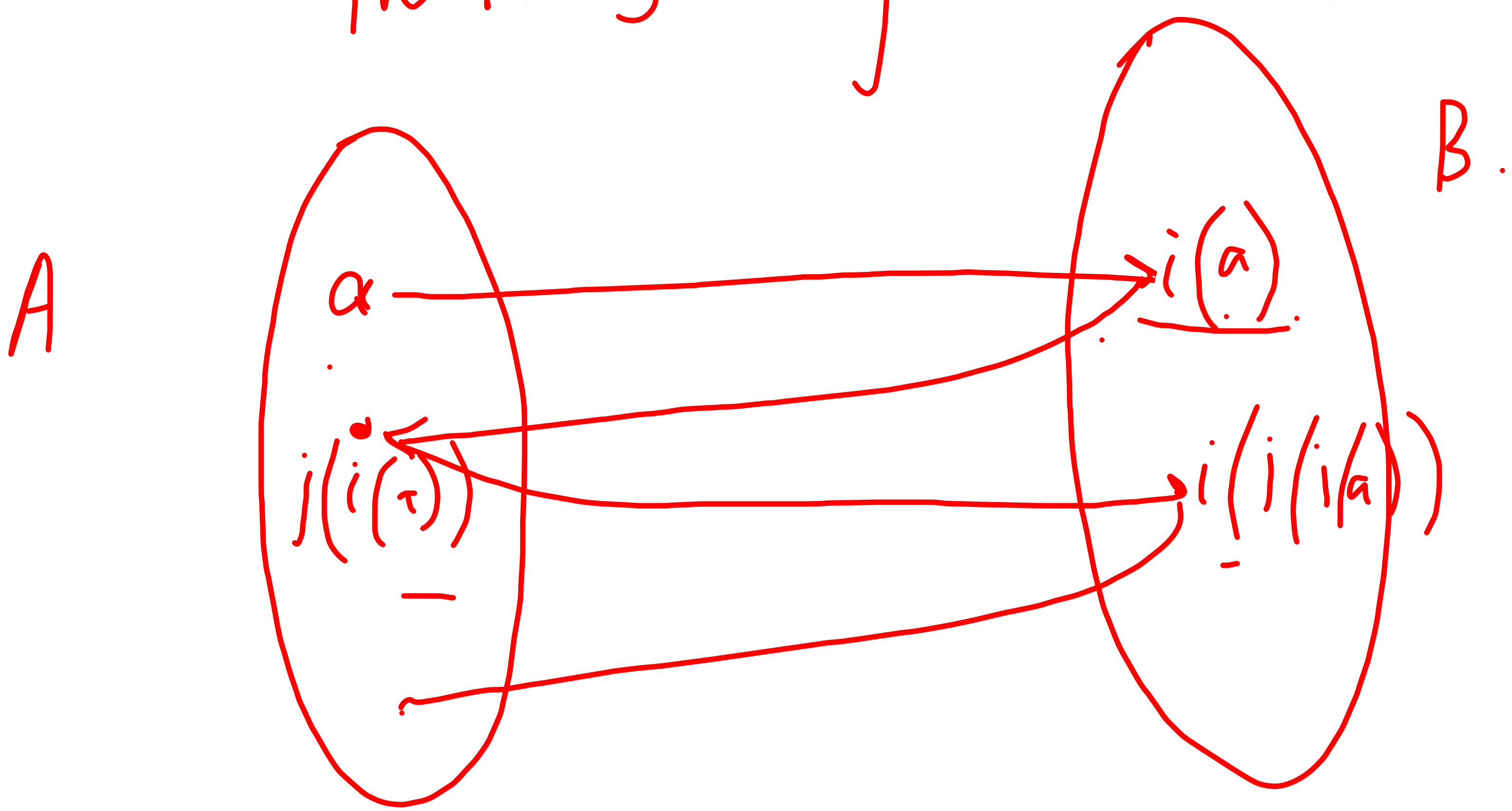
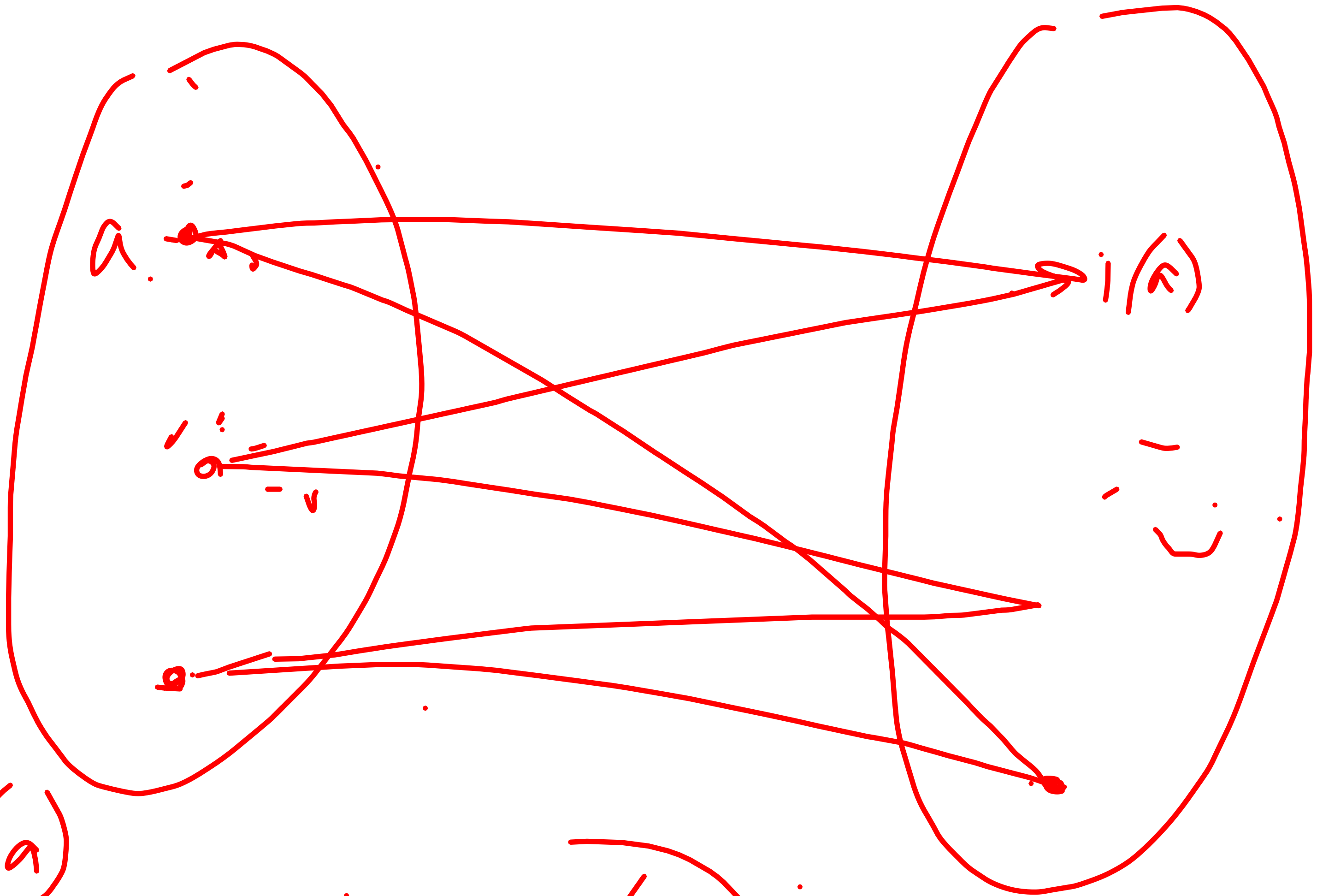
SCHROEDER-BERNSTEIN theorem:

