Graph theory (Mondays)

References :- ① Graph theory → Bondy, Murty

② ↳ Bollobas

③ Algebraic graph theory

↳ Godsil, Royle

Expander graphs → good error-correcting codes

Daniel Spielman Venkatesan Guruswami

Anup Rao

Graph theory

$G = (V, E)$

$V \rightarrow$ set of vertices

$E \rightarrow$ set of edges

$\phi : E \longrightarrow V \times V$

$V \times V = \{ (a,a), (a,b) \ldots \}$

$V = \{a, b, c\}$,

$E = \{1, 2, 3, 4\}$

$\phi(1) = (a, b)$

$\phi(2) = (b, c)$, $\phi(3) = (a, a)$

$\phi(4) = (a, c)$

Graph theory

$G = (V, E)$

$V \to$ set of vertices

$E \to$ set of edges

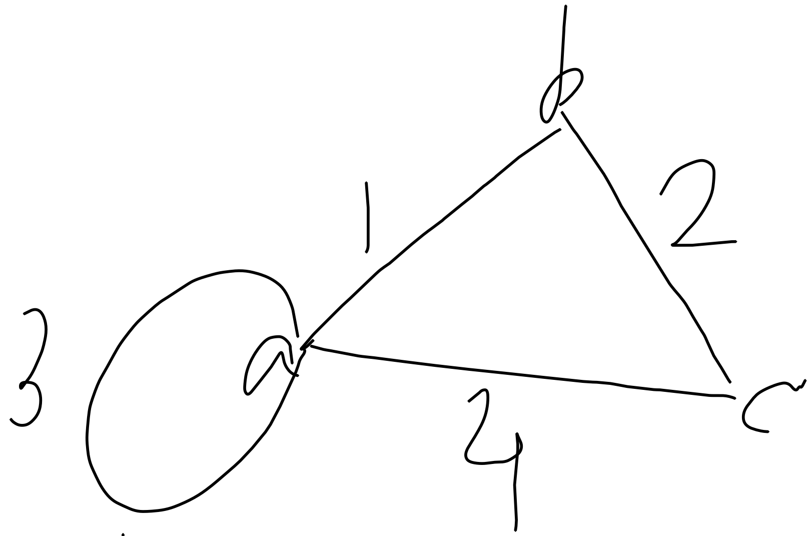
$\phi : E \longrightarrow V \times V$

$V \times V = \{ (a,a), (a,b) \quad - \quad — \}$

$V = \{ a, b, c \}$, $\phi(1) = (a, b)$

$E = \{ 1, 2, 3, 4 \}$ $\phi(2) = (b, c)$, $\phi(3) = (a, a)$

$\phi(4) = (a, c)$



