Dijkstra's Algorithm

Given a connected graph.
Eedges have +ve weights
Q:- To find out the shortest distance from a Single source (vertex) to all other ~~v~~vertices.

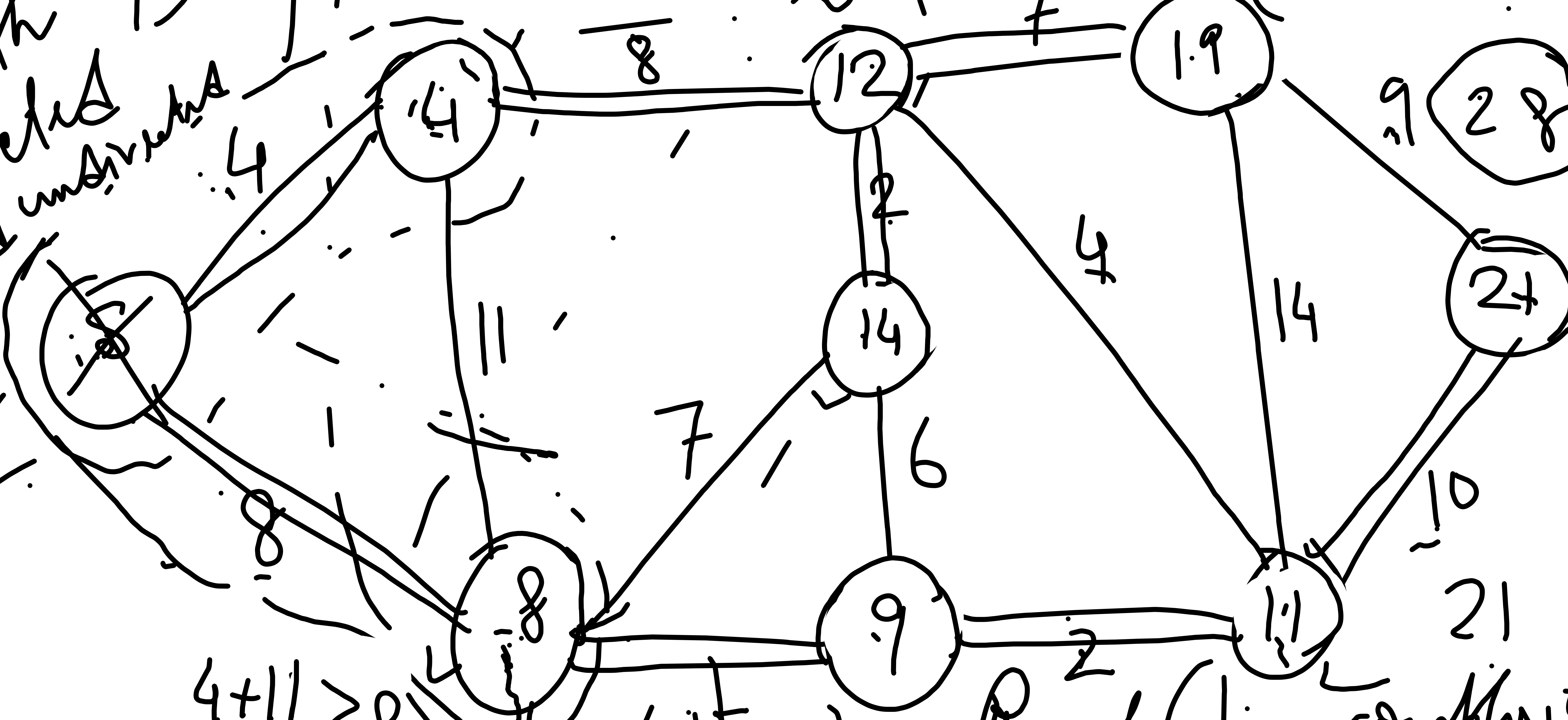
(1) What if the weights are all same?
Ans:- Run a BFS

Weights are different

Shortest path tree (BFS tree)