A, B. $\quad i : A \hookrightarrow B$

$j : B \hookrightarrow A$

then $\exists$ bijection $f : A \longrightarrow B$





$f(a) = i(a)$

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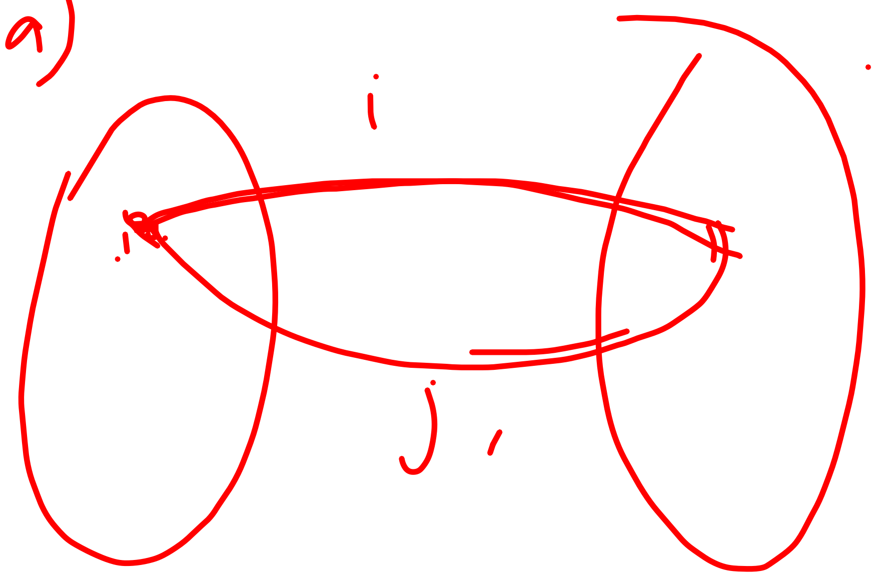
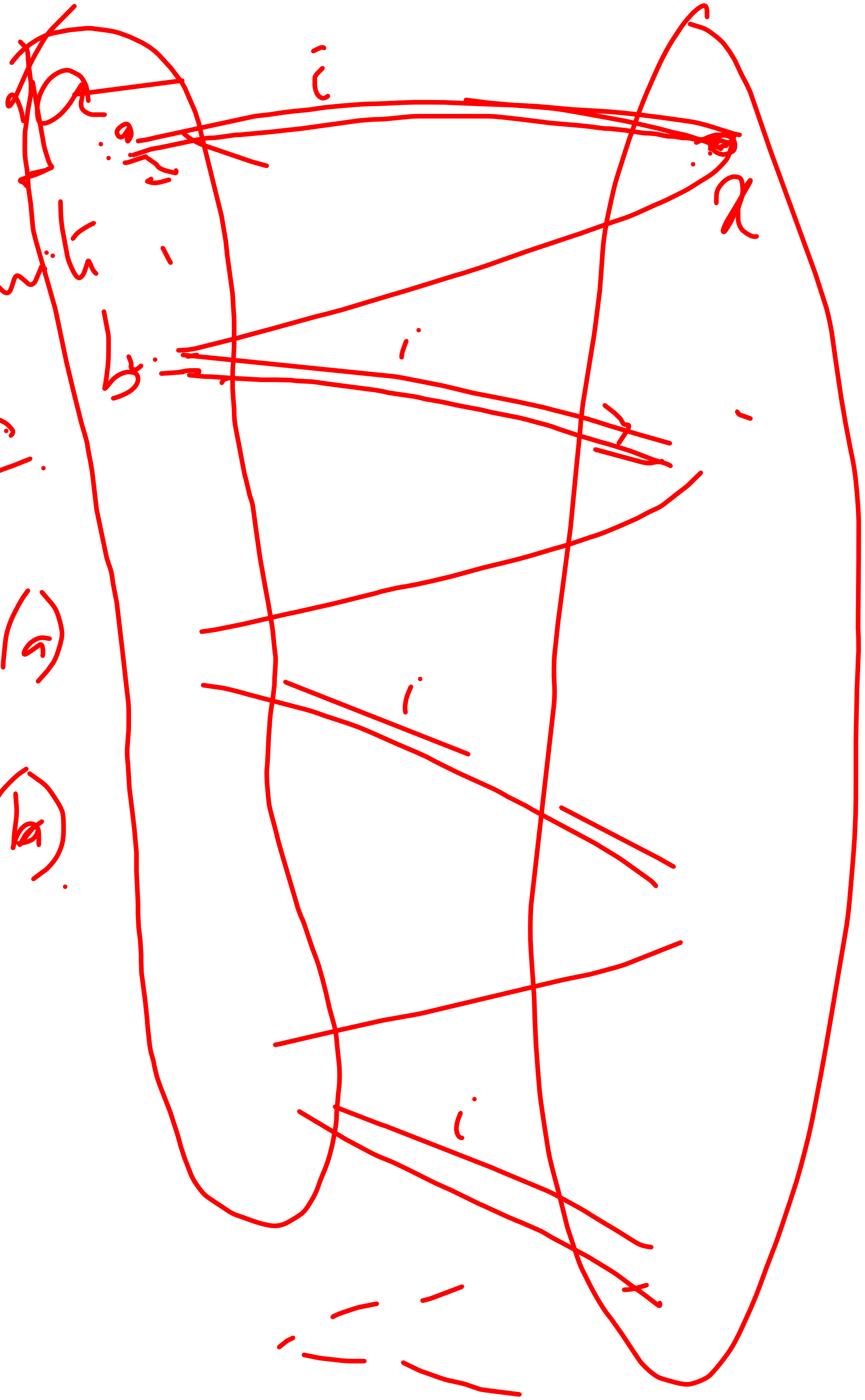


Infinite path



$f(a) = i(a)$

$f(b) = i(b)$

f

$\vdots$

$g(x) = i^{-1}(x)$

$f = g^{-1}$

1.

2. cycle.

3. Infinite path.

Distance :- $d(u,v) = \underset{\substack{\text{all paths} \\ \text{joining} \\ (u,v)}}{\text{Min}} |P(u,v)|$

Diameter :- $\text{Diam}(G) = \max\limits_{u,v} d(u,v)$

Eccentricity of a vertex u, $\varepsilon(u) = \max\limits_{v \in V} d(u,v)$

Radius : $\text{Radius}(G) : \min\limits_{u \in V} \underline{\varepsilon(u)}$

Distance :- $d(u,v) = \text{Min}\,|P(u,v)|$

(metric ? Verify)

(all paths joining (u,v))

Diameter :- $\boxed{\text{Diam}(G) = \max_{u,v} d(u,v)}$

Eccentricity of a vertex u, $\varepsilon(u) = \max_{v \in V} d(u,v)$

Radius : $\text{Radius}(G) : \min_{u \in V} \underline{\varepsilon(u)}$

Hypercube (boolean) $|V| = 2^k$

$|E|$? $(k 2^{k-1})$ $V = \{ (x_1, \ldots, x_k) : x_i = 0, 1 \}$

Exc. two vertices are adjacent if they differ at exactly one co-ordinate

Q: What is the diameter $= k$?

