Graphs : $g \equiv 2d+1$ (Moore graphs)

(connected) (girth) (diameter)

Claim :- Moore graphs are regular.

$\deg(u) = \deg(v)$ find a bijection

$\deg(u) = \deg(x) = \deg(v)$ between $N(u)$ and $N(v)$

(*) Moore graphs are regular.

Next : Moore graphs for $g=5$.

Degree sequence:

G : $v_1, v_2, \ldots, v_n$

$(\deg v_1, \deg v_2, \ldots, \deg v_n)$ and sort them in a non-increasing order.

Suppose you are given a non-increasing finite list of integers: Is it the degree sequence of some graph?

Claim:- $\sum_{v_i \in V} \deg(v_i) = 2 \times |E|$.

$\Rightarrow \sum_{\deg(v_i) \text{ is Even}} \deg(v_i) + \sum_{\deg(v_i) \text{ is Odd}} \deg(v_i) = \underline{2 \times |E|}$ (Even)

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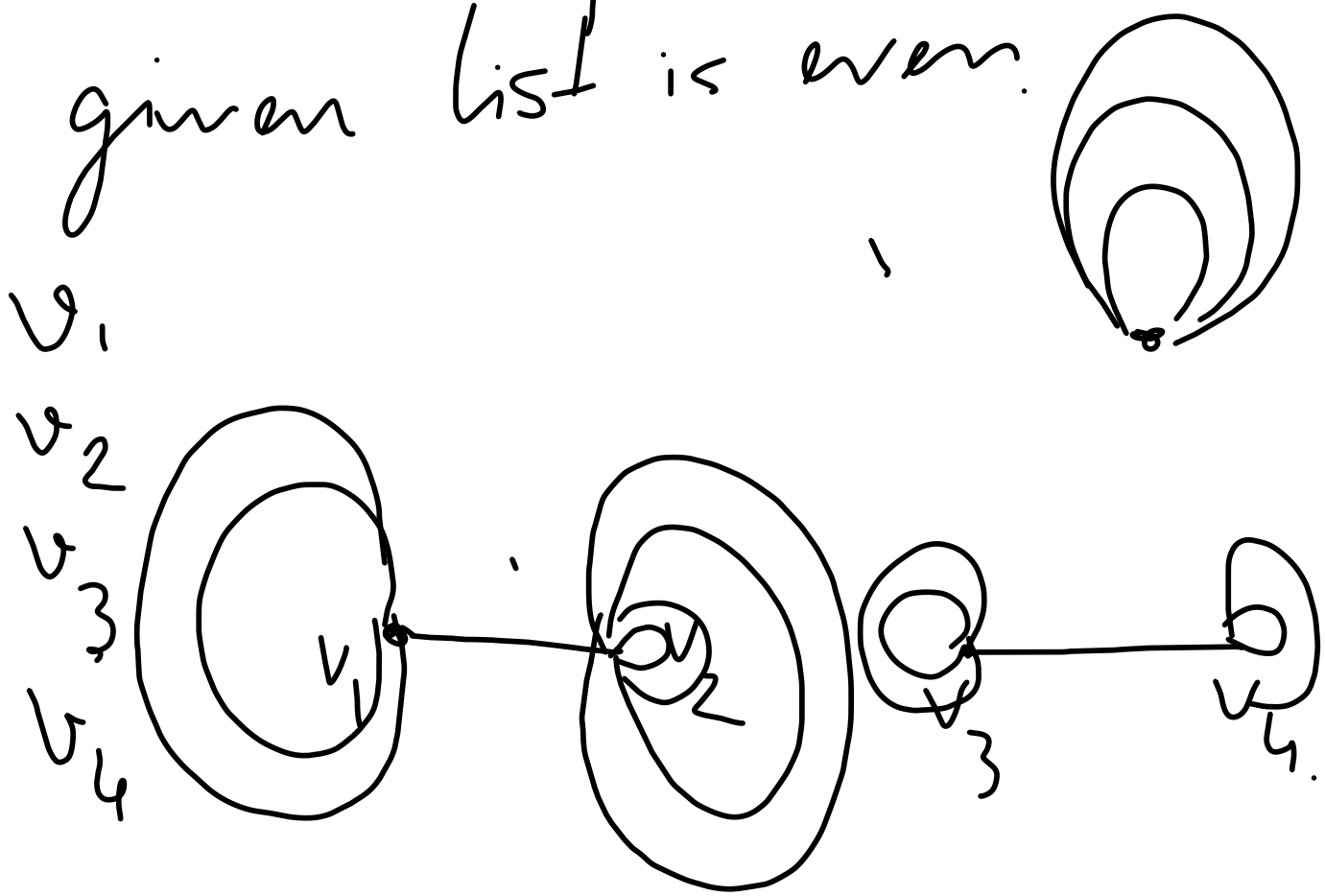
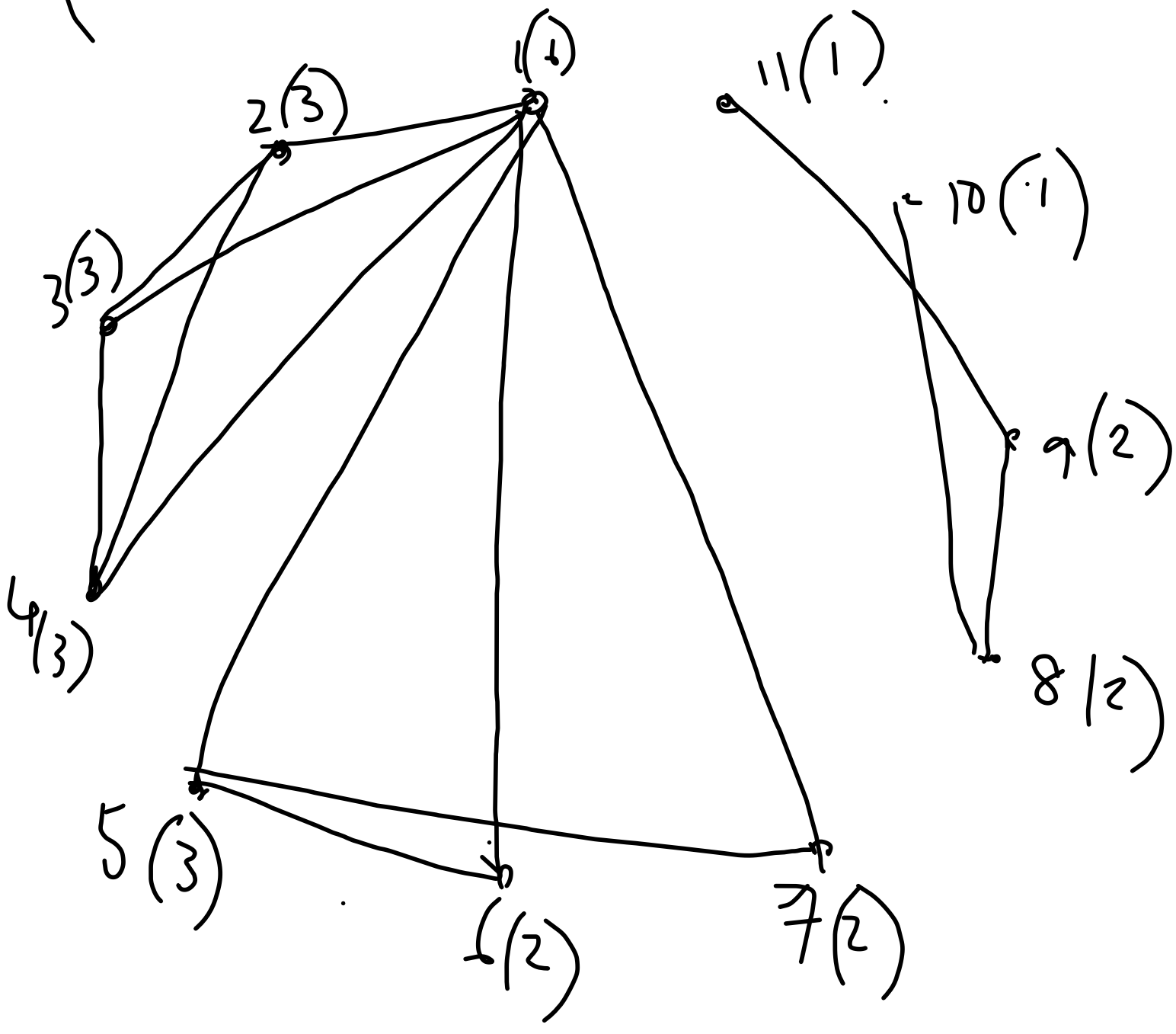


A degree sequence is called a graphic sequence if it is the degree sequence of a simple graph. (no loops, no parallel edges)