Dijkstra

works for both directed and undirected graph



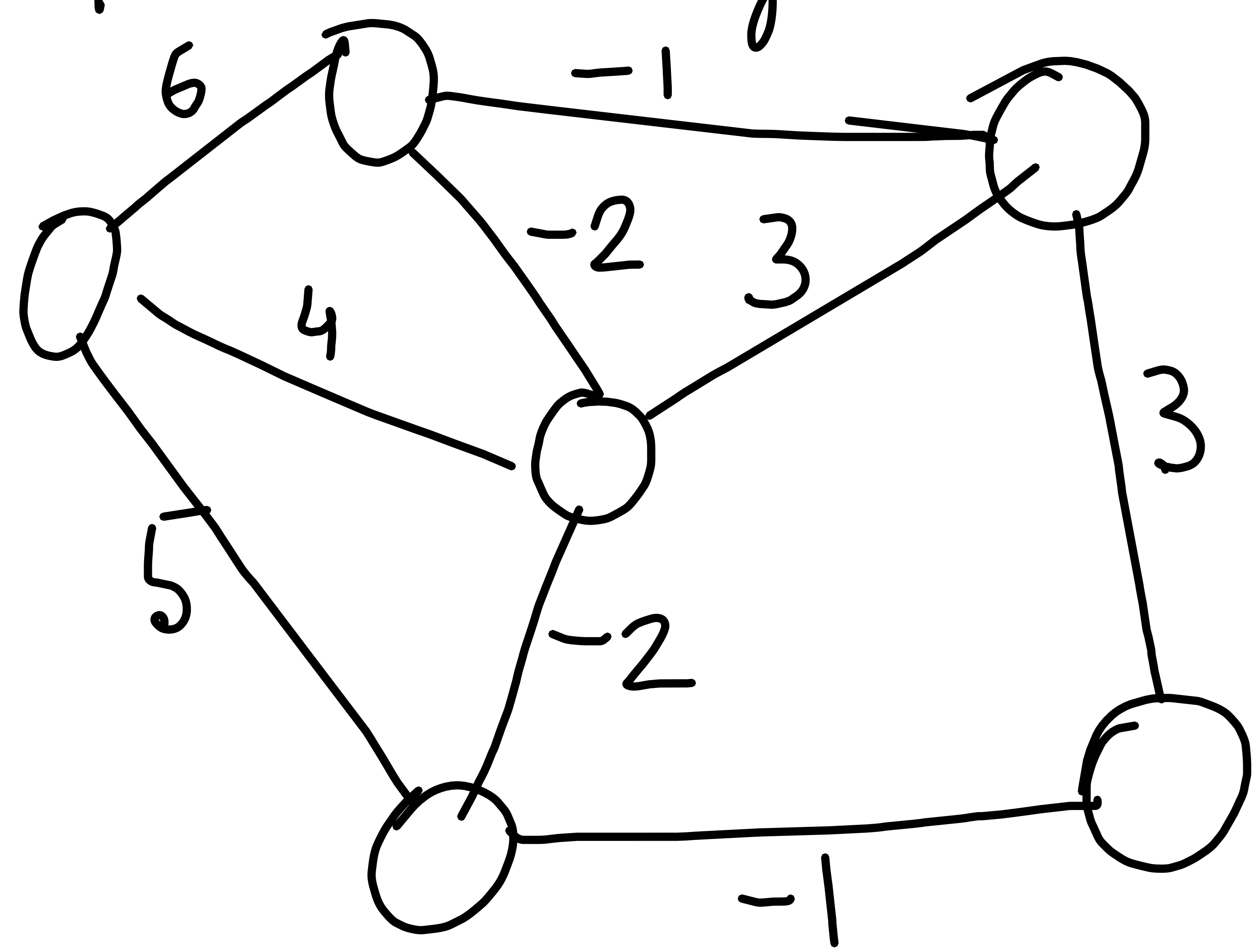
4+11 > 8

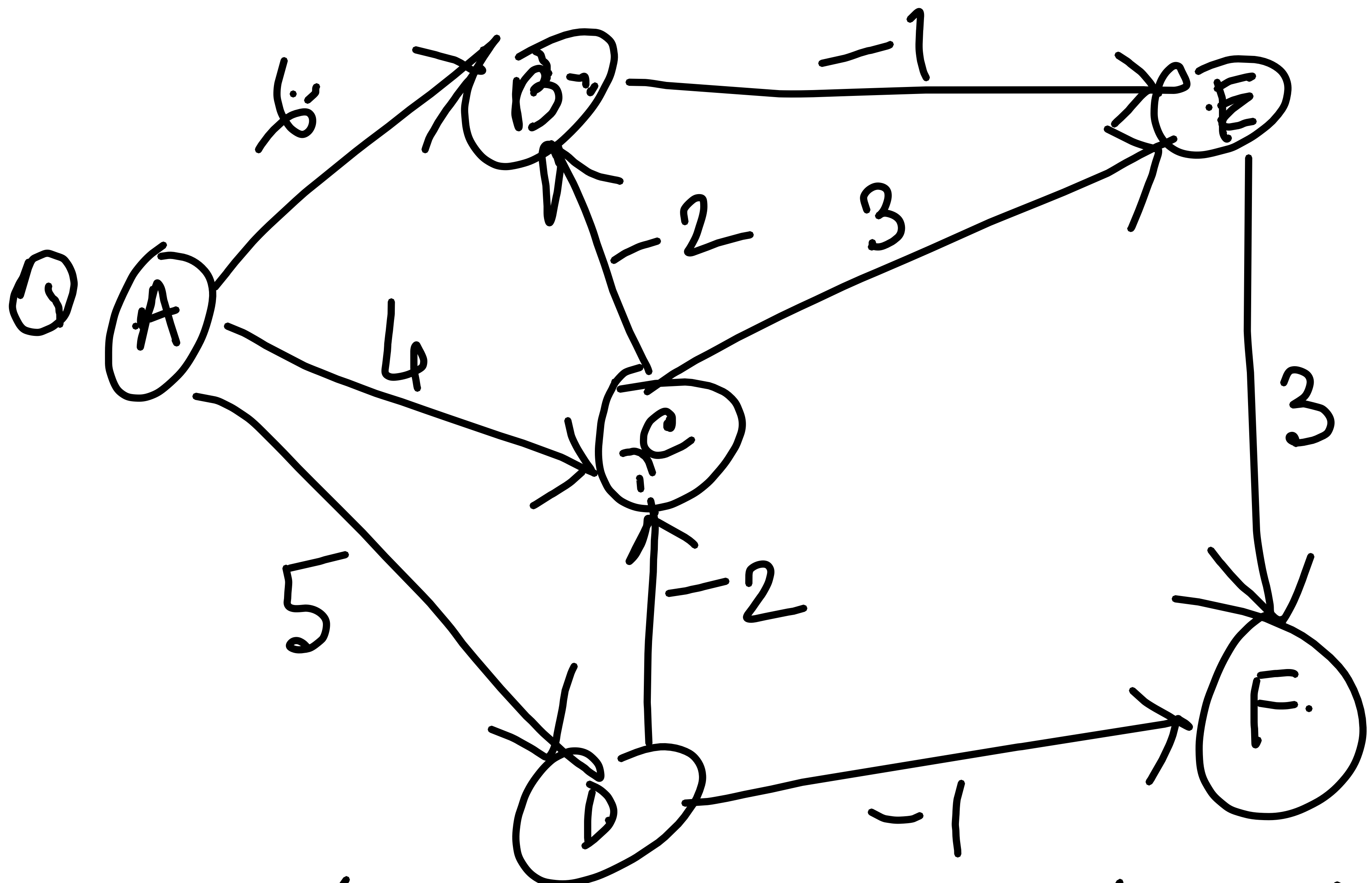
Read (time complexity)

Single source shortest path with negative edge weights.

Bellman-Ford algorithm.

Example.