$$(0, 0, \ldots, 0)$$

$$\downarrow$$

$$(1, 0, 0, \ldots, 0)$$

$$\downarrow$$

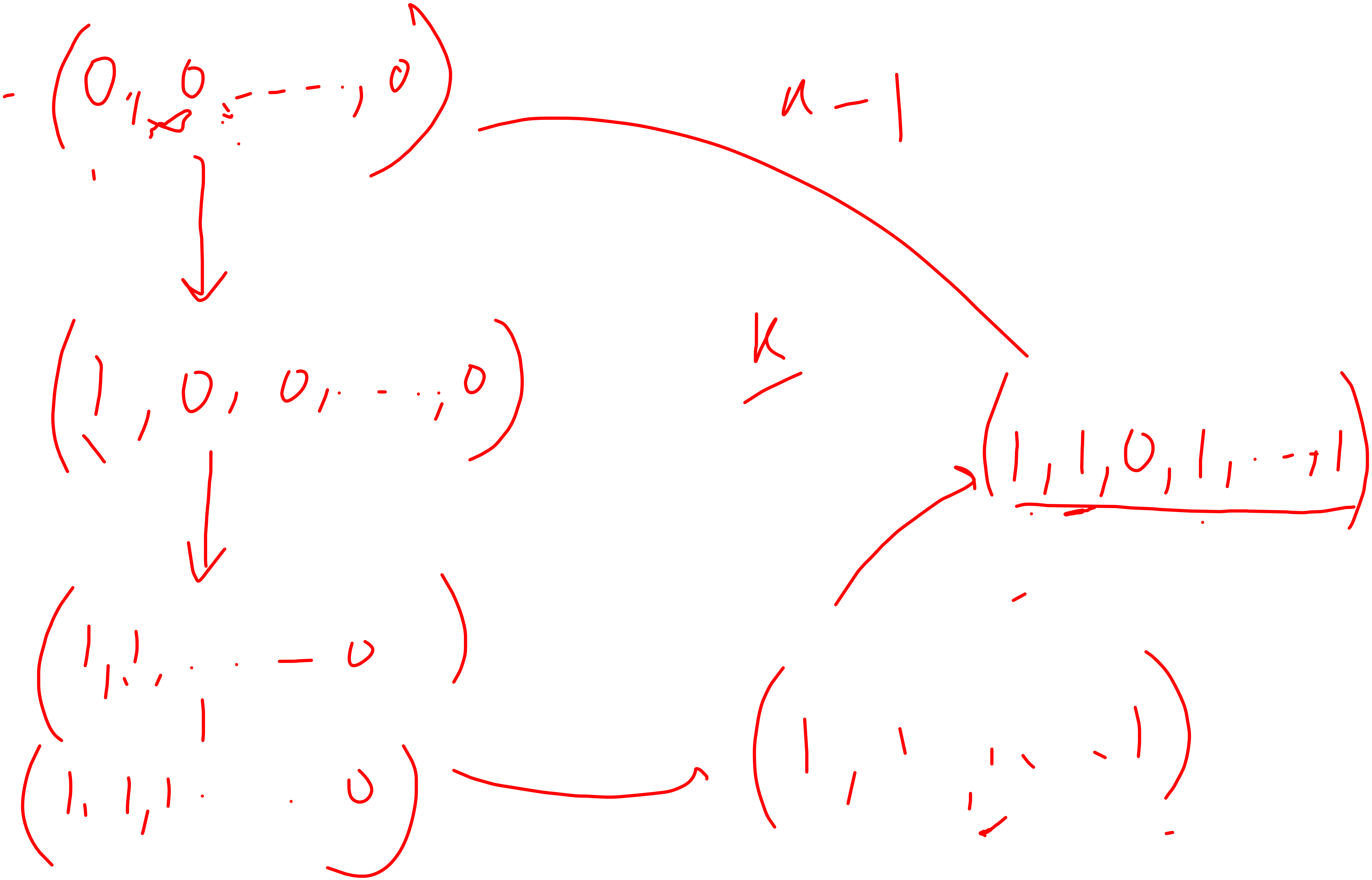
$$(1, 1, \ldots, 0)$$

$$(1, 1, 1, \ldots, 0) \longrightarrow (1, 1, \ldots, 1)$$

metric

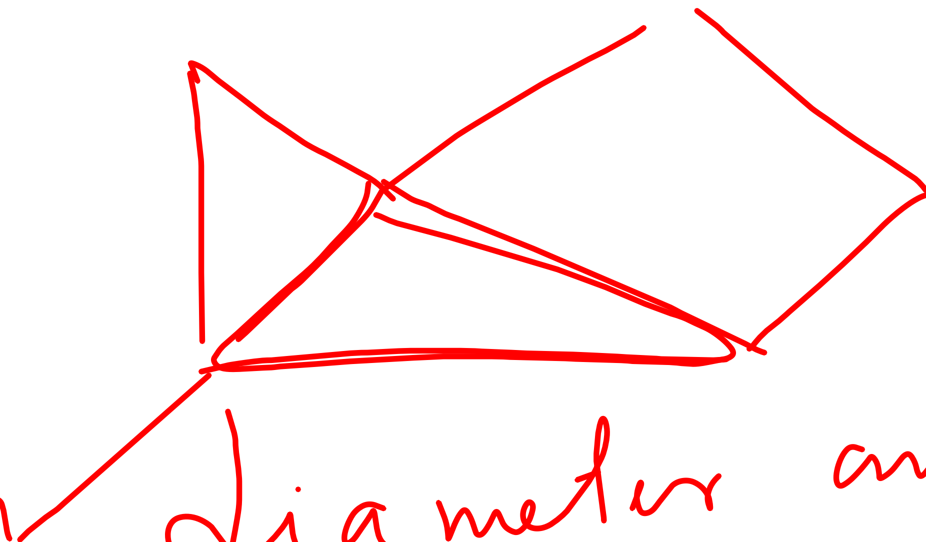
$d(u, v)$

$\underline{k}$



(0, 0, ..., 0)
(1, 0, 0, ..., 0)
(1, 1, ..., 0)
(1, 1, ..., 0)
(1, 1, ..., 1)
(1, 1, 0, 1, ..., 1)
n-1
k

