

Propositional Logic

\mathcal{P} = set of atomic propositions / atoms
atomic formulas

$\mathcal{P} = \{ p_0, p_1, p_2, \dots \}$

Well-formed formulas / propositional

Connectives : \neg (Negation)

\vee (OR / disjunction)

\wedge (AND / conjunction)

\Rightarrow (Implication)

\Leftrightarrow (double implication)

The set of WFF is the smallest set ~~consis~~ containing \mathcal{P} and satisfying the following.

• If $\alpha \in \text{WFF}$ then $(\neg \alpha) \in \text{WFF}$

• If $\alpha, \beta \in \text{WFF}$, then so are $(\alpha \wedge \beta), (\alpha \vee \beta) \in \text{WFF}$

• If α, β are in WFF then so are $(\alpha \rightarrow \beta), (\alpha \leftrightarrow \beta)$

Ex Show That WFF is countable

Defⁿ A valuation / truth assignment
is a fn

$$v : P \rightarrow \{ \text{TRUE}, \text{FALSE} \} = \{ 1, 0 \}$$

We extend v to WFF as
follows:

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We extend v to WFF as
follows:

(i) If $\alpha = p$ an atom

$$v(\alpha) = v(p)$$

(ii) If $\alpha = \neg \beta$, then

$$v(\alpha) = \begin{cases} 0 & \text{if } v(\beta) = 1 \\ 1 & \text{if } v(\beta) = 0 \end{cases}$$

(iii) If $\alpha = \beta \vee \gamma$, then

$$v(\alpha) = \begin{cases} 1 & \text{if } v(\beta) = 1 \text{ or } v(\gamma) = 1 \\ 0 & \text{otherwise} \end{cases}$$

(iv) If $\alpha = \beta \wedge \gamma$, then

$$v(\alpha) = \begin{cases} 1 & \text{if } v(\beta) = 1 \text{ and } v(\gamma) = 1 \\ 0 & \text{o.w.} \end{cases}$$

(v) If $\alpha = \beta \rightarrow \gamma$, then

$$v(\alpha) = \begin{cases} 0 & \text{if } v(\beta) = 1 \text{ and } v(\gamma) = 0 \\ 1 & \text{o.w.} \end{cases}$$

(2:1) $\alpha := \beta \leftrightarrow \delta$

$$v(\alpha) = \begin{cases} 1 \\ 0 \end{cases} \quad \text{if } v(\beta) = v(\delta) \quad \text{O.W.}$$

Defⁿ A set $X \subseteq WFF$ is

satisfiable if \exists an assignment v

$$\text{s.t. } v(\alpha) = 1 \quad \forall \alpha \in X$$

If $X = \{\alpha\}$ we say that α is satisfiable

X/α is unsatisfiable if it is not satisfiable

α is called a tautology/valid

In this case we write $\vDash \alpha$

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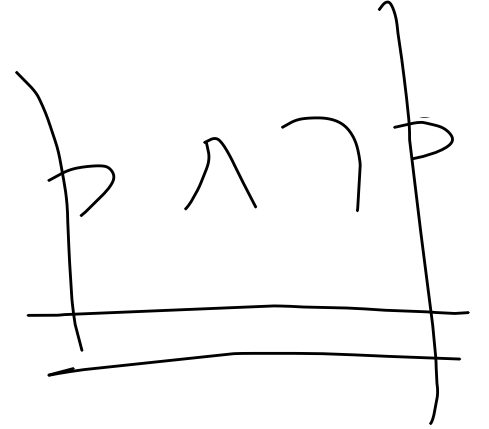
if for any valuation v .

$$v(\alpha) = 1$$

satisfiable

unsatisfiable

Example



tautology

Proposition α is a tautology iff

$\neg \alpha$ is unsatisfiable.

pf $\exists x$.

$\exists x$

There are $n+1$ pigeons
& n pigeonholes.

For $1 \leq i \leq n+1, 1 \leq j \leq n$

p_{ij} : i th pigeon is occupying the
 j th pigeonhole.

Write down a WFF that
expresses the Peirce's Law Principle.

Logical Equivalence

Let α, β be WFFs.

α is said to be logically equivalent to β ,
if for any valuation ν ,

$\alpha \equiv \beta$

$$\nu(\alpha) = \nu(\beta)$$

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$$\alpha \rightarrow \beta \equiv \neg \alpha \vee \beta$$

$$\alpha \leftrightarrow \beta \equiv (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$$

Ex

$$(i) \quad \alpha \wedge \alpha \equiv \alpha, \quad \alpha \vee \alpha \equiv \alpha$$

$$(ii) \quad \alpha \wedge \beta \equiv \beta \wedge \alpha, \quad \alpha \vee \beta \equiv \beta \vee \alpha$$

$$(iii) \quad (\alpha \wedge (\beta \wedge \gamma)) \equiv (\alpha \wedge \beta) \wedge \gamma$$

$$\alpha \vee (\beta \vee \gamma) \equiv (\alpha \vee \beta) \vee \gamma$$

$$(iv) \quad \alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$$

$$\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$$

$$(2) \neg(\alpha \wedge \beta) \equiv (\neg\alpha) \vee (\neg\beta)$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha) \wedge (\neg\beta)$$

$$(3) \neg\neg\alpha \equiv \alpha.$$