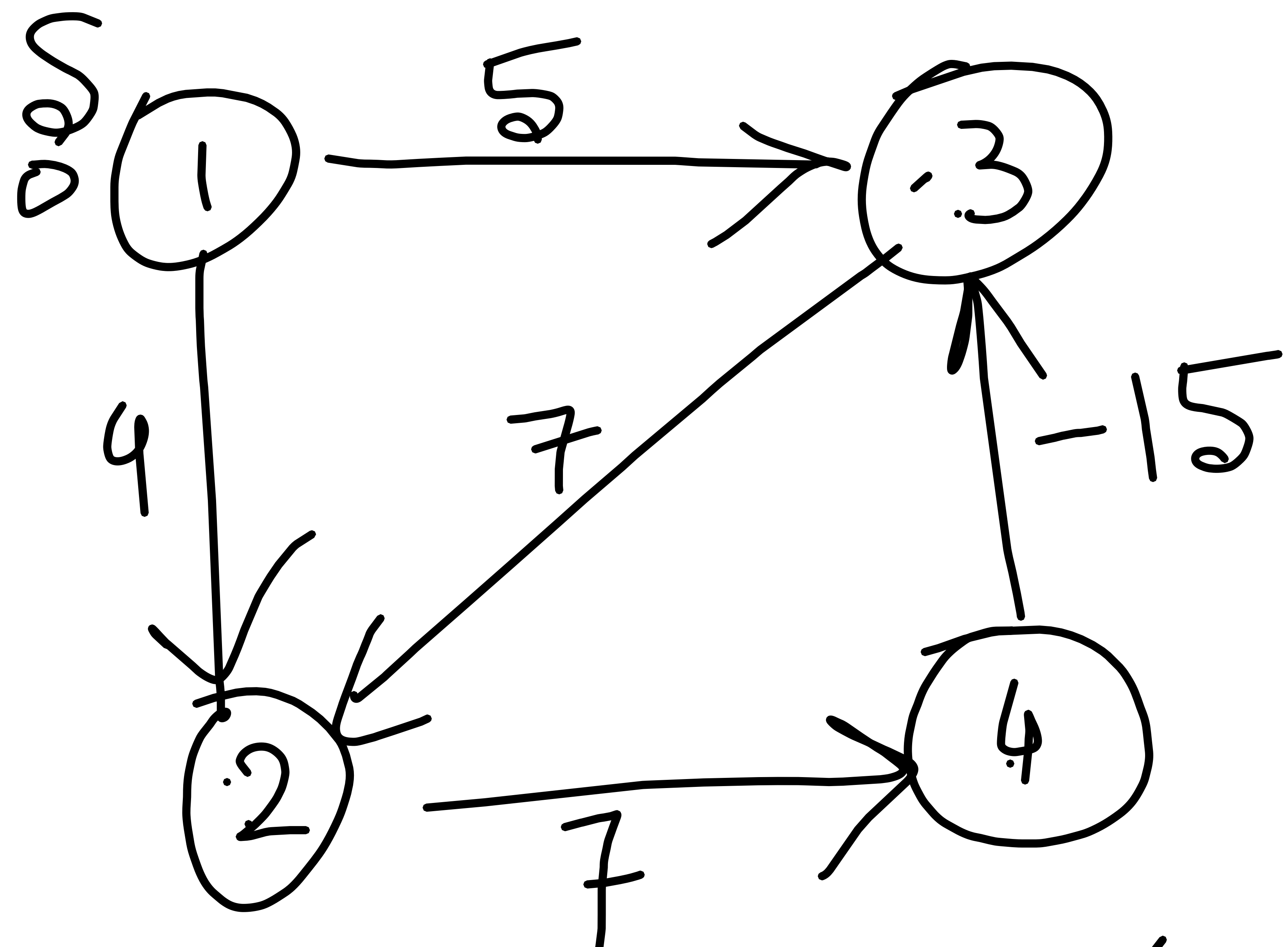


B. F algorithm runs for $|V| - 1$ number of iterations

(A B) (A C) (A D) (D C) (C B) (C E) (E F) (B E) D F

	A	B	C	D	E	F
0	0	∞	∞	∞	∞	∞
1	0	1	3	5	0	4
2	0	1	3	5	0	4
3						
4						
5						

Graph with a negative cycle