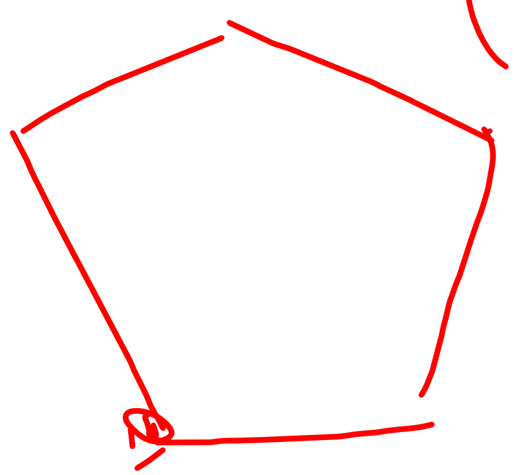
Girth :- length of the smallest cycle



$g = 3$

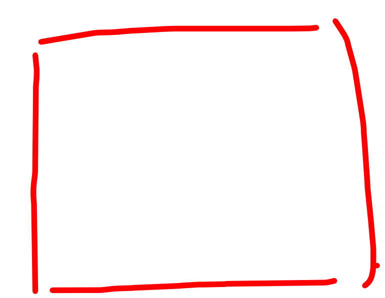
Relation between diameter and girth?

$(g < \infty)$



diameter $d = 2$

girth $= 5$

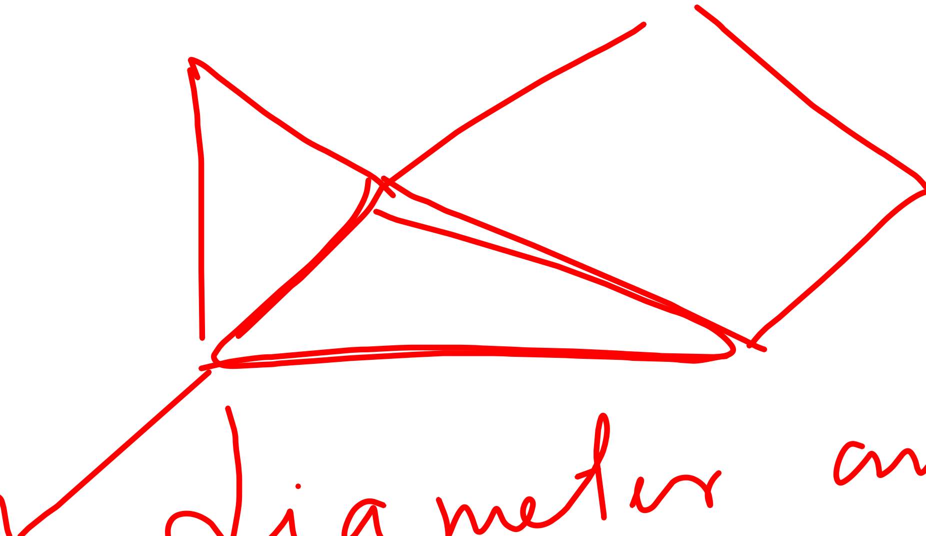


$d = 2$

$g = 4$

$g \leq 2d+1$?

Girth :- length of the smallest cycle



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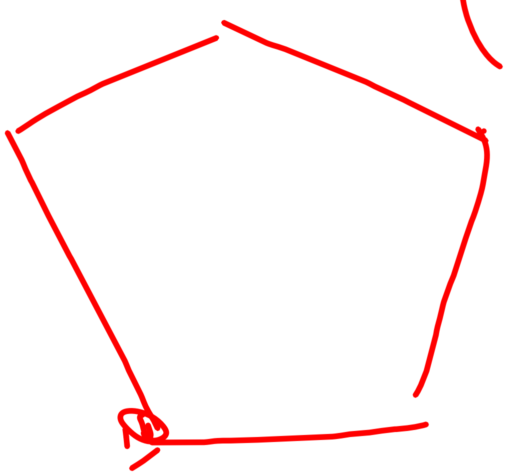
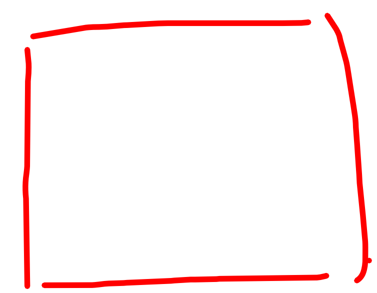


diameter $d = 2$

girth $= 5$

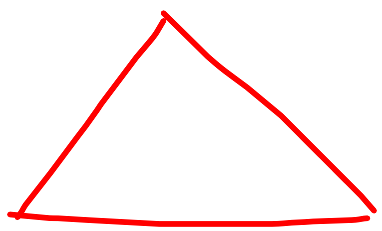


$d = 2$

$g = 4$

$g \leq 2d+1$?

