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Literature Study

Feedback Vertex Set and Connected Feedback Vertex Set

## Feedback vertex set

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Literature Study

Feedback Vertex Set and Connected Feedback Vertex Set



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### Figure: A feedback vertex set: $\{1,5,7,6\}$

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Feedback Vertex Set and Connected Feedback Vertex Set



### Figure: A feedback vertex set: {1,5,7,6}



Figure: A minimum feedback vertex set: {3,7,6}

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Feedback Vertex Set and Connected Feedback Vertex Set



Figure: A feedback vertex set: {1,5,7,6}



Figure: A connected feedback vertex set: {3,4,7,6}

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Figure: A minimum feedback vertex set: {3,7,6}

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Figure: A connected feedback vertex set: {3,4,7,6}



Figure: A minimum feedback vertex set: {3,7,6}



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Figure: A minimum connected feedback vertex set: {3,4,6}

 Directed graphs are often used in path generating devices and correctness of computer programs.

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- Directed graphs are often used in path generating devices and correctness of computer programs.
- Cyclic structure makes path finding complicated and difficult to compute.

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- The usual approach is to make the graph cycle free and then analyze the paths.

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- Directed graphs are often used in path generating devices and correctness of computer programs.
- Cyclic structure makes path finding complicated and difficult to compute.
- The usual approach is to make the graph cycle free and then analyze the paths.
- Similarly this approach is also used in Constraint Satisfaction and Baysian Inference.

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■ Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs [8].

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- Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs [8].
- A 2-approximation algorithm is known for Feedback vertex set in general graphs [5].

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- Polynomial time algorithm for Feedback vertex set is known for some special graph classes [1],[2],[3].

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- To the best of our knowledge there is no approximation known for Connected Feedback vertex set problem for general graphs. A polynomial time algorithm is known for planar graphs [4].

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- Polynomial time algorithm for Feedback vertex set is known for some special graph classes [1],[2],[3].
- To the best of our knowledge there is no approximation known for Connected Feedback vertex set problem for general graphs. A polynomial time algorithm is known for planar graphs [4].
- We study the Connected Feedback vertex set problem in some classes of perfect graphs and AT Free graphs. Following we discuss those graph classes.

Yearly Progression	Report
Graph classes	

Permutation Graph

## Permutation graphs

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Permutation Graph



Figure: A Permutation diagram



Figure: The Permutation graph

 $l_i$  line joining i of upper chain to i of lower chain.

Permutation graph obtained from permutation diagram.

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Permutation Graph



Figure: (1,2,3,4) is a 4-cycle



Figure: Permutation graph with 3-cycle



Figure: Permutation diagram



Figure: Permutation diagram

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 Vertices of permutation graph can be ordered linearly. That is the permutation ordering.

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- A vertex u is said to be to the left of vertex v that is  $u \leq_{\pi} v$  if  $\pi_u < \pi_v$ .

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A vertex v in subgraph H of G (H ⊆ G), is said to be left most vertex if for every other vertex u ∈ V(H), v ≤<sub>π</sub> u. Permutation Graph

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Figure: Vertex 3 is the left most and vertex 5 is the right most



Figure: Permutation diagram

Yearly Progression Report
Graph classes

## AT Free graphs

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Yearly Progression Report
└─ Graph classes
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Figure: Example of Asteroidal Triple: consider vertices (1, 4, 8)

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└─ Graph classes
AT 6



Figure: The path (4, 5, 6, 2, 1) does not contain any neighbour of 2

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└─ Graph classes
AT 6



Figure: The path (8, 7, 6, 2, 1) does not contain any neighbour of 4

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└─ Graph classes
AT 6



Figure: The path (4, 5, 6, 7, 8) does not contain any neighbour of 1

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Graph	classes	

- Asteroidal Triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third.
- A graph that does not contain any Asteroidal Triple is called AT-free graph.



Figure: Example of AT free graph

**Dominating set** *D* is a subset of the vertices such that every vertex not in *D* is adjacent to at least one member of *D*.

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Figure: Example of dominating set

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• **Dominating pair** is a pair of vertices such that all path between them is a dominating set.

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Figure: (3,7) is a dominating pair

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Figure: (3,7) is a dominating pair

 Every AT Free graph contains at least one dominating pair, and finding such pair is easy [7].