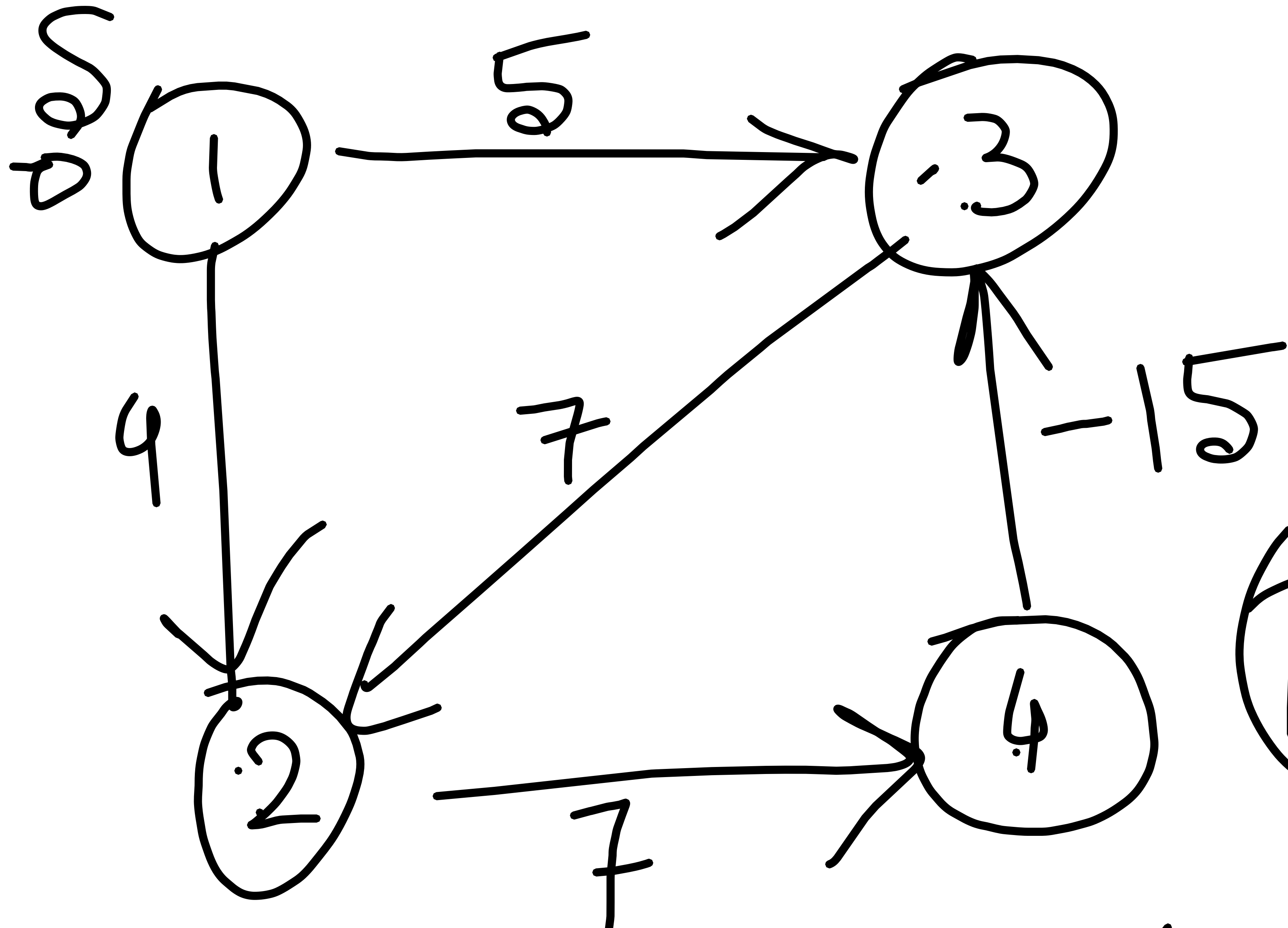
-1

(13) (12) (32) (24) (43)

	1	2	3	4
0	0	∞	∞	∞
1	0	4	-4	11
2	0	3	-5	10
3	0	2	-6	9

4 0 1 -7 8

Graph with a negative cycle



to detect a negative cycle

−1

(Time complexity)

(13) (12) (32) (24) (43)

