Yearly Progression Report

Tamojit Saha

Computer Science Department
RKMVERI (TCG-CREST)

August 29, 2022

Feedback vertex set

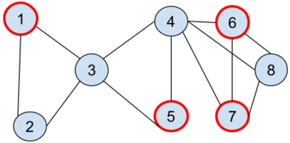

Figure: A feedback vertex set: $\{1,5,7,6\}$

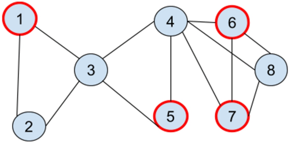

Figure: A feedback vertex set: *f*1,5,7,6*g*

Figure: A minimum feedback vertex set: *f*3,7,6*g*

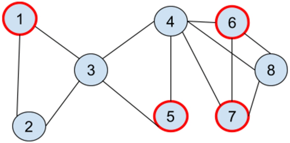

Figure: A feedback vertex set: *f*1,5,7,6*g*

Figure: A connected feedback vertex set: *f*3,4,7,6*g*

Figure: A minimum feedback vertex set: *f*3,7,6*g*

Figure: A feedback vertex set: {1,5,7,6}

Figure: A connected feedback vertex set: {3,4,7,6}

Figure: A minimum feedback vertex set: {3,7,6}

Figure: A minimum connected feedback vertex set {3,4,6}

- Directed graphs are often used in path generating devices and correctness of computer programs.

- Directed graphs are often used in path generating devices and correctness of computer programs.
- Cyclic structure makes path nding complicated and di cult to compute.

- Directed graphs are often used in path generating devices and correctness of computer programs.
- Cyclic structure makes path nding complicated and di cult to compute.
- The usual approach is to make the graph cycle free and then analyze the paths.

- Directed graphs are often used in path generating devices and correctness of computer programs.
- Cyclic structure makes path nding complicated and di cult to compute.
- The usual approach is to make the graph cycle free and then analyze the paths.
- Similarly this approach is also used in Constraint Satisfaction and Baysian Inference.

- Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs. [8]

- Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs [8].
- A 2 approximation algorithm is known for Feedback vertex set in general graphs [5].

- Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs [8].
- A 2 approximation algorithm is known for Feedback vertex set in general graphs [5].
- Polynomial time algorithm for Feedback vertex set is known for some special graph classes [1],[2],[3].

- Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs [8].
- A 2 approximation algorithm is known for Feedback vertex set in general graphs [5].
- Polynomial time algorithm for Feedback vertex set is known for some special graph classes [1],[2],[3].
- To the best of our knowledge there is no approximation known for Connected Feedback vertex set problem for general graphs. A polynomial time algorithm is known for planar graphs [4].

- Finding Feedback vertex set and Connected feedback vertex set are NP-hard problem on general graphs [8].
- A 2 approximation algorithm is known for Feedback vertex set in general graphs [5].
- Polynomial time algorithm for Feedback vertex set is known for some special graph classes [1],[2],[3].
- To the best of our knowledge there is no approximation known for Connected Feedback vertex set problem for general graphs. A polynomial time algorithm is known for planar graphs [4].
- We study the Connected Feedback vertex set problem in some classes of perfect graphs and AT Free graphs. Following we discuss those graph classes.

l_i line joining i of upper chain to i of lower chain.

Figure: A Permutation diagram

Permutation graph obtained from permutation diagram.

Figure: The Permutation graph

Figure: (1,2,3,4) is a 4-cycle

Figure: Permutation diagram

Figure: Permutation graph with 3-cycle

Figure: Permutation diagram

- Vertices of permutation graph can be ordered linearly. That is the permutation ordering.

- Vertices of permutation graph can be ordered linearly. That is the permutation ordering.
- A vertex u is said to be to the left of vertex v that is $u \quad v$ if ${}_u < {}_v$.

- Vertices of permutation graph can be ordered linearly. That is the permutation ordering.
- A vertex u is said to be to the left of vertex v that is u v if $_u < _v$.
- A vertex v in subgraph H of G (H G); is said to be left most vertex if for every other vertex u 2 $V(H)$; v u.

- Vertices of permutation graph can be ordered linearly. That is the permutation ordering.
- A vertex u is said to be to the left of vertex v that is $u \prec v$ if $\pi_u < \pi_v$.
- A vertex v in subgraph H of G $(H \subseteq G)$; is said to be left most vertex if for every other vertex $u \in V(H)$; $v \prec u$.
- $v \in V(H)$ is the right most if for every other vertex $u \in V(H)$; $u \prec v$.