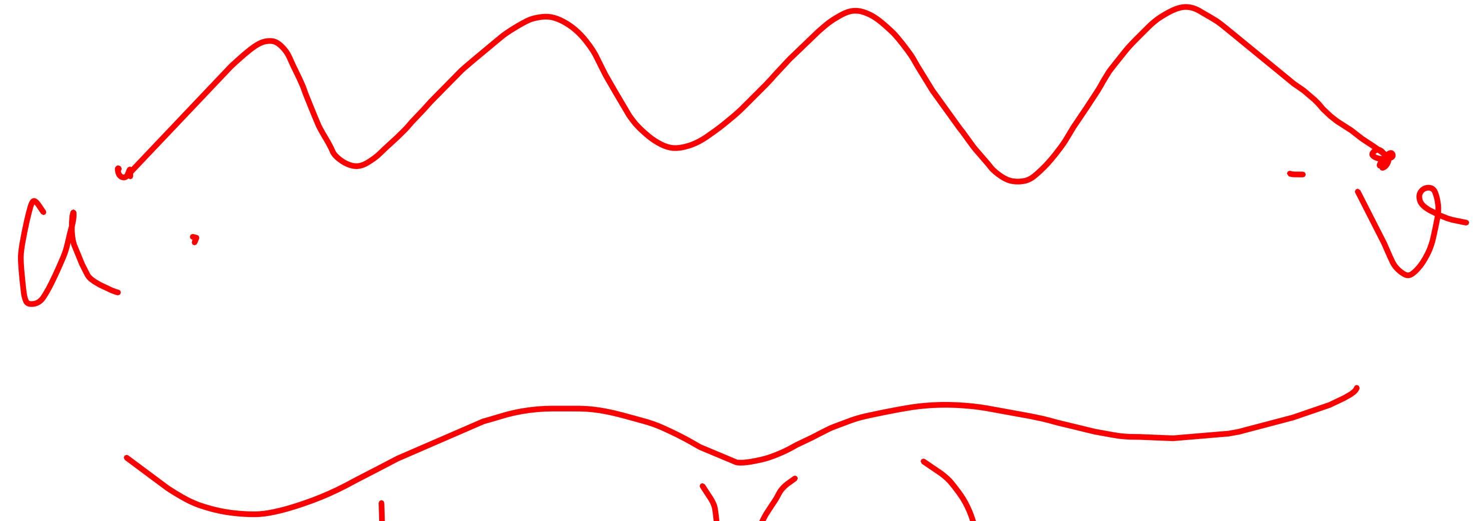
Theorem: For any simple graph with at least one cycle $g \leq 2d+1$.



$g = 3$

$d = 1$

Proof choose two vertices which are at a distance d apart.