	1	2	3	4
0	0	∞	∞	∞
1	0	4	−4	11
2	0	3	−5	10
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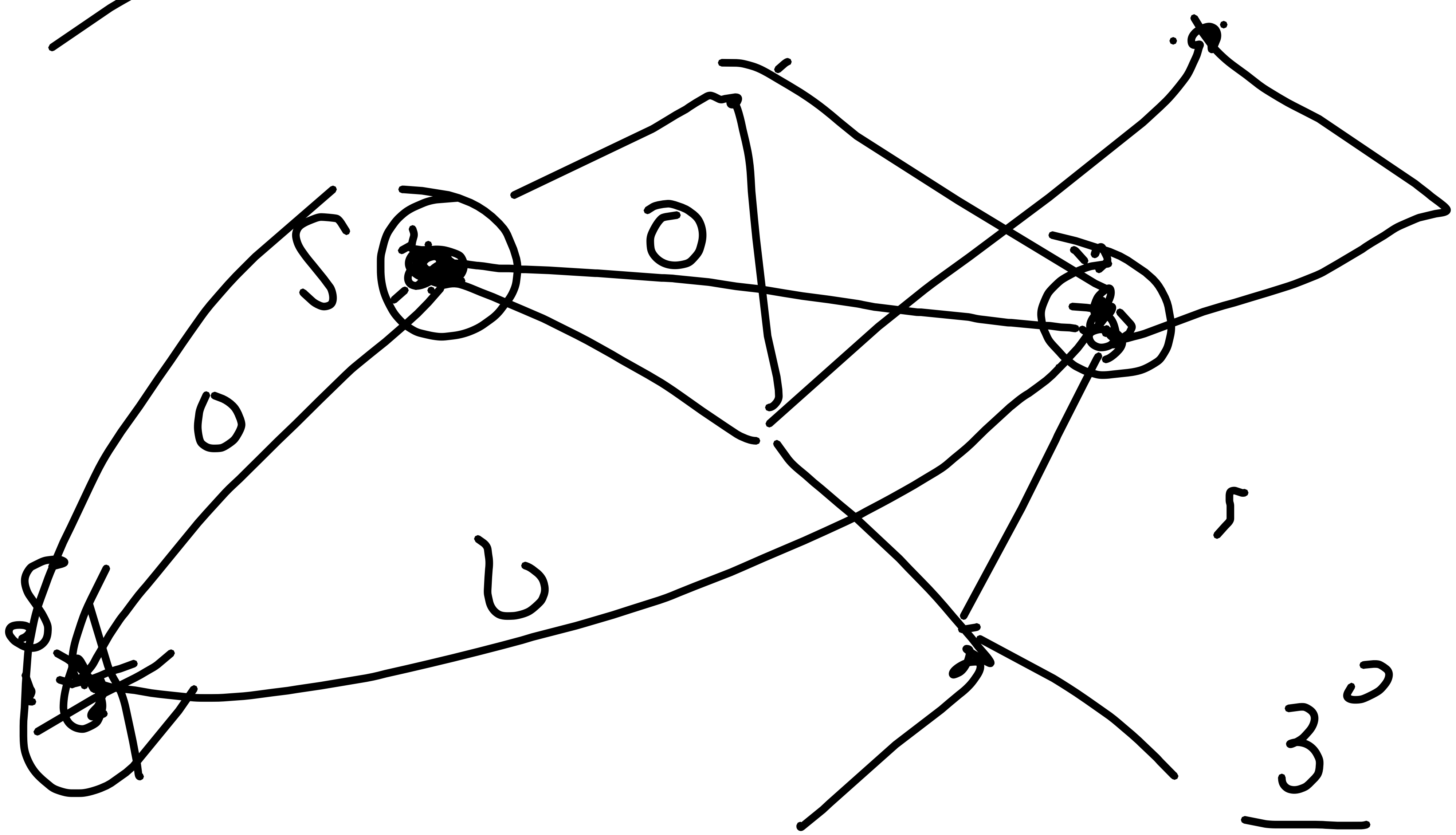
4 0 1 −7 8

Q. Multiple Source shortest path?

1° unweighted graph?



2° weighted graphs (all weights are +ve)

3° negative edge?

Q. multiple source (negative edge) no negative cycle

1. add another vertex (q)
2. join q with the Source vertices by 0-weight edges
3. Bellman-Ford v $h(v)$
4. Remove q
5. Update the weight of the edges

Q. multiple source (negative edge) no negative cycle.

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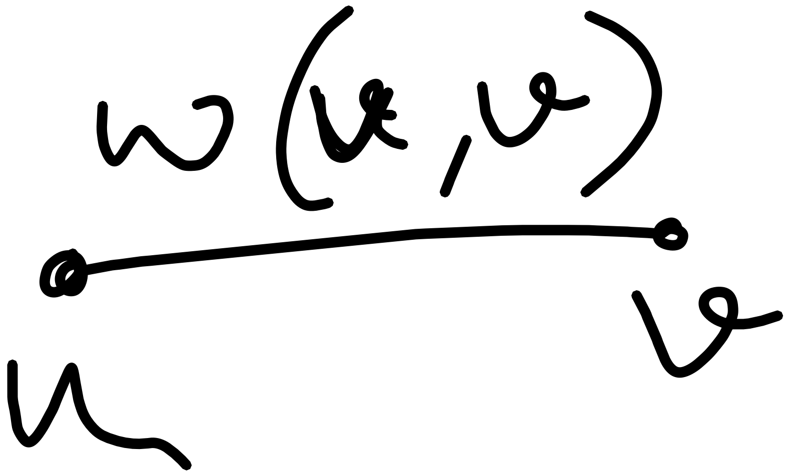