- A vertex v in subgraph H of G $(H \quad G)$; is said to be left most vertex if for every other vertex $u \in V(H)$; $v \quad u$.
- $v \in V(H)$ is the right most if for every other vertex $u \in V(H)$; $u \quad v$.

Figure: Vertex 3 is the left most and vertex 5 is the right most

Figure: Permutation diagram

- Asteroidal Triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third.

Figure: Example of Asteroidal Triple: consider vertices (1; 4; 8)

- Asteroidal Triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third.

Figure: The path (4; 5; 6; 2; 1) does not contain any neighbour of 2

- Asteroidal Triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third.

Figure: The path (8; 7; 6; 2; 1) does not contain any neighbour of 4

- Asteroidal Triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third.

Figure: The path $(4; 5; 6; 7; 8)$ does not contain any neighbour of 1

- Asteroidal Triple is an independent set of three vertices such that each pair is joined by a path that avoids the neighborhood of the third.
- A graph that does not contain any Asteroidal Triple is called AT-free graph.

Figure: Example of AT free graph

- Dominating set D is a subset of the vertices such that every vertex not in D is adjacent to at least one member of D.

- Dominating set D is a subset of the vertices such that every vertex not in D is adjacent to at least one member of D.

Figure: Example of dominating set

- **Dominating set** D is a subset of the vertices such that every vertex not in D is adjacent to at least one member of D.
- **Dominating pair** is a pair of vertices such that all path between them is a dominating set.

- Dominating set D is a subset of the vertices such that every vertex not in D is adjacent to at least one member of D.
- Dominating pair is a pair of vertices such that all path between them is a dominating set.

Figure: $(3, 7)$ is a dominating pair

- **Dominating set** D is a subset of the vertices such that every vertex not in D is adjacent to at least one member of D.
- **Dominating pair** is a pair of vertices such that all path between them is a dominating set.

Figure: $(3, 7)$ is a dominating pair

- Every AT Free graph contains at least one dominating pair, and finding such pair is easy [7].

- In chordal graph all cycles of length four or more have a chord, which is an edge that is not part of the cycle but connects two vertices of the cycle.

- In chordal graph all cycles of length four or more have a chord, which is an edge that is not part of the cycle but connects two vertices of the cycle.
- That implies that maximum induced cycle length is 3.

Figure: A chordal graph

Figure: Every cycle of length more than 4 has a chord

- Vertex elimination : Elimination of vertex v is as follows.
  - Add edges in $N(v)$ such that $N(v)$ is pairwise adjacent.
  - Delete v and its incident edges.

Figure: Consider this graph

- Elimination of vertex v is as follows.
  - Add edges in $N(v)$ such that $N(v)$ is pairwise adjacent.
  - Delete v and its incident edges.

Figure: Eliminating vertex 7

- Elimination of vertex v is as follows.
  - Add edges in $N(v)$ such that $N(v)$ is pairwise adjacent.
  - Delete v and its incident edges.

Figure: Eliminating vertex 7

- Elimination of vertex v is as follows.
  - Add edges in $N(v)$ such that $N(v)$ is pairwise adjacent.
  - Delete v and its incident edges.

Figure: Eliminating vertex 7

- The Sequence in which we eliminate the vertices of some graph is called elimination ordering.

- The Sequence in which we eliminate the vertices of some graph is called elimination ordering.
- For vertex v the $D(v)$ denotes the set of edges that is added while eliminating v.

- The Sequence in which we eliminate the vertices of some graph is called elimination ordering.
- For vertex v the $D(v)$ denotes the set of edges that is added while eliminating v.
- For an elimination ordering $S = v_1, \ldots, v_{n-1}$, $D(S) = \cup_{i=1}^{n} D(v_i)$.

- The Sequence in which we eliminate the vertices of some graph is called elimination ordering.
- For vertex v the $D(v)$ denotes the set of edges that is added while eliminating v.
- For an elimination ordering $S = v_1, \ldots, v_{n-1}$, $D(S) = \cup_{i=1}^{n} D(v_i)$.
- An elimination ordering S is called minimum if there is no other ordering S', for which $|D(S')| < |D(S)|$.

- The Sequence in which we eliminate the vertices of some graph is called elimination ordering.
- For vertex v the $D(v)$ denotes the set of edges that is added while eliminating v.
- For an elimination ordering $S = v_1, \ldots, v_{n-1}$, $D(S) = \cup_{i=1}^{n} D(v_i)$.
- An elimination ordering S is called minimum if there is no other ordering S', for which $|D(S')| < |D(S)|$.
- An elimination ordering S is called perfect if $D(S) = \emptyset$.