$E \subseteq V \times V$

loop

$\varphi : E \longrightarrow V \times V$

$G : V, E, \phi$

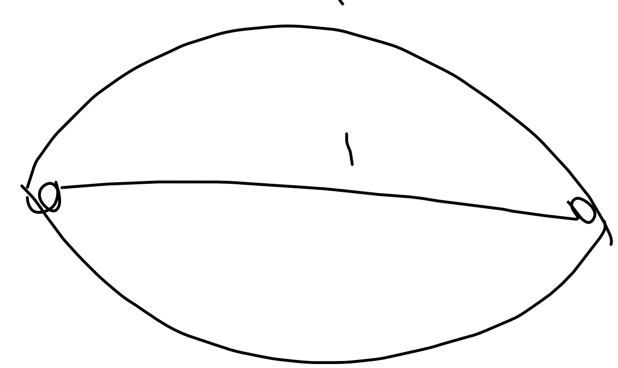
$\phi(e) = \{\phi\}$

$\phi : V \times V \longrightarrow E$

$E \to V \times V$

$1 \to ab)$

$2 \to$



unordered pairs → undirected graphs

$(a, b) = (b, a)$

$(a, b) \neq (b, a)$

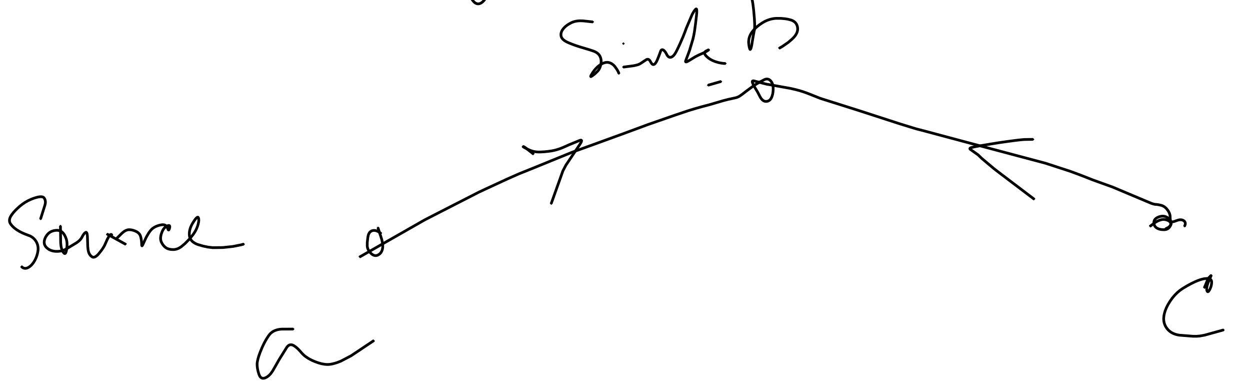
1 → (a, b)

2 → (b, a)

ordered pairs → directed graph

digraphs

directed graph G is called an orientation

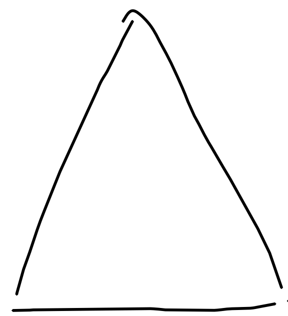
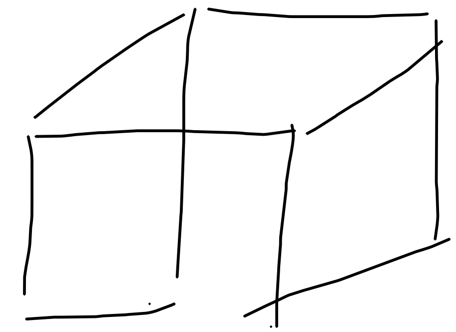


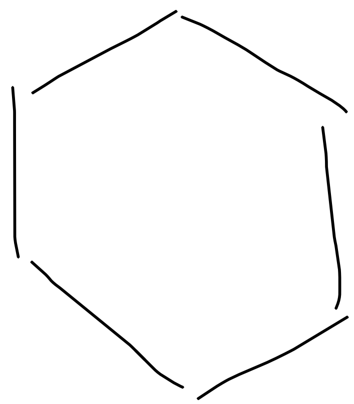
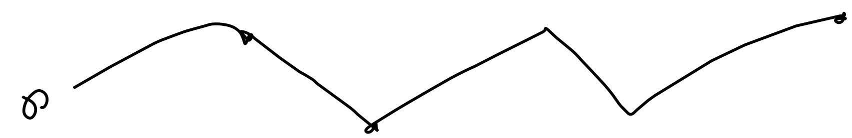
Definitions - 1 degree - no of edges incident on a vertex.

digraph ⟶ indegree → no of edges for which the said vertex is a sink.

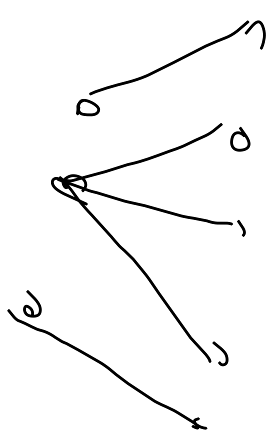
outdegree ⟶ vertex is a source.