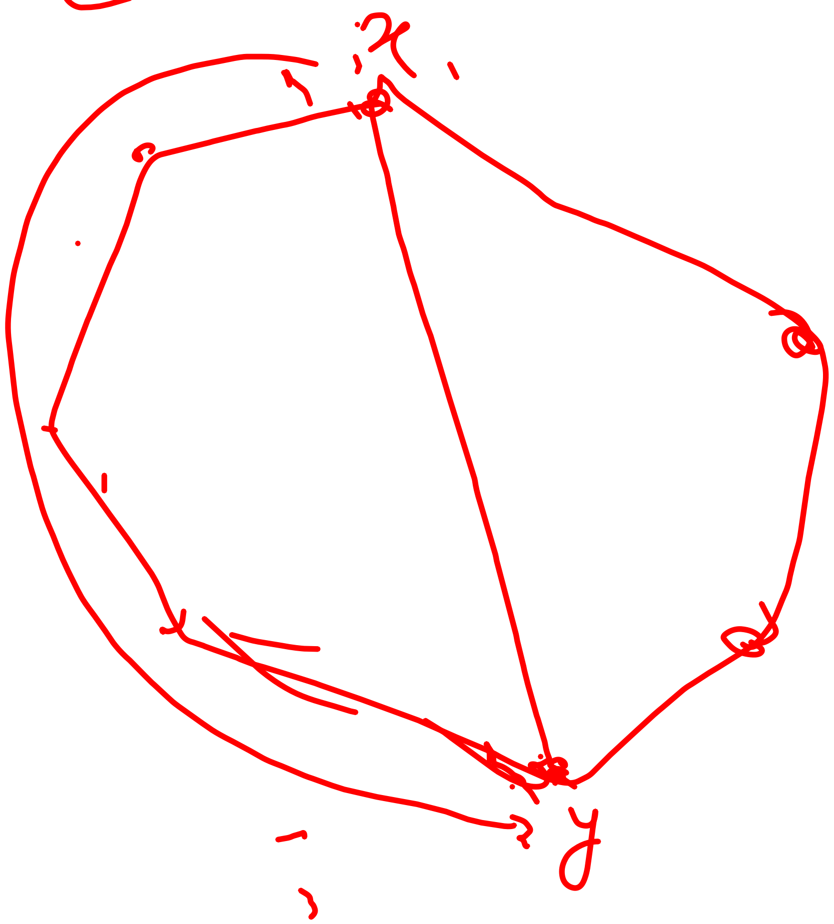
$d = d(u,v)$

$\boxed{g \le 2d+1}$

$g > 2d+1$

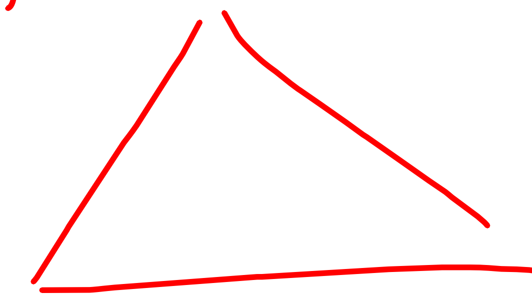
$\therefore \lfloor g/2 \rfloor$

$\lfloor g/2 \rfloor < d$



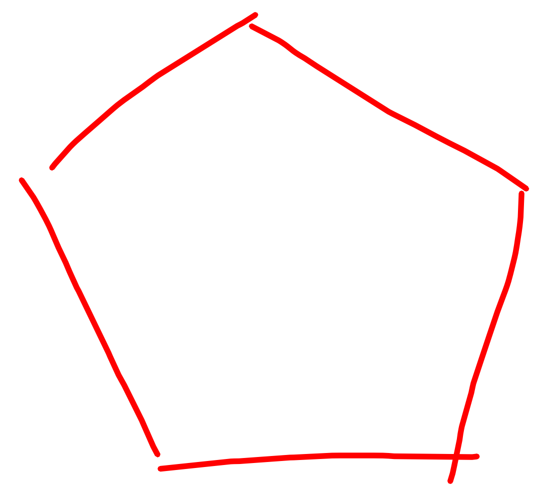
Characterize all graphs s.t

$$\underline{g = 2d+1}$$



$g=3$

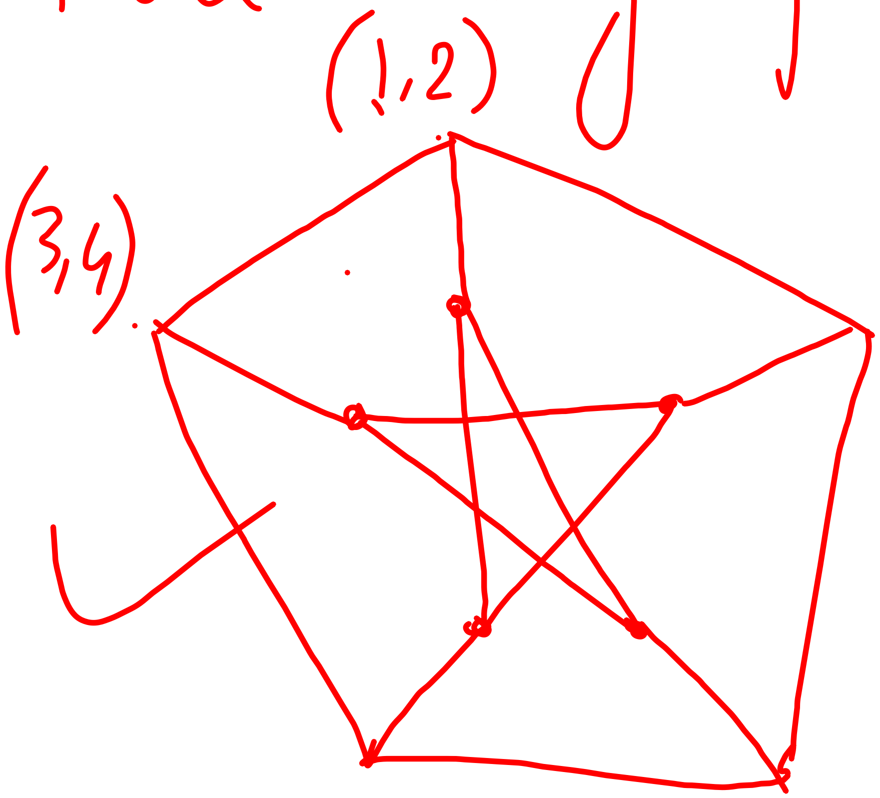
$d=1$



$d=2$

$g=5$

Petersen Graph

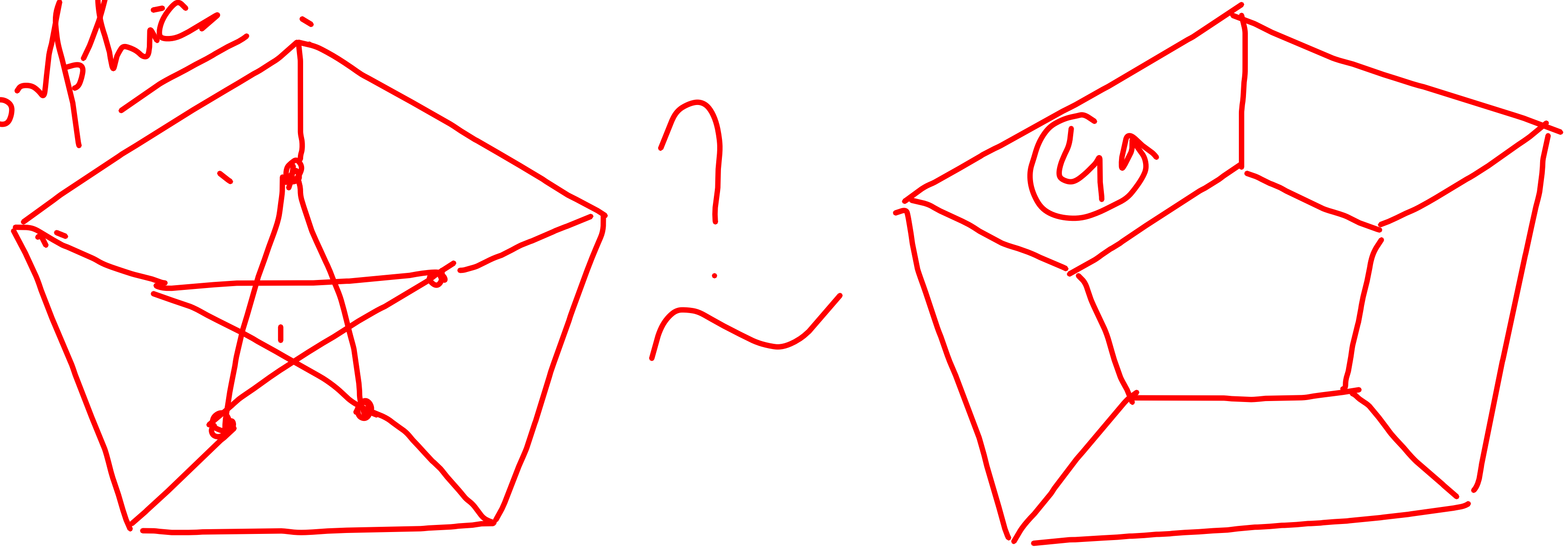


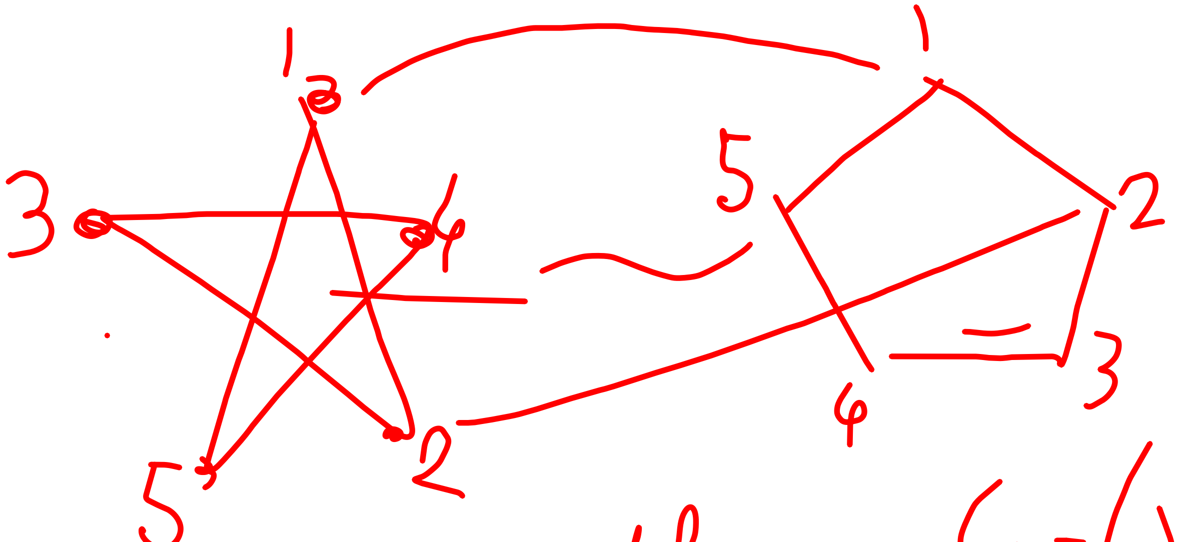
$g=5$

$d=2$

Not isomorphic

(5, 2, 0)





Graph isomorphism

$G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$

$g_1 : V_1 \longrightarrow V_2$.

$g_2 : E_1 \longrightarrow E_2$.