- An elimination ordering S is called minimum if there is no other ordering S', for which $D(S') < D(S)$.
- An elimination ordering S is called perfect if $D(S) =$.
- Not all graphs admits a perfect ordering. consider the following example.

Figure: There is no perfect elimination ordering

- An elimination ordering S is called minimum if there is no other ordering S^0, for which $D(S^0) < D(S)$.
- An elimination ordering S is called perfect if $D(S) =$.
- Not all graphs admits a perfect ordering. consider the following example.
- Chordal graphs has at least one perfect elimination ordering.
- Following we give an example.

Figure: A chordal graph

Figure: A chordal graph

Figure: (1)eliminate 3

Figure: A chordal graph

Figure: (2)eliminate 7

Figure: (1)eliminate 3

Figure: A chordal graph

Figure: (2)eliminate 7

Figure: (1)eliminate 3

Figure: (3)eliminate 8

Figure: A chordal graph

Figure: (2)eliminate 7

Figure: (4)eliminate 1

Figure: (1)eliminate 3

Figure: (3)eliminate 8

Figure: A chordal graph

Figure: (2)eliminate 7

Figure: (4)eliminate 1

Figure: (1)eliminate 3

Figure: (3)eliminate 8

Figure: (5)eliminate 2

- An elimination ordering S is called minimum if there is no other ordering S', for which $D(S') < D(S)$.
- An elimination ordering S is called perfect if $D(S) =$.
- Not all graphs admits a perfect ordering. consider the following example.
- Chordal graphs has at least one perfect elimination ordering.
- For the above graph ; 3; 7; 8; 1; 2 is a perfect elimination ordering.
- Perfect ordering can be used to decompose a chordal graph into various ordered subgraphs which satis es certain properties. Like maximal cliques[6].

- Interval graph is a subclass of chordal graphs.
- An interval graph is an undirected graph formed from a set of intervals on the real line, with a vertex for each interval and an edge between vertices whose intervals intersect. Consider the following interval graph and its interval representation.

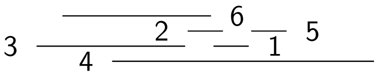

Figure: An interval representation of 6 intervals

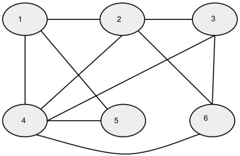

Figure: Corresponding intervel graph

- We are studying this problem on the above graph classes from the point of view of approximation algorithm.

- I have done the following courses.

- Automata and Formal Languages
- Discrete Mathematics
- Graph Theory and Matroid
- Cryptology
- Design and Analysis of Algorithms
- Graph Theory and Matroid – II
- Research Methodology
- Introduction to Probability and Statistics

[1] Y. Daniel Liang, On the feedback vertex set problem in permutation graphs, Information Processing Letters, Volume 52, Issue 3, 1994, Pages 123-129, ISSN 0020-0190.

[2] Dieter Kratsch, Haiko Müller, Ioan Todinca, Feedback vertex set on AT-free graphs, Discrete Applied Mathematics, Volume 156, Issue 10, 2008, Pages 1936-1947, ISSN 0166-218X.

[3] Chin Lung Lu, Chuan Yi Tang, A linear-time algorithm for the weighted feedback vertex problem on interval graphs, Information Processing Letters, Volume 61, Issue 2, 1997, Pages 107-111, ISSN 0020-0190.

[4] Grigoriev, A., Sitters, R. (2010). Connected Feedback Vertex Set in Planar Graphs. In: Paul, C., Habib, M. (eds) Graph-Theoretic Concepts in Computer Science. WG 2009. Lecture Notes in Computer Science, vol 5911. Springer, Berlin, Heidelberg.

[5] Bafna, Vineet and Berman, Piotr and Fujito, Toshihiro, A 2-Approximation Algorithm for the Undirected Feedback Vertex Set Problem, SIAM Journal on Discrete Mathematics,volume 12,number 3,289-297, 1999.

[6] Rose, Donald J. and Tarjan, R. Endre and Lueker, George S.,Algorithmic Aspects of Vertex Elimination on Graphs,SIAM Journal on Computing, volume 5, number 2, pages 266-283,1976.

[7] Corneil, Derek G. and Olariu, Stephan and Stewart, Lorna, Asteroidal Triple-Free Graphs, SIAM Journal on Discrete Mathematics, volume 10, number 3, pages 399-430, 1997.

[8] Garey, M. R.; Johnson, D. S. (1979). Victor Klee (ed.). Computers and Intractability: A Guide to the Theory of NP-Completeness. A Series of Books in the Mathematical Sciences. San Francisco, Calif.: W. H. Freeman and Co. pp.