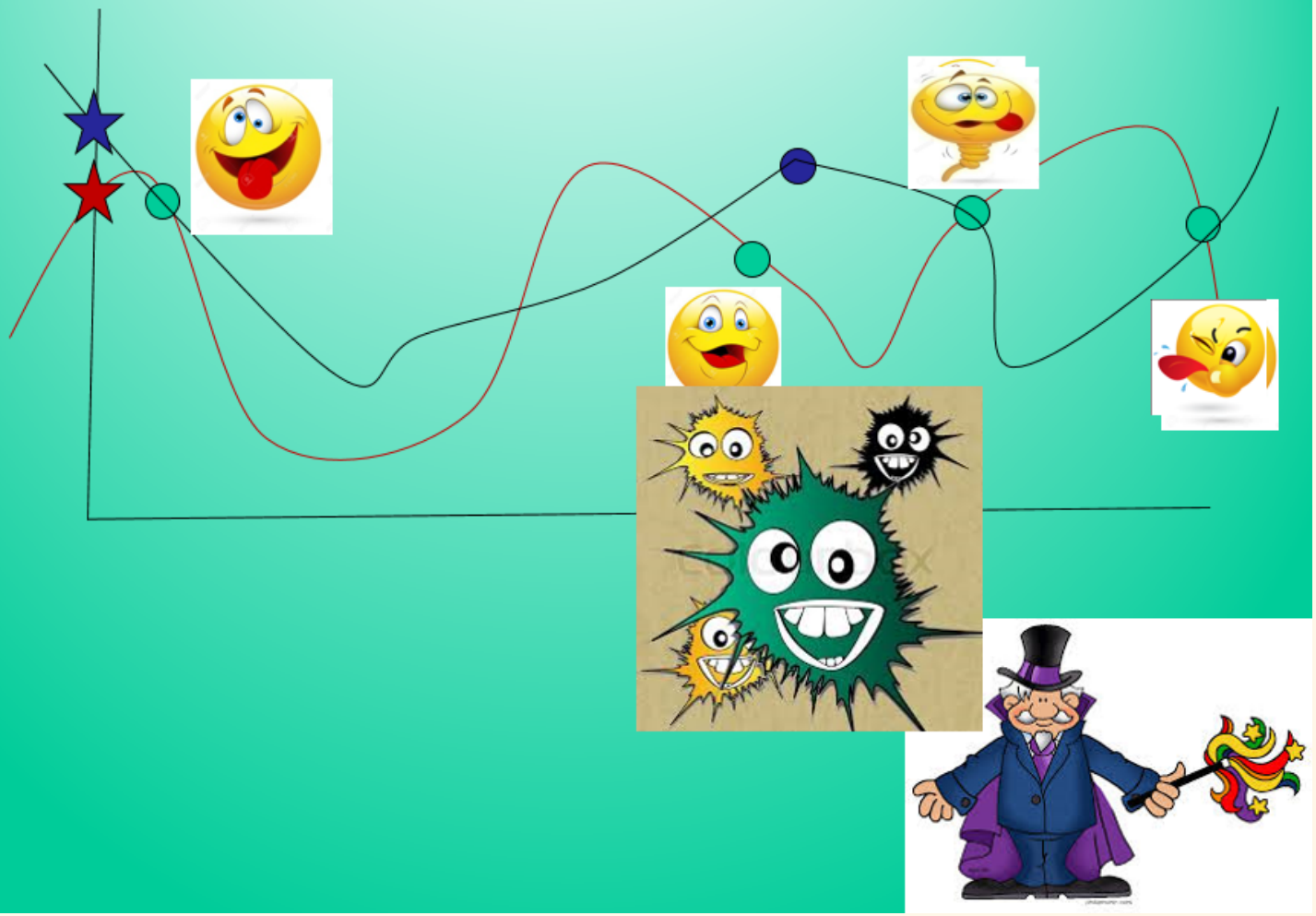
Attacks on (k, n)-SISS

Attacks on (k, n)-SISS

Attacks on (k, n)-SISS

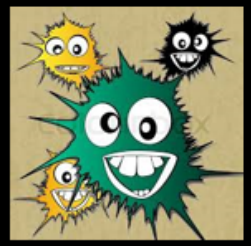
Attacks on (k, n)-SISS

Twist in the Story!