Suppose u and v are adjacent in G_1 iff $g(u)$ and $g(v)$ are adjacent in G_2.

Set systems, incidence systems.

Johnson graphs. $J(n, k, \mu)$

$[n] = \{\underbrace{1, 2, 3, \ldots, n}\}$

$V = \binom{[n]}{k}$: all k-element subsets of $[n]$

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Set systems, incidence systems.

Johnson graphs. $J(n, k, \mu)$

$[n] = \{\underbrace{1, 2, 3, \ldots, n}\}$

$V = \binom{[n]}{k}$: all k-element subsets of $[n]$

E : any two vertices u, v are adjacent if $|u \cap v| = \mu$.

$[5] = \{1, 2, 3, 4, 5\}$ $\qquad$ $J(5, 2, 0)$

$V = \binom{[5]}{2}$: set of all 2-element subsets of $[5]$

$$= \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\} \neq [5] \times [5]$$

E : any two vertices u, v are adjacent if $|u \cap v| = 0$

$[5] = \{1, 2, 3, 4, 5\}$ $J(5, 2, 0)$

$V = \binom{[5]}{2}$: set of all 2-element subsets of $[5]$

$= \{(1,2), (1,3), (1,4), (1,5), \quad = [5]\times[5]/\sim$

$(2,3), (2,4), (2,5),$

$(3,4), (3,5), (4,5)\}$

E : any two vertices u, v are adjacent if $|u \cap v| = 0$

Strongly regular graphs.

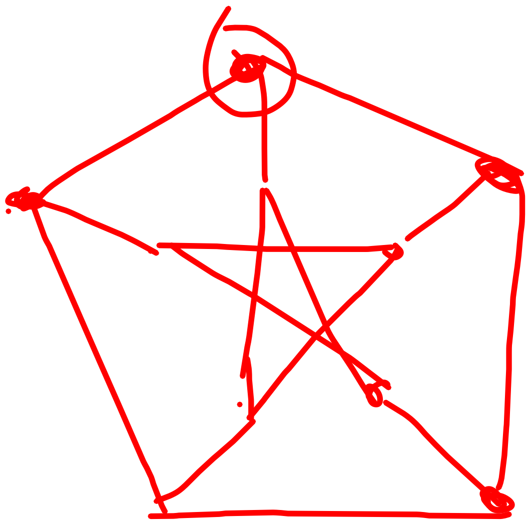
s.r.g(n, k, λ, μ)

μ → any two non-adjacent vertices have μ common neighbours

n → no of vertices

k → common degree of all the vertices

λ → any two adjacent vertices have λ common neighbours



s.r.g$(10, 3, 0, 1)$