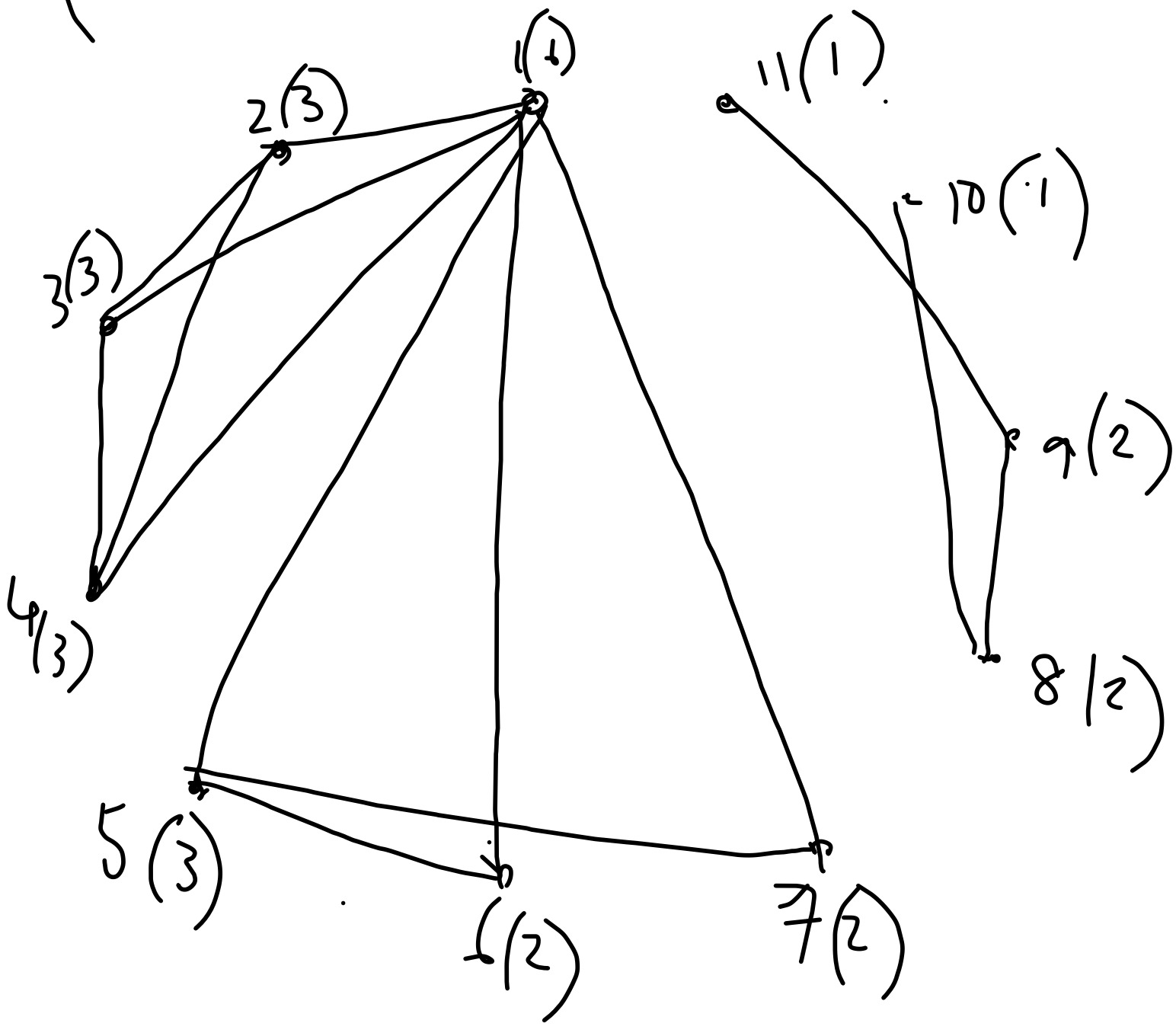
Q: When is a degree sequence a graphic sequence?

(6, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1)

(6, 3, 3, 3, 3)



- $(6, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1)$



$(6, 3, 3, 3, 3)$

$A = (S, t_1, t_2, \ldots, t_S, d_1, d_2 \ldots, d_n)$

$A' = (t_1 - 1, t_2 - 1, \ldots, t_{S-1}, d_1, d_2, \ldots d_n)$

$\hookrightarrow A'' \longrightarrow A''' \longrightarrow \ldots \emptyset$

A is a graphic sequence iff

A' is " "

Havel-Hakimi Algorithm. (Exc:- running time)

Q:- How to decide if a graph is connected or not?

Ans:- by definition:- if there are two vertices which are not joined by any path.

What is a smarter way! a polynomial time algorithm (if it exists).

Spanning Subgraph of a graph G is a subgraph which is obtained by edge deletion.

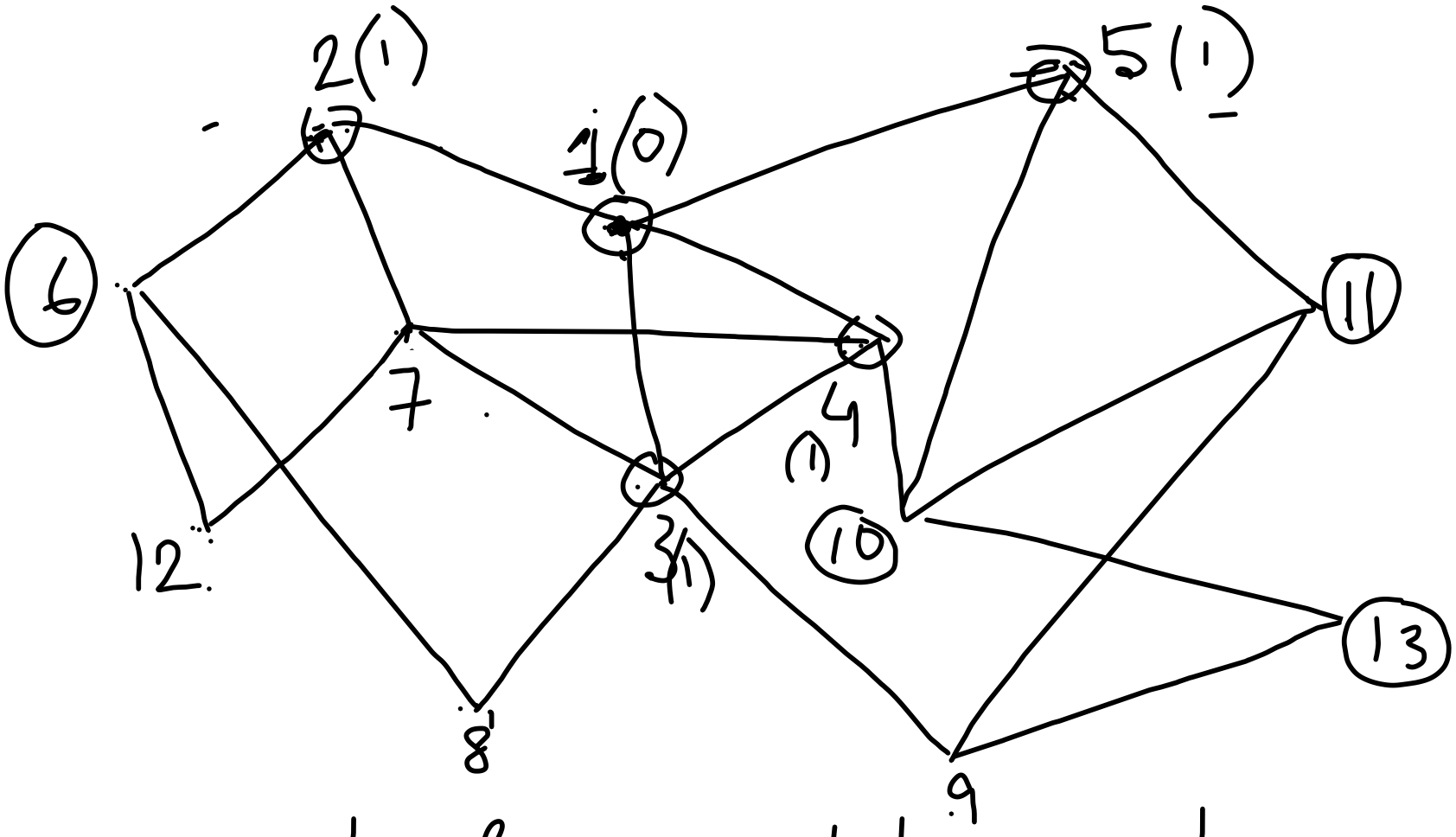
Spanning tree :— Spanning subgraph which is a tree.

(*) A graph is connected iff it has a spanning tree —

Q. How to find Spanning trees inside a graph.

(1) Breadth first search.