Regular → if all vertices have the same degree

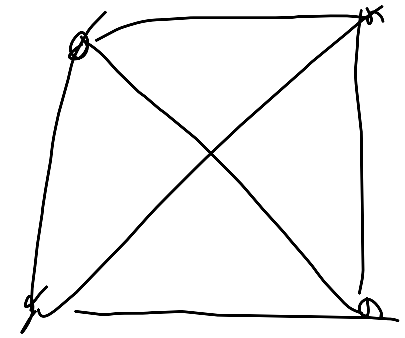
triangle —  cube - 

cycle  Path 

Bipartite. $V = V_1 \sqcup V_2$ such that there are no edges, completely in V_1 or V_2

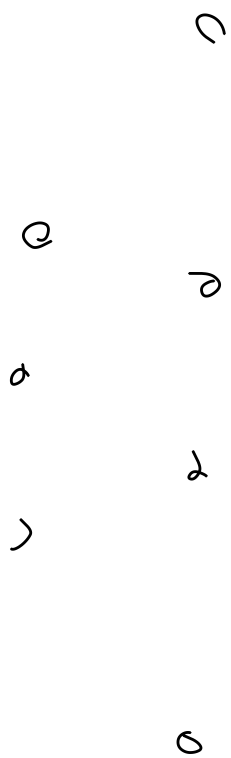


Complete graph : K_n

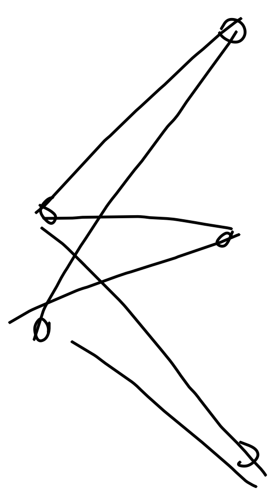


Complete bipartite graph - $K_{m,n}$

$|V_1| = m, \ |V_2| = n$



$K_{2,3} \rightarrow$



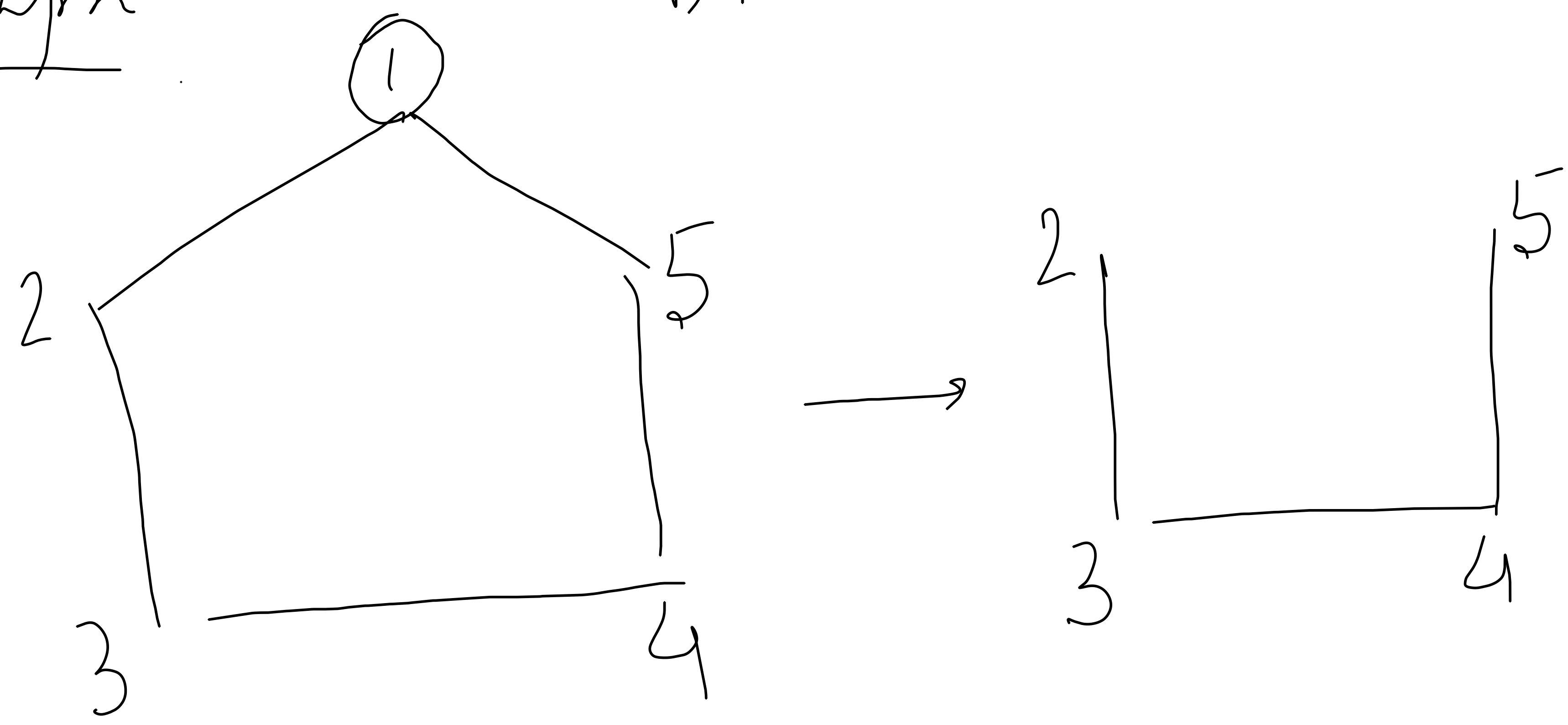
Tree :- a connected graph which does not contain a cycle (as a subgraph).