Johnson's algorithm

$$\underline{w'(u,v)} = w(u,v) + h(u) - h(v)$$

$$> 0$$

6 Multiple source for +ve weight edges

$$D(u,v)$$

7 update $D(u,v) = D(u,v) + h(v) - h(u)$

Q. Suppose you have to find out two disjoint paths joining a pair of vertices such that the total weight is minimum

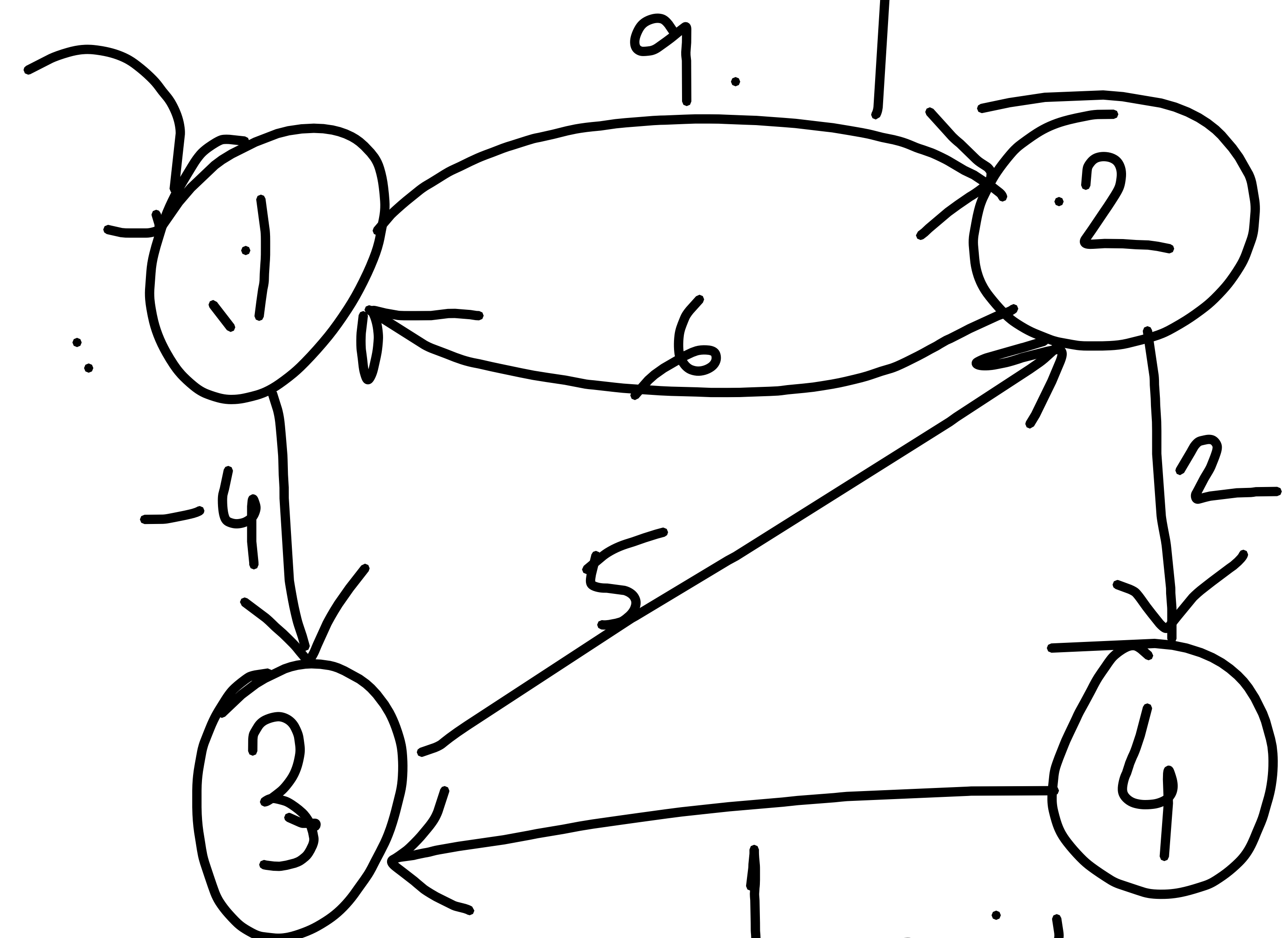
Suurballe's algorithm (Networking)

Q. All pairs shortest path.

Floyd-Warshall algorithm

1 2   1 3   1 4

2 3   2 4   3 4



Adjacency matrix

$$D^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & \infty \\ 2 & 6 & 0 & \infty & 2 \\ 3 & \infty & 5 & 0 & \infty \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

$$D^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & \infty \\ 2 & 6 & 0 & 2 & 2 \\ 3 & \infty & 5 & 0 & \infty \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