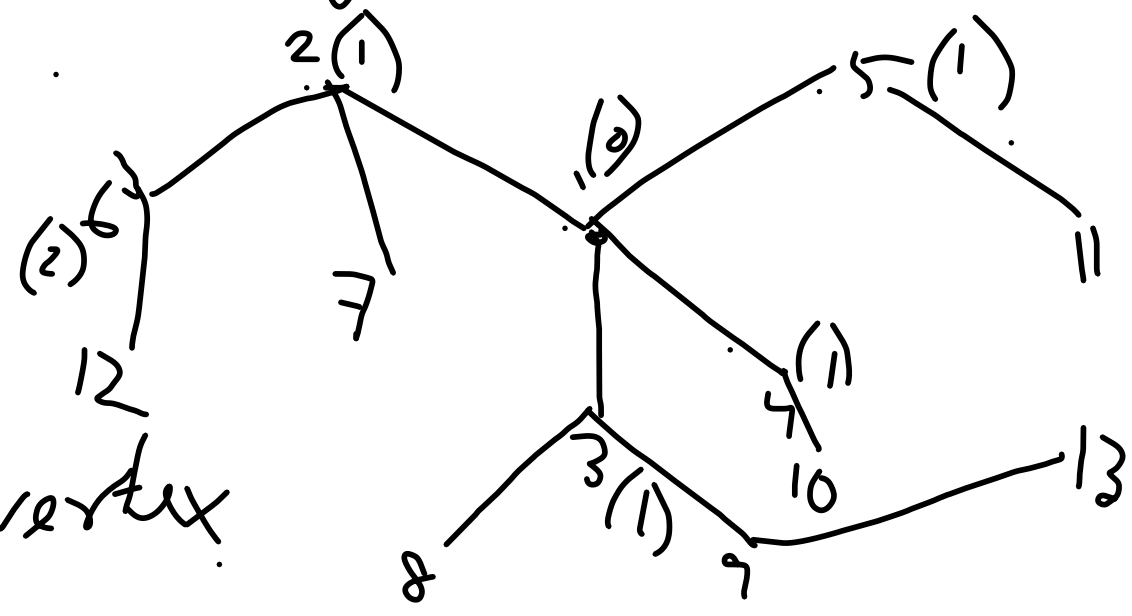
(2) Depth first search.



Breadth first search

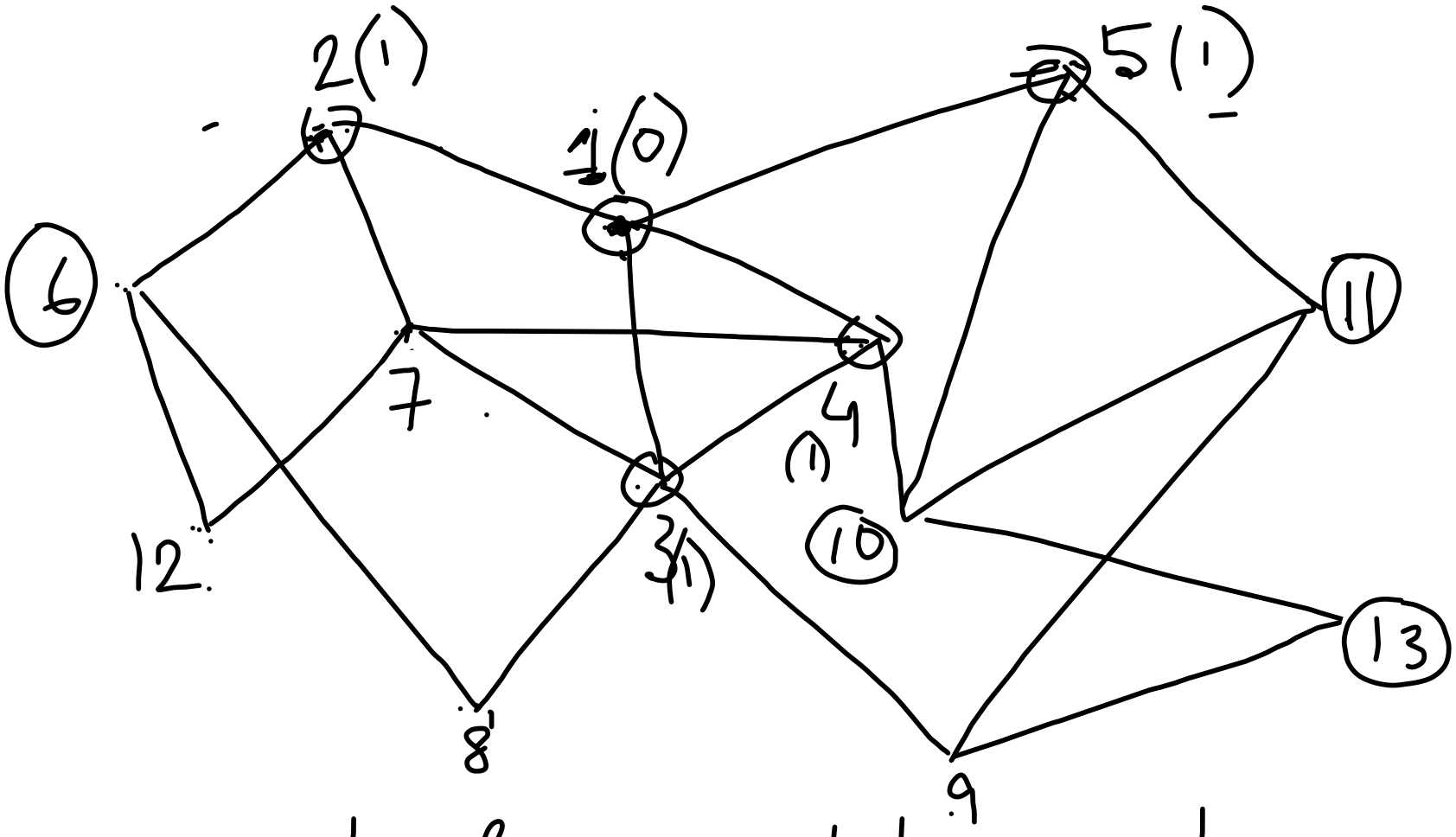
(search along the breadth and then proceed)

idea:- keep on adding edges until there is no cycle.



(spanning tree)

can also get the distance from the starting vertex.



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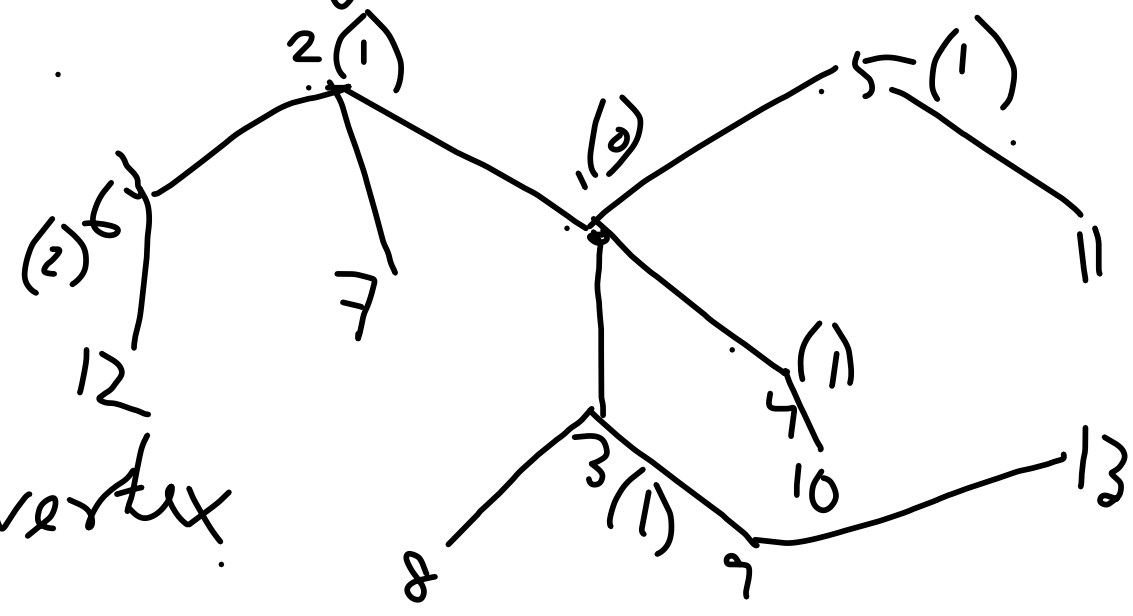