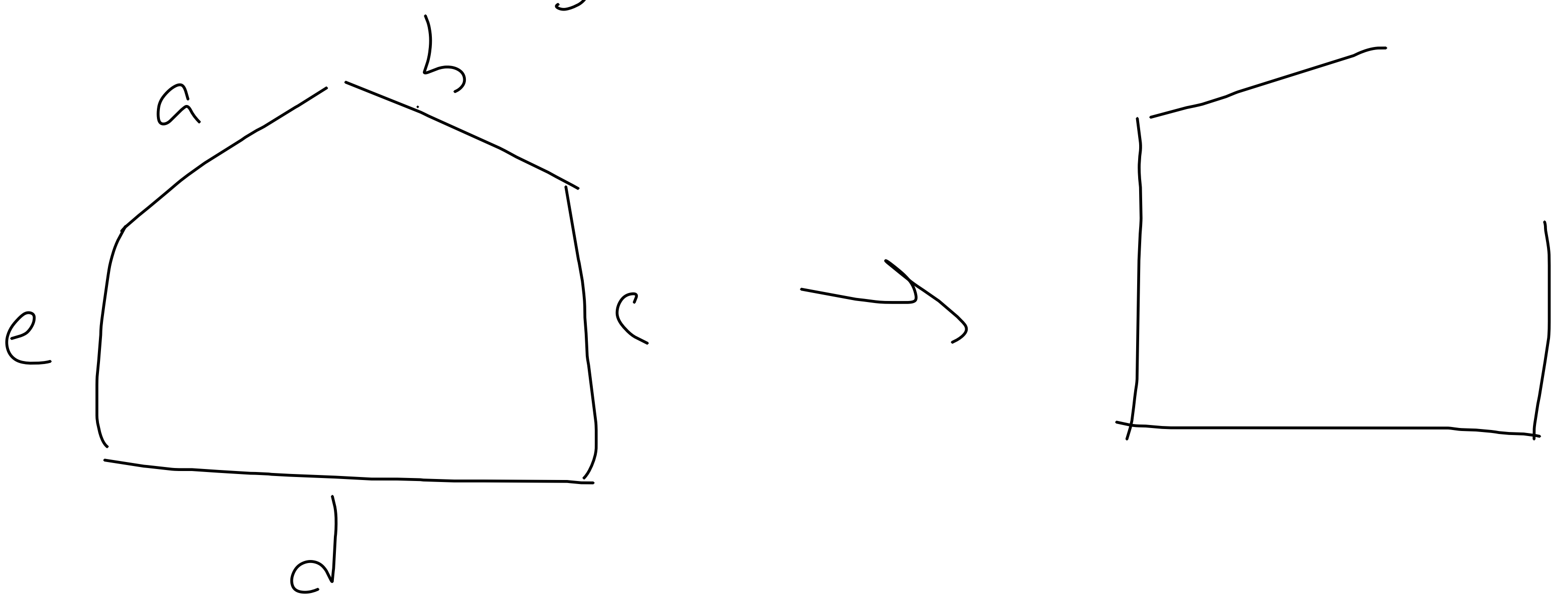
A graph is called connected :- if any two vertices can be joined by a path

A graph which is not connected is called disconnected and maximally connected subgraph is called a connected component.