$[2,3] \approx [2,1] + [1,3]$

Q. All pairs $O(|V|^3)$ shortest path.

Floyd-Warshall algorithm

1 2   1 3   1 4

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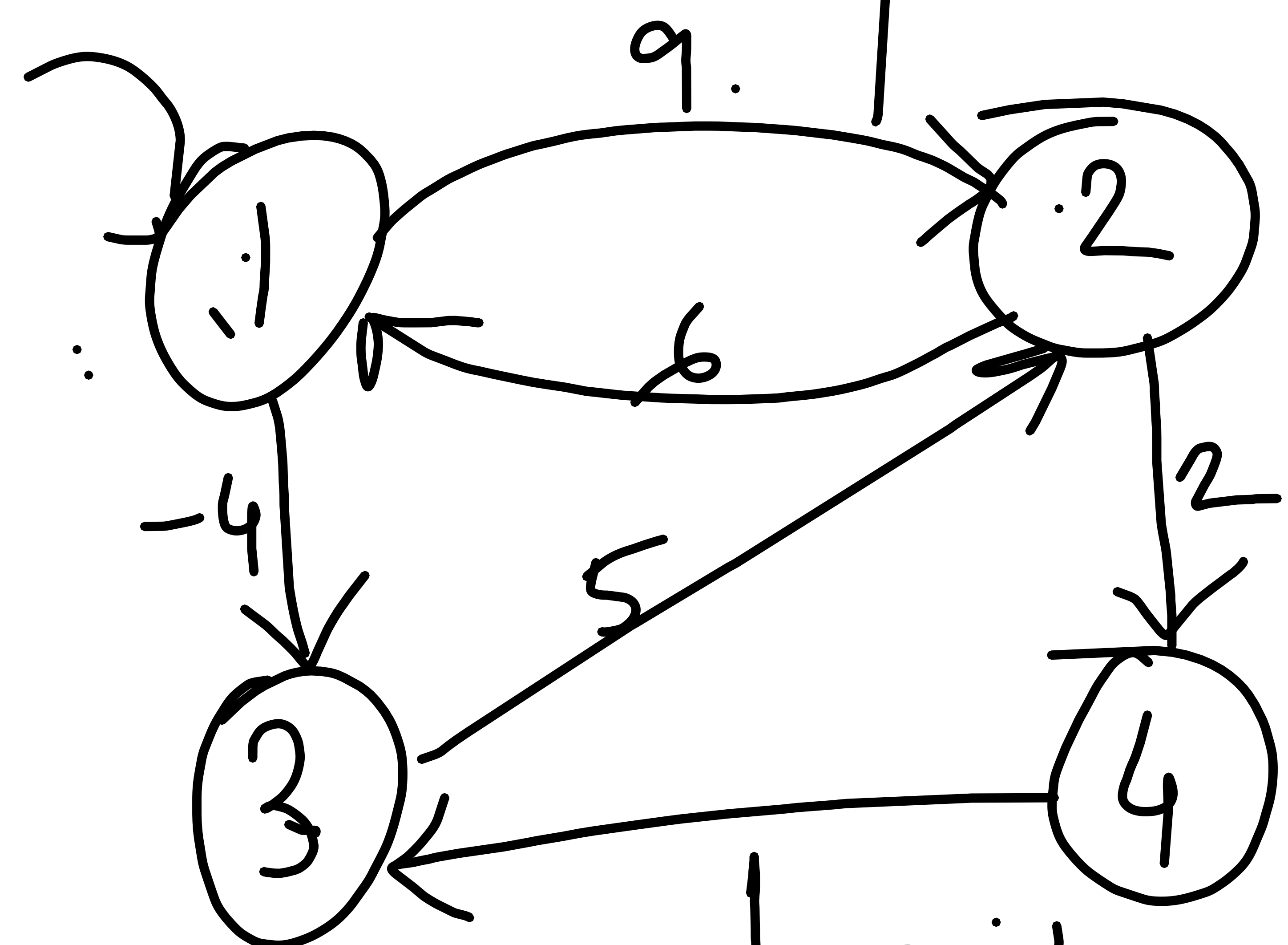

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$[2,3] \approx [2,1] + [1,3]$

after midsem

Ford - Fulkerson algorithm

$D^0 \quad D^1 \quad D^2 \quad D^3 \quad \underline{D^4}.$

$$D^k[i,j] = \min\left[D^{k-1}[i,j],\ D^{k-1}