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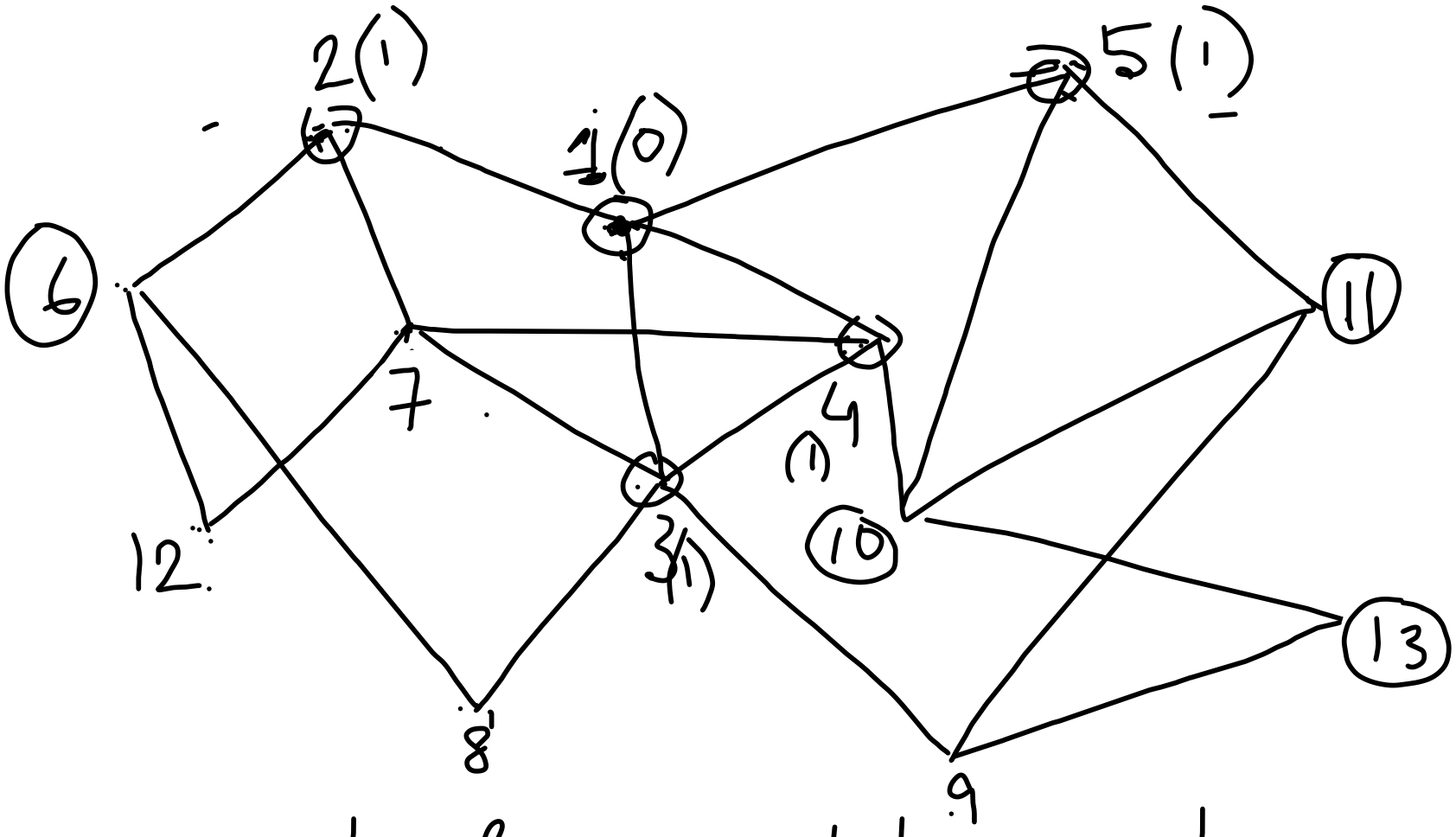

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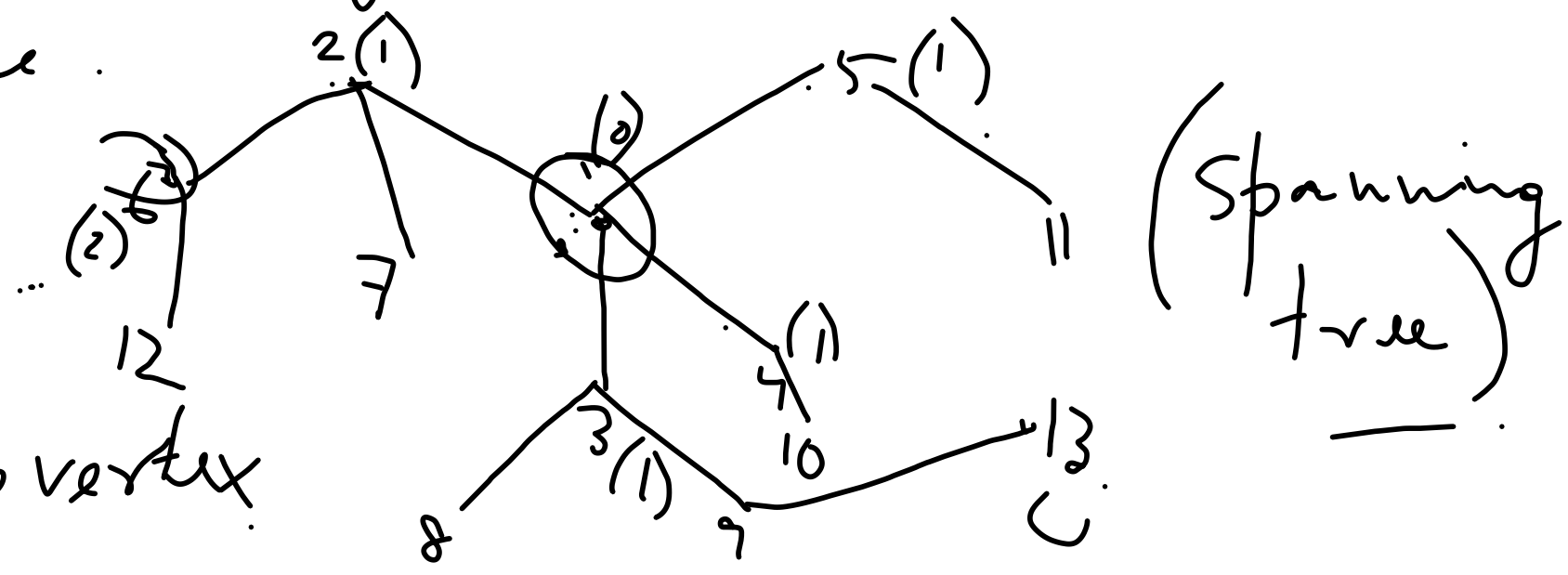


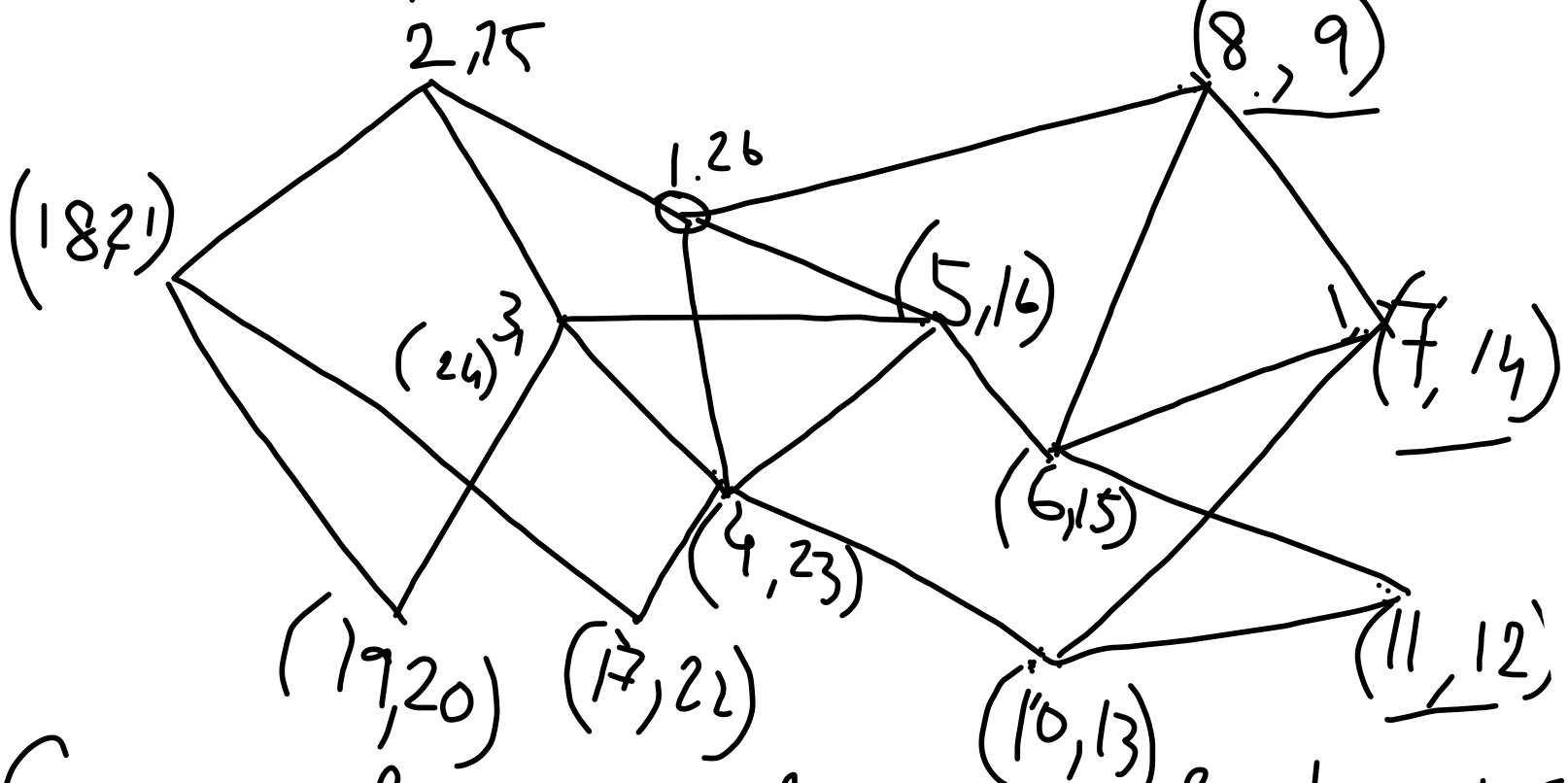
(Spanning tree)