Subgraph

Vertex deletion





edge deletion

$G \longrightarrow G_1$ by a sequence of edge/vertex deletions.

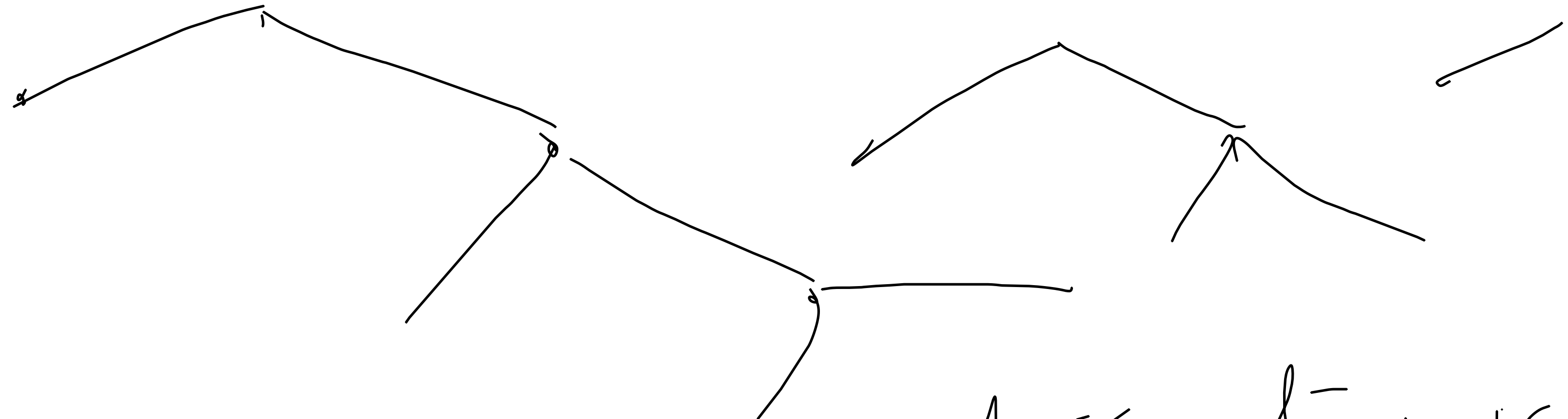
G_1 is called a subgraph of G.

If G_1 is obtained from G using only vertex deletions $\longrightarrow$ induced subgraph.

If G_1 is obtained from G using only edge deletions,

$\hookrightarrow$ spanning subgraph.

Tree :-



A graph whose every component is a tree is called a forest

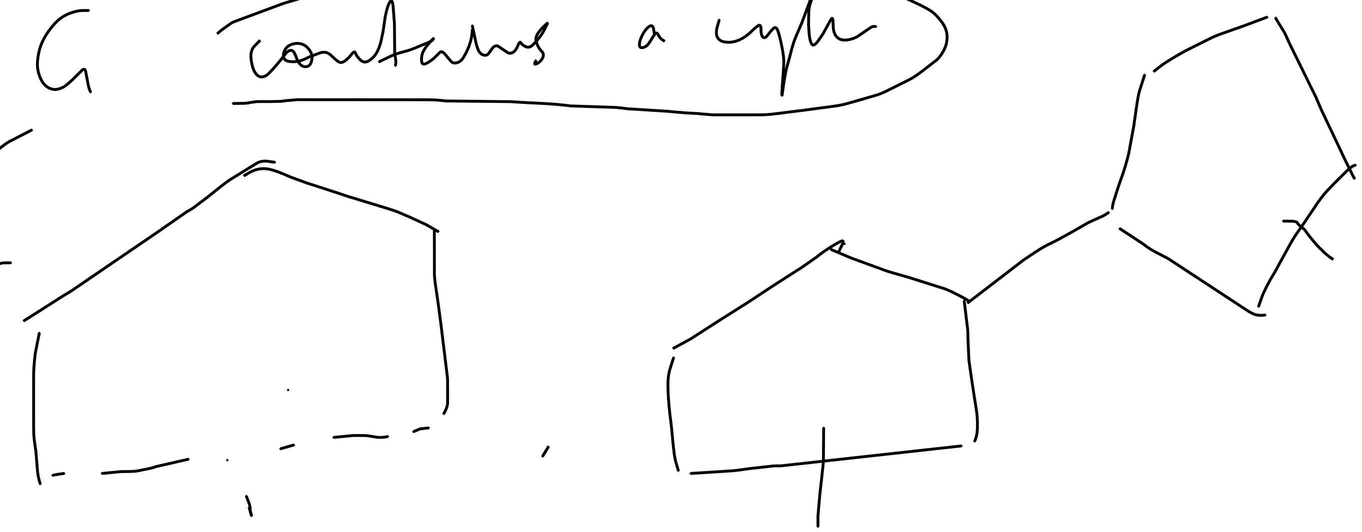
(2) Tree :- (A connected graph which has n vertices and (n-1) edges)