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Yearly Progression Report
└─ Graph classes

## Chordal graphs

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In chordal graph all cycles of length four or more have a chord, which is an edge that is not part of the cycle but connects two vertices of the cycle.

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Graph classes

- In chordal graph all cycles of length four or more have a chord, which is an edge that is not part of the cycle but connects two vertices of the cycle.
- That implies that maximum induced cycle length is 3.



Figure: A chordal graph

Figure: Every cycle of length more than 4 has a chord

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└─Vertex elimination

• Vertex elimination : Elimination of vertex v is as follows.

- Add edges in N(v) such that N(v) is pairwise adjacent.
- Delete *v* and its incident edges.



Figure: Consider this graph

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Elimination of vertex v is as follows.

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Figure: Eliminating vertex 7

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Yearly Progression Report
Graph classes
Elimination Ordering

 The Sequence in which we eliminate the vertices of some graph is called elimination ordering.

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- The Sequence in which we eliminate the vertices of some graph is called elimination ordering.
- For vertex v the D(v) denotes the set of edges that is added while eliminating v.

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• For an elimination ordering  $S = v_1, \dots v_{n-1},$  $D(S) = \bigcup_{i=1}^n D(v_i).$  Elimination Ordering

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- For vertex v the D(v) denotes the set of edges that is added while eliminating v.
- For an elimination ordering  $S = v_1, \dots v_{n-1},$  $D(S) = \bigcup_{i=1}^n D(v_i).$
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• An elimination ordering S is called perfect if  $D(S) = \phi$ .

Perfect Elimination Ordering

- An elimination ordering S is called minimum if there is no other ordering S', for which D(S') < D(S).
- An elimination ordering S is called perfect if  $D(S) = \phi$ .
- Not all graphs admits a perfect ordering. consider the following example.



Figure: There is no perfect elimination ordering

- An elimination ordering S is called minimum if there is no other ordering S', for which D(S') < D(S).
- An elimination ordering S is called perfect if  $D(S) = \phi$ .
- Not all graphs admits a perfect ordering. consider the following example.
- Chordal graphs has at least one perfect elimination ordering.

• Following we give an example.

Graph classes

Perfect Elimination Ordering



Figure: A chordal graph



Graph classes

Perfect Elimination Ordering



#### Figure: A chordal graph



Figure: (1)eliminate 3

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Perfect Elimination Ordering





Figure: A chordal graph



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Figure: (1)eliminate 3

Perfect Elimination Ordering





Figure: A chordal graph







Figure: (1)eliminate 3

Figure: (3)eliminate 8

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Perfect Elimination Ordering







Figure: A chordal graph

Figure: (2)eliminate 7

Figure: (4)eliminate 1

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Figure: (1)eliminate 3

Figure: (3)eliminate 8

Perfect Elimination Ordering







Figure: A chordal graph

Figure: (2)eliminate 7

Figure: (4)eliminate 1





Figure: (1)eliminate 3

Figure: (3)eliminate 8

Figure: (5)eliminate 2

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- An elimination ordering S is called minimum if there is no other ordering S', for which D(S') < D(S).
- An elimination ordering S is called perfect if  $D(S) = \phi$ .
- Not all graphs admits a perfect ordering. consider the following example.
- Chordal graphs has at least one perfect elimination ordering.
- For the above graph 3,7,8,1,2 is a perfect elimination ordering.
- Perfect ordering can be used to decompose a chordal graph into various ordered subgraphs which satisfies certain properties. Like maximal clique [6].

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Graph classes
L Interval Graphs

# Interval graphs

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- Graph classes
LInterval Graphs

- Interval graph is a subclass of chordal graphs.
- An interval graph is an undirected graph formed from a set of intervals on the real line, with a vertex for each interval and an edge between vertices whose intervals intersect. Consider the following interval graph and its interval representation.

$$3 - 2 - 6 - 5$$

Figure: An interval representation of 6 intervals

Figure: Corresponding intervel graph



└─Our work

• We are studying this problem on the above graph classes from the point of view of approximation algorithm.

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I have done the following courses.

- Automata and Formal Languages
- Discrete Mathematics
- Graph Theory and Matroid
- Cryptology

- Design and Analysis of Algorithms
- Graph Theory and Matroid II
- Research Methodology
- Introduction to Probability and Statistics

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