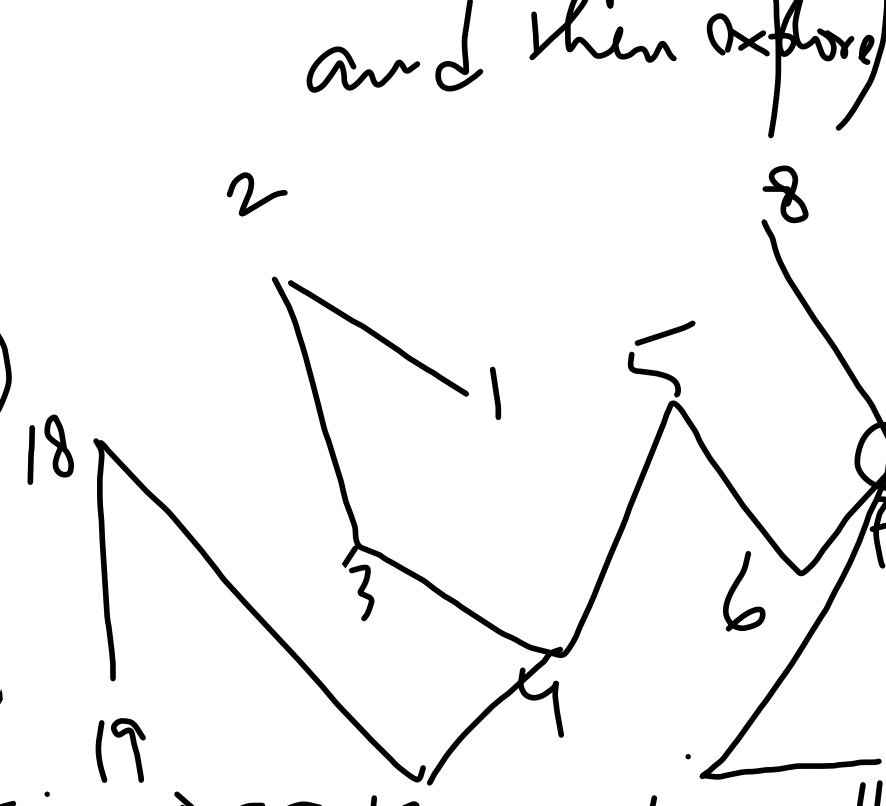
can also get the distance from the starting vertex.

Algorithm for detecting a cycle?

(union finding algorithm)