(1) Tree (a connected graph with no cycle)

Tree (a graph s.t for any two vertices, there is a unique path joining them)

Suppose G contains a cycle

is a cycle



n

W.L.O.G $\longrightarrow$ G contains exactly one cycle. remove one edge

(Exc)

n, $(n-1)$ edges

Tree :- a connected graph with no cycle

① to prove that n vertices and $(n-1)$ edges

$$|V| = n \;,\; |E| = q$$

② connected n vertices and $(n-1)$ edges $\Longrightarrow$ no cycles

Tree :- a connected graph with no cycle

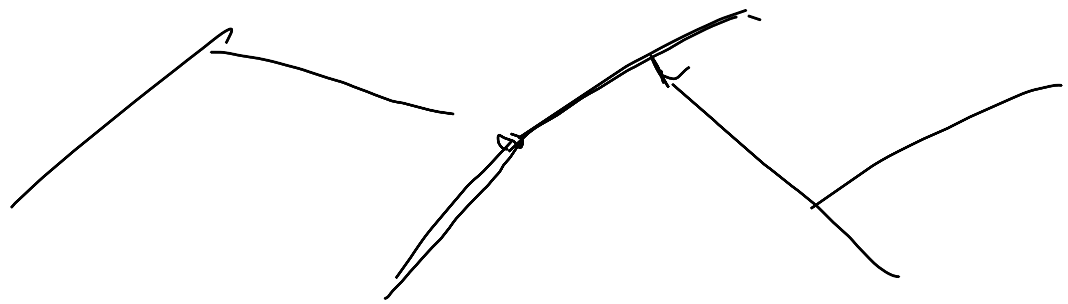
(1) to prove that n vertices and $(n-1)$ edges

$n = m$, $m-1$ edges (Exc)

$n = m+1 \longrightarrow m$ edges

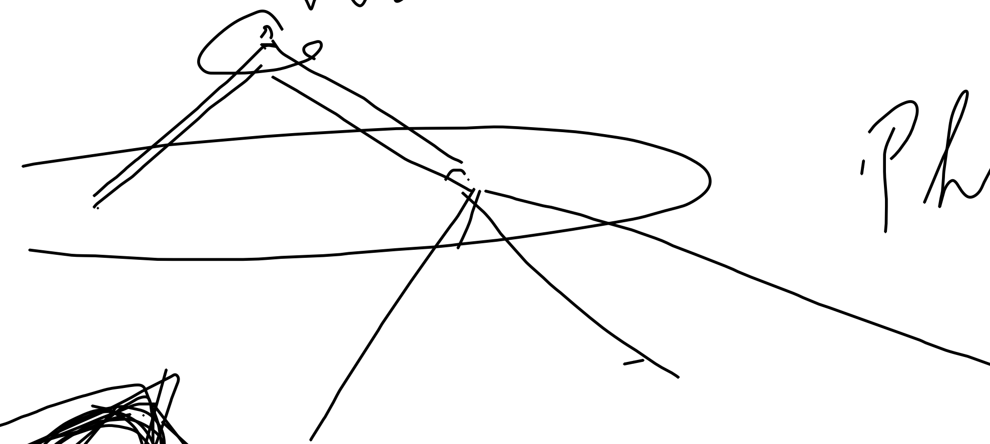
(2) Connected n vertices and $(n-1)$ edges $\Longrightarrow$ no cycles

Graph theory proof of Schroeder Bernstein theorem



rooted tree

Level - 1



Phylogenetic tree

Reconstruction number