Characterization of Adversarial Activities

Adversary



Honest but Curious

Malicious

Non-Rushing

Rushing

Share Generation Algo for SIS using SSSS

Algorithm 1: Share Generation of Shamir's (k, n) - SIS scheme Using Public ID

Input: A secret image S of size $M \times N$, i.e., $S[i, j], 0 \leq i \leq M-1, 0 \leq j \leq N-1$, and three integers k and n, such that $1 < k \leq n$.

Output: n shadow images $S_1, S_2, \ldots, S_n$ with a proxy share S'_t each of size $M \times N$.

Assume a bijective mapping $\phi : \mathbb{Z}_{2^8} \rightarrow \mathrm{GF}(2^8)$.
Choose an irreducible polynomial $q(x) \in F_p[x]$ of degree 8.
for $(i = 0; i < M; i++)$ **do**
  for $(j = 0; j < N; j++)$ **do**
    Assign the coefficients $a_0, a_1, \ldots, a_{k-1} \in \mathrm{GF}(2^8)$ as follows:
    $a_0 \leftarrow \phi(S[i, j])$
    for $(q = 1; q < k; q++)$ **do**
      Generate $a_q \xleftarrow{\$} \mathrm{GF}(2^8)$ uniformly at random.
    end
    for $(r = 1; r \leq n; r++)$ **do**
      $f(\alpha_r) \leftarrow (a_0 + a_1 \cdot \alpha_r + a_2 \cdot \alpha_r^2 + \cdots + a_{k-1} \cdot \alpha_r^{k-1}) \pmod{q(x)}$
      $S_r[i, j] = \phi^{-1}(f(\alpha_r))$
    end
  end
end

Attack on SSSS

Algorithm 4: An Attack on Shamir's (k, n) - CSIS scheme Using Public ID

Input: A secret image S of size $M \times N$, i.e., $S[i, j], 0 \leq i \leq M-1, 0 \leq j \leq N-1$, and three integers k and n, such that $1 < k \leq n$.

Output: n shadow images $S_1, S_2, \ldots, S_n$ with a proxy share S'_t each of size $M \times N$.

1 **for** $(i = 0; i < M; i++)$ **do**
2 **for** $(j = 0; j < N; j++)$ **do**
3 $g(y) = (y - \alpha_1)(y - \alpha_2) \cdots (y - \alpha_{t-1})(y - \alpha_{t+1}) \ldots (y - \alpha_k) \pmod{q(x)}$
4 $S'_t[i, j] = \phi^{-1}(g(\alpha_t) + f(\alpha_t))$
5 **end**
6 **end**

Share Generation Algo for Meaningful Images

Algorithm 5: Share generation of Shamir's (k, n)-SIS scheme with Private ID.

Input: A secret image S and a meaningful proxy image P each of of size $M \times N$, i.e., $S[i, j], 0 \leq i \leq M-1, 0 \leq j \leq N-1$, and three integers k and n, such that $1 < k \leq n$.

Output: n shadow images $S_1, S_2, \ldots, S_n$ with a proxy share S'_t each of size $M \times N$.

Choose an irreducible polynomial $q(x) \in F_p[x]$ of degree 8.
for $(i = 0; i < M; i++)$ **do**
  for $(j = 0; j < N; j++)$ **do**
    Assign the coefficients $a_0, a_1, \ldots, a_{k-1} \in \mathrm{GF}(2^8)$ as follows:
    $a_0 \leftarrow \phi(S[i, j])$
    for $(q = 1; q < k; q++)$ **do**
      randomly choose $a_q \in \mathrm{GF}(2^8)$
    end
    for $(r = 1; r \leq n; r++)$ **do**
      $f(\alpha_r) \leftarrow (a_0 + a_1 \cdot \alpha_r + a_2 \cdot \alpha_r^2 + \cdots + a_{k-1} \cdot \alpha_r^{k-1}) \pmod{q(x)}$
    end
    $$g(y) = \frac{(-1)^{k-1}}{\prod_{v=1, v \neq t}^{k} \phi(\alpha_v)} (\phi(P[i, j]) - \phi(S[i, j]))\,(y - \alpha_1)(y - \alpha_1) \cdots (y - \alpha_{t-1})(y - \alpha_{t+1}) \ldots (y - \alpha_k) \pmod{q(x)}$$
    for $(r = 1; r \leq n; r++)$ **do**
      $S_r[i, j] = \phi^{-1}(f(\alpha_r))$
    end
    $S'_t[i, j] = \phi^{-1}(g(\alpha_t) + f(\alpha_t))$
  end
end