(2) Depth first search. idea → first go deep. and then explore



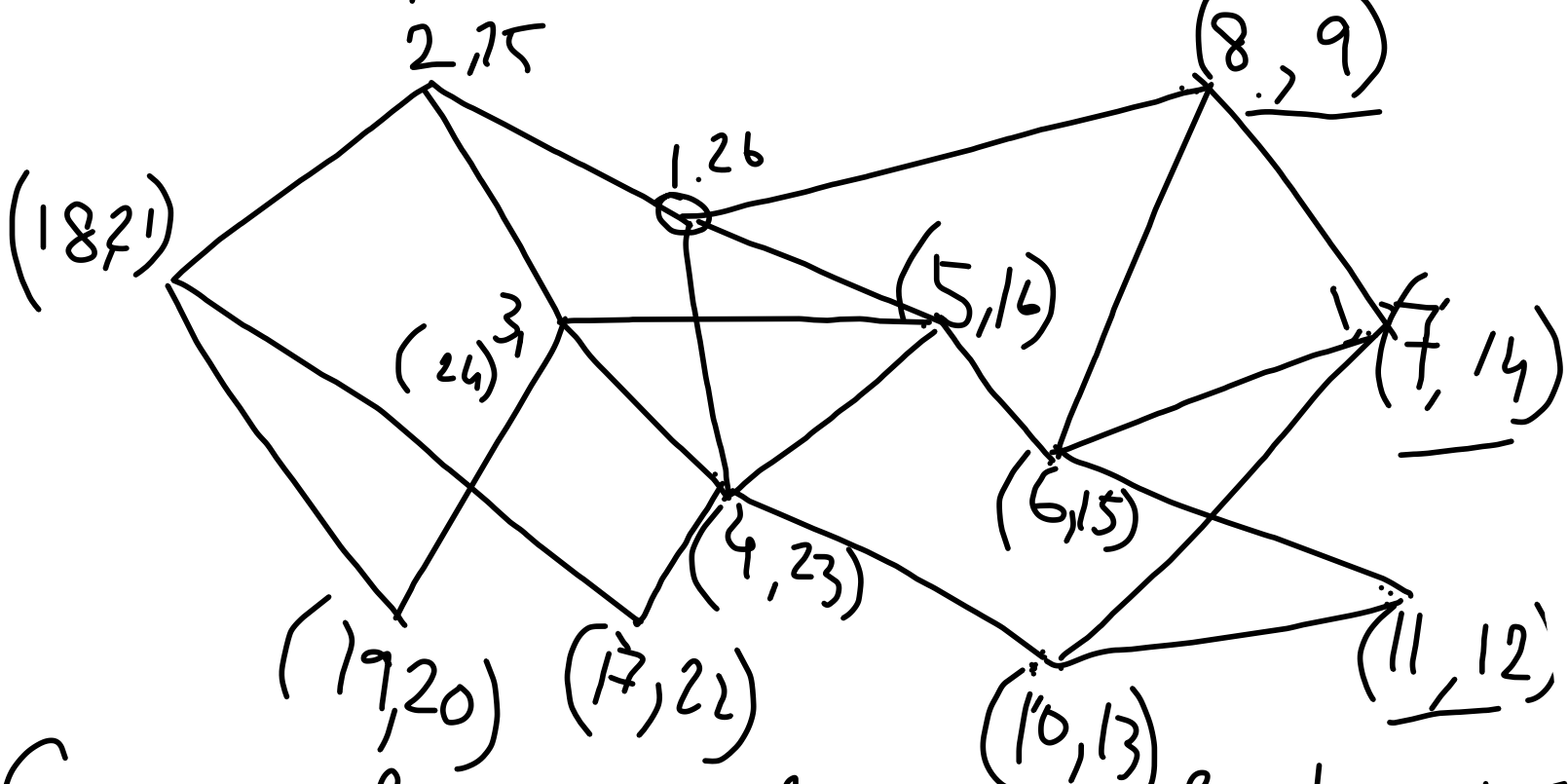


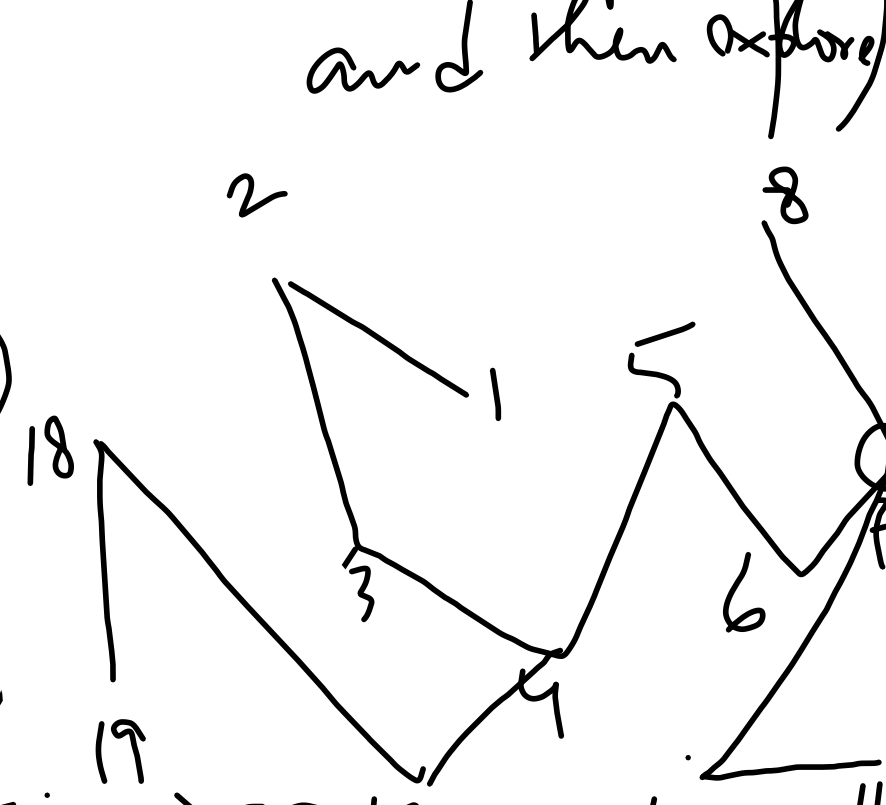
Can you find the diameter of a tree using DFS ? (Assignment)

Algorithm for detecting a cycle?

(union finding algorithm)

② Depth first search. idea → first go deep. and then explore

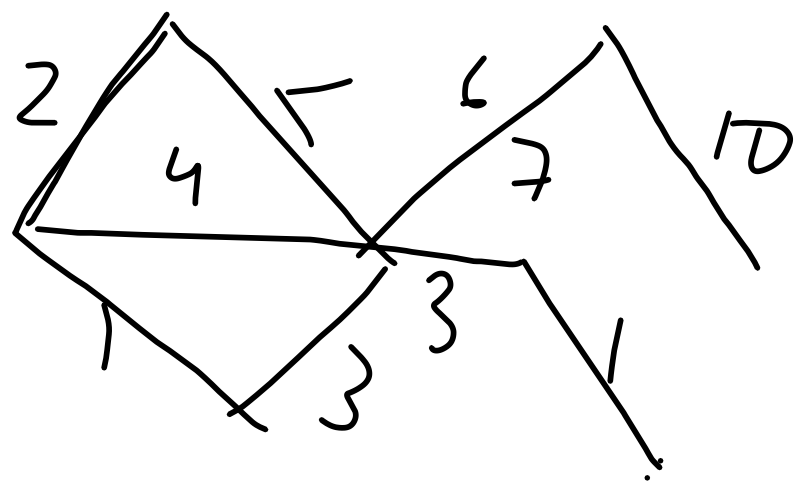




Can you find the diameter of a tree using DFS ? (Assignment)

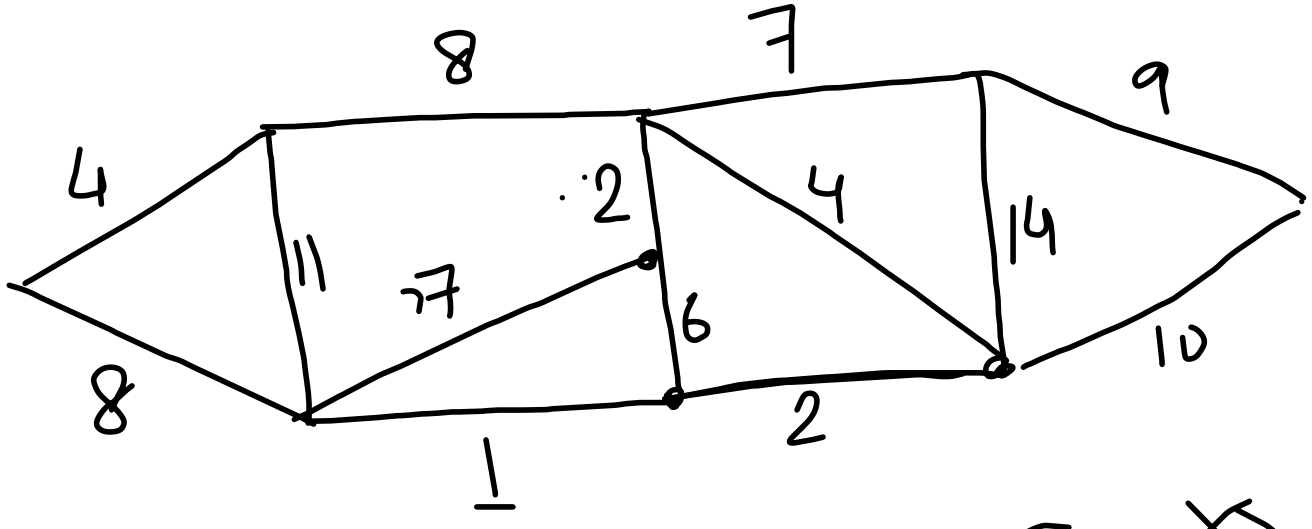
(*) If you run B.F.S on a tree twice, you can end up with the diameter. (Exc)

Suppose your graph has weights on the edges.

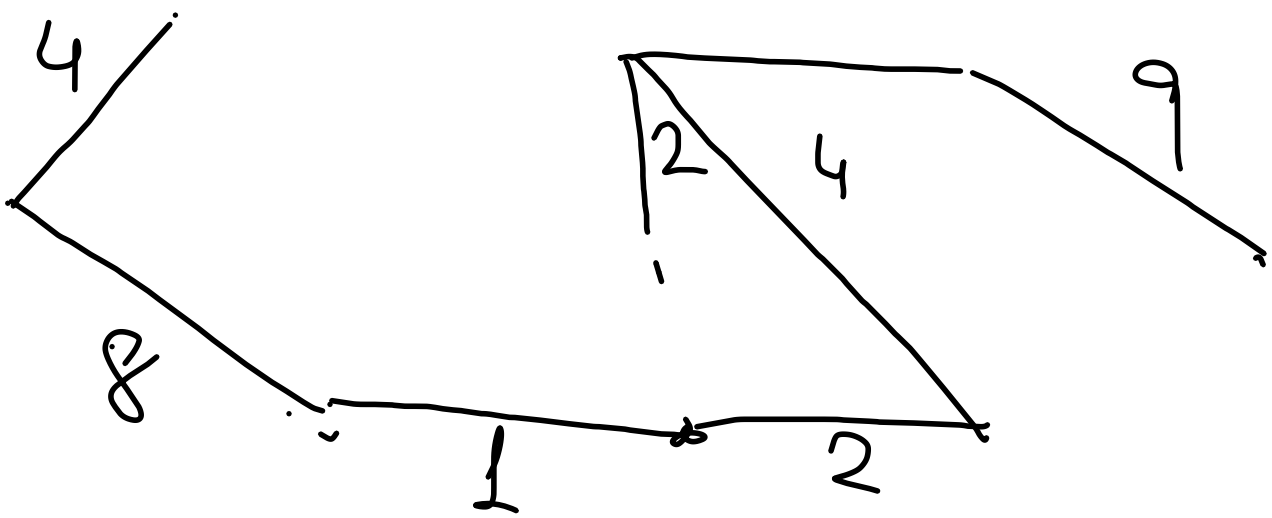


How can you find the minimum weight spanning tree?

Kruskal's algorithm ! Prim's algorithm.



1 2 2 4 4 ~~6~~ ~~7~~ 7 8 ~~8~~ 9 ~~10~~ ~~11~~ ~~14~~



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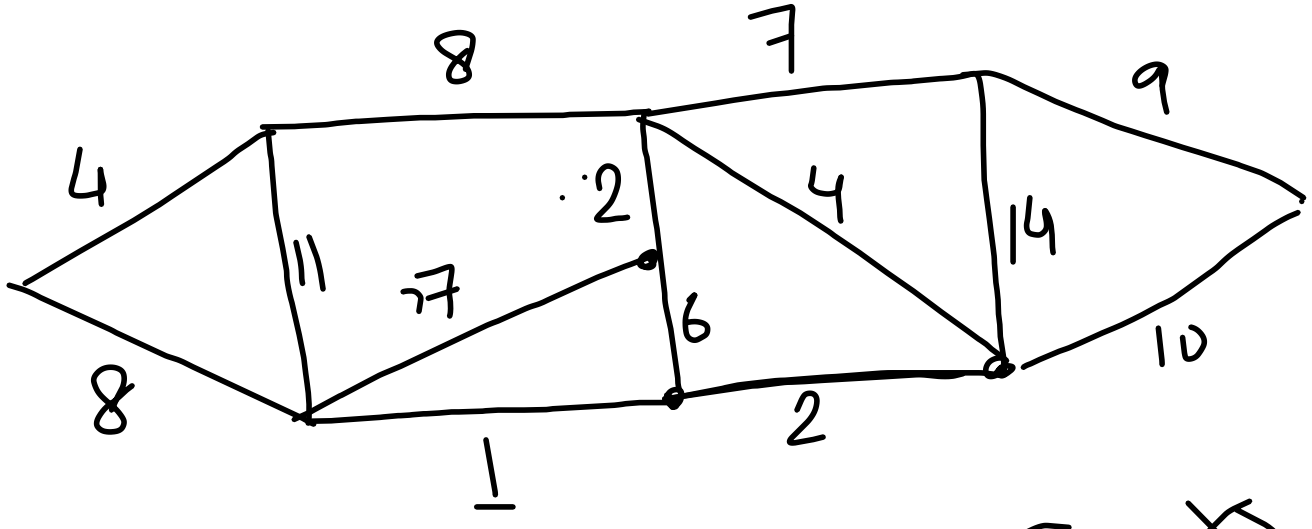

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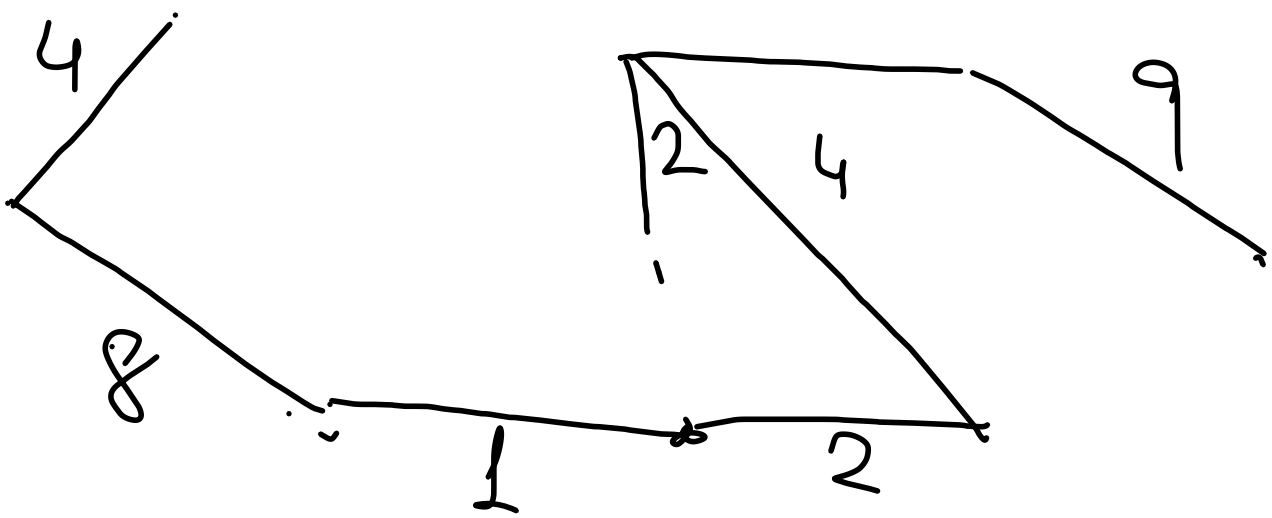
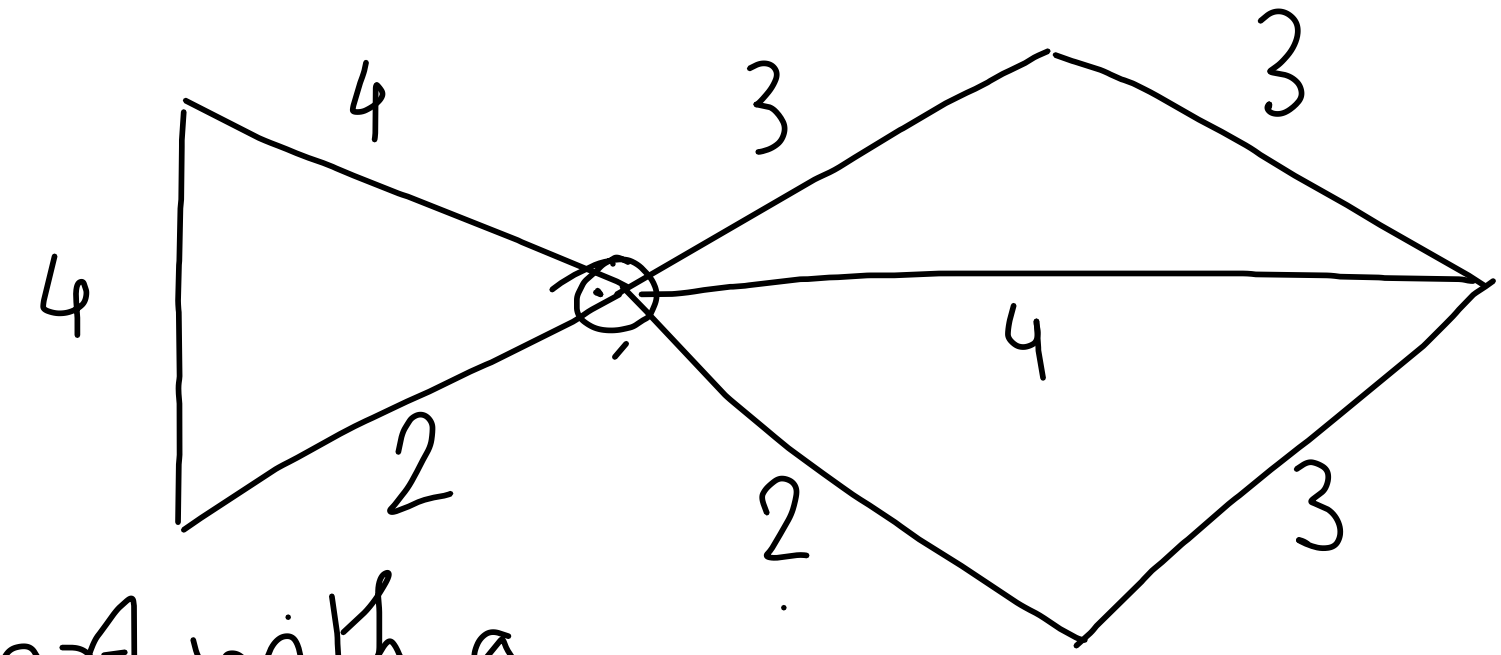
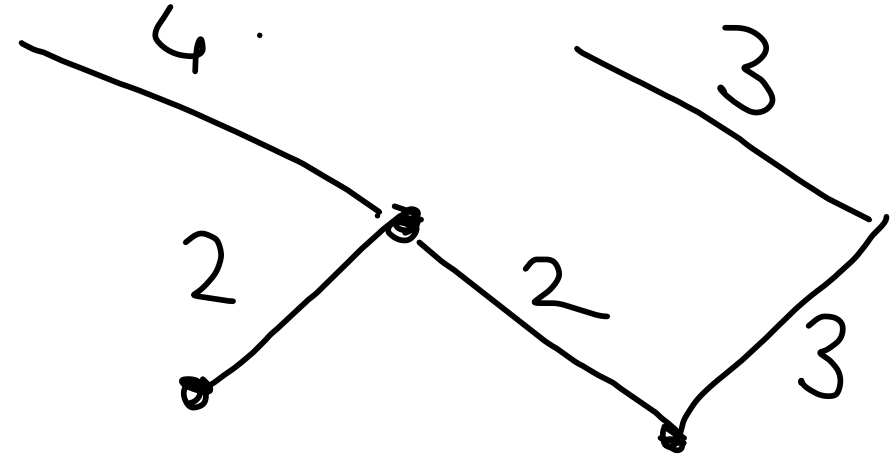


Prim's algorithm.



1. Start with a random vertex.
2. choose the adjacent edge with the lowest wt
3. Add it till it does not form a cycle.



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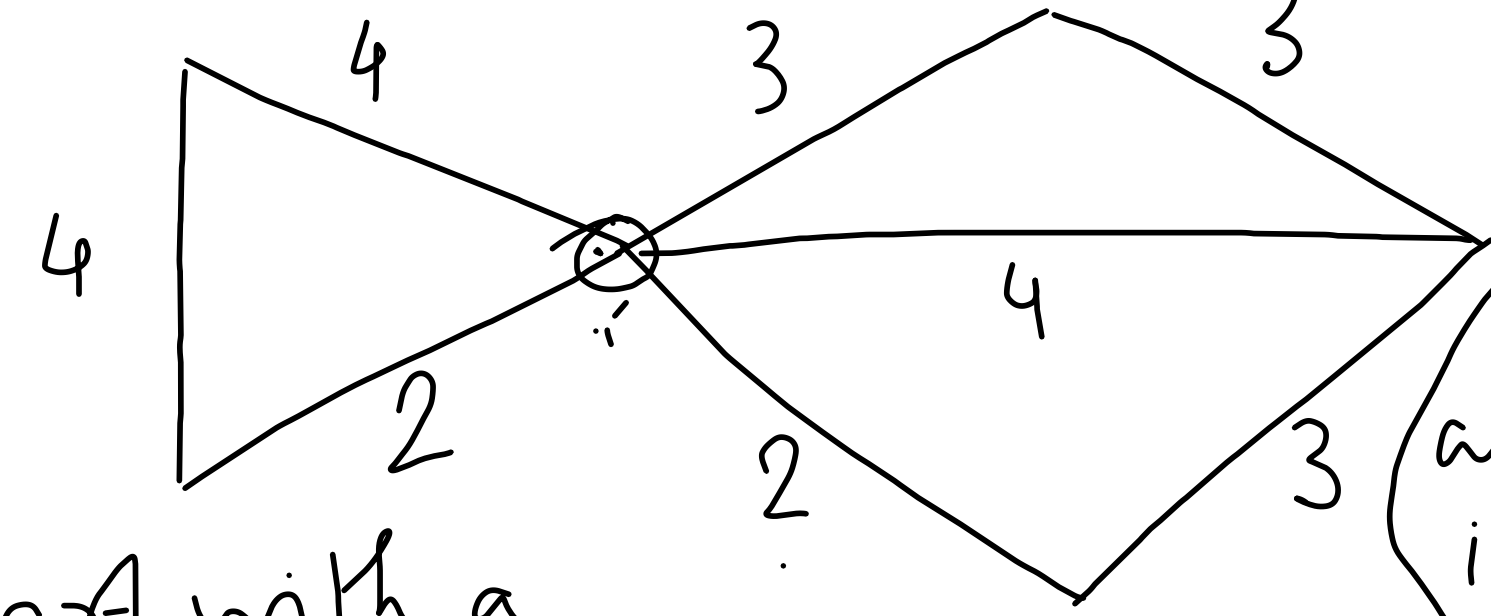
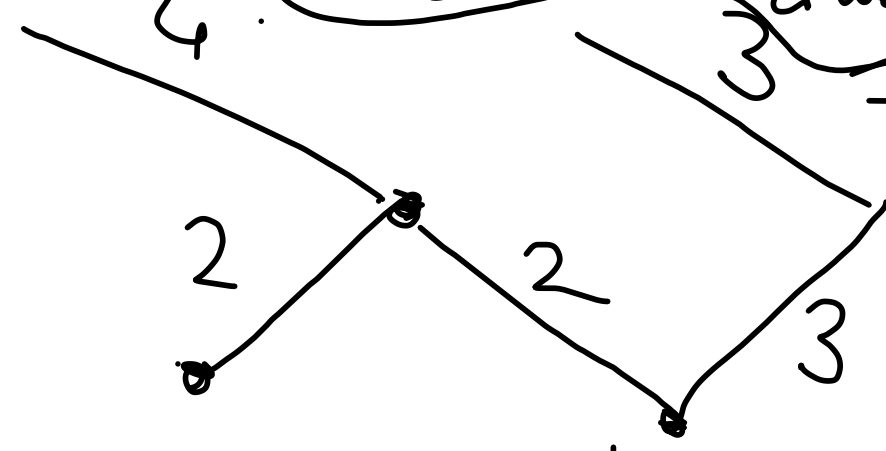


Greedy algorithms

will give an optimal solution if the underlying structure is a matroid



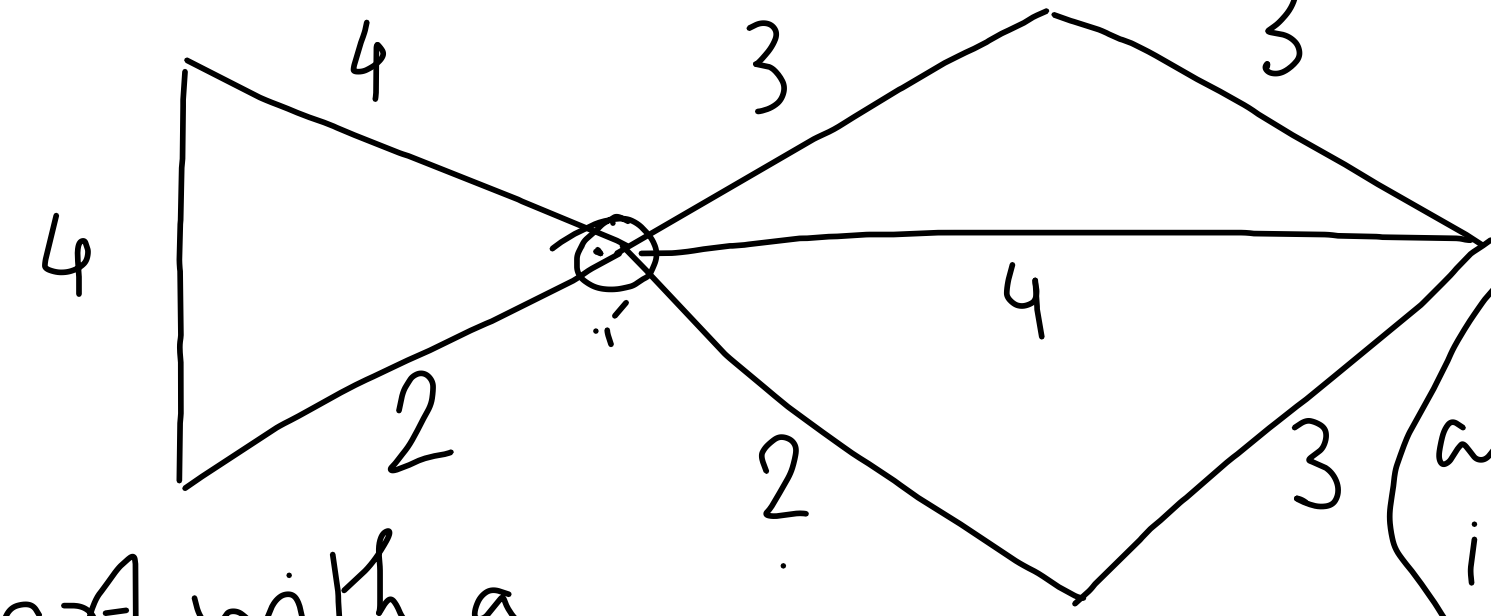
① Start with a random vertex.

② choose the adjacent edge with the lowest wt

③ Add it till it does not form a cycle.

ring times

Prim's algorithm.



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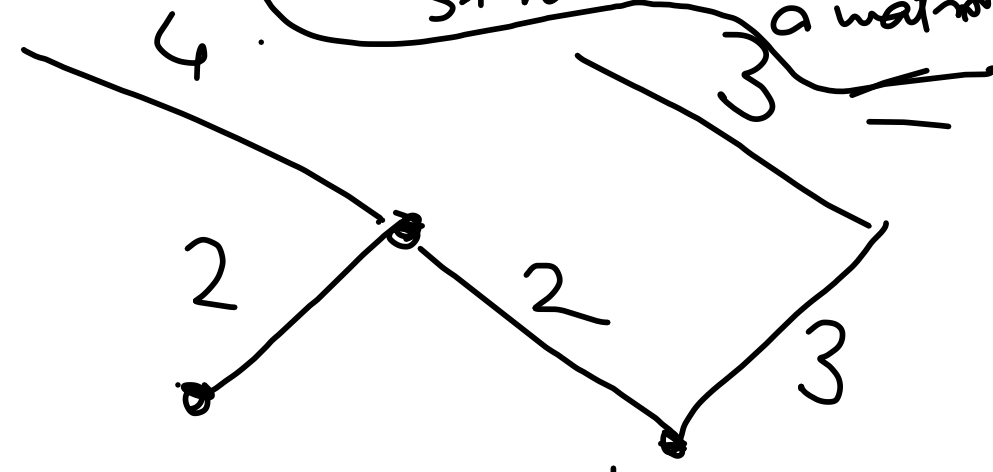

ring theory

(*) Union finding algorithm.
(to detect cycles)

Dijkstra's algorithm