Recovery Algo for Meaningful Image

Algorithm 6: Recovery phase of Shamir's (k, n)-SIS scheme with Private ID

Input: Suppose k participants $P_1, \ldots, P_t, \ldots, P_k$ submit their shares $S_1, \ldots, S'_t, \ldots, S_k$ each of size $M \times N$.

Output: The meaningful proxy image P of size $M \times N$.

Choose an irreducible polynomial $q(x) \in \mathrm{F}_p[x]$ of degree 8.

for $(i = 0; i < M; i++)$ **do**

  for $(j = 0; j < N; j++)$ **do**

    Lagrange's interpolation formula as follows:

$$h(0) = \sum_{r=1}^{k} \phi(S_{\alpha_r}[i,j]) \prod_{t=1, t\neq r}^{k} \frac{-\alpha_t}{\alpha_r - \alpha_t} \pmod{Q(x)}$$

    $P[i,j] = \phi^{-1}(h(0))$

  end

end

Example to Show how the attack works

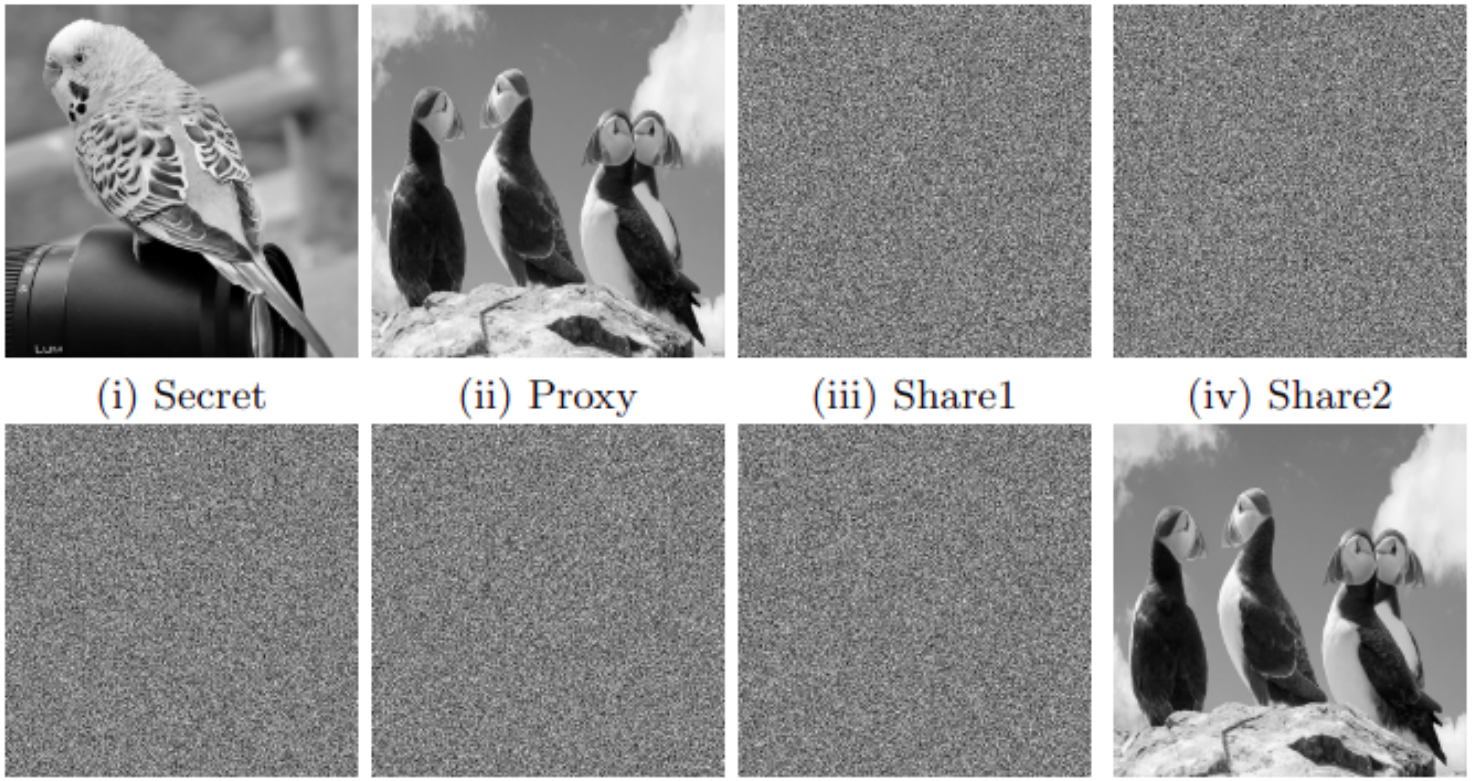


Fig. 1. Experimental results of our (3, 4)-CDSIS scheme. (i) Original secret image, (ii) Proxy image (iii)-(vii) shadow images, (viii) random proxy share generate by participant and (ix) secret recovered by 123 after cheating.

Acknowledgment and Bibliography

- Thankful to **Google** for helping in finding important information and necessary images.

- I am thankful to my PhD Scholars Jyotirmoy Pramanik, Md Kutubuddin Sardar and my student Anisha Dutta and my Masters Students for helping me in making the slide.
- Thanks to **Department of Information Technology, Government of India, DRDO and WESEE (Ministry of Defense), DST-SERB, DST-MATRICS, NBHM, JSPS and JST, Government of Japan** for providing me financial support towards my research.

Bioliography

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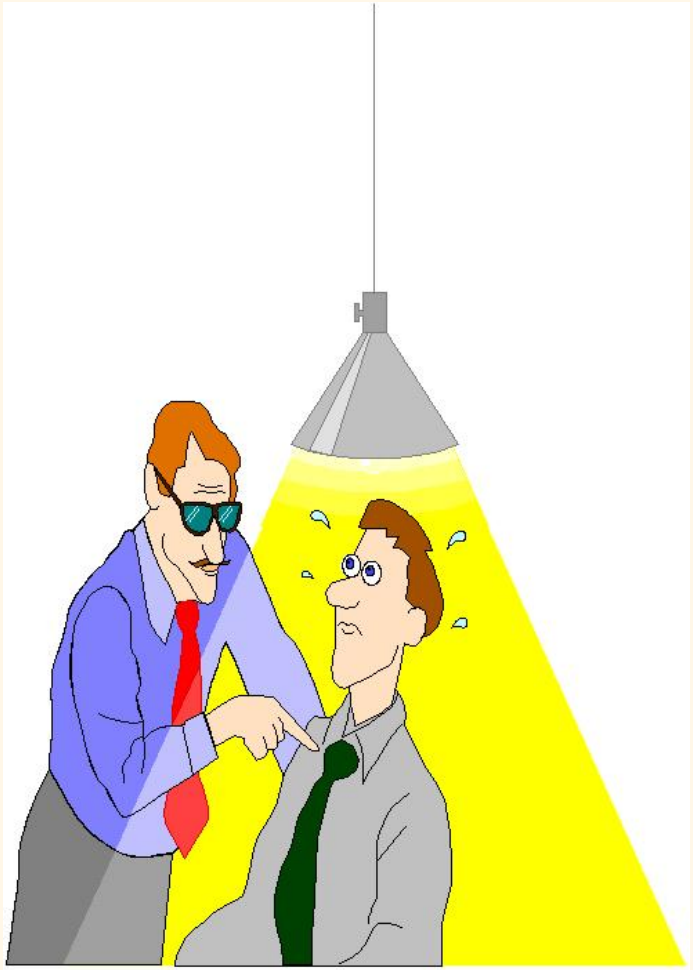
Questions or Comments!



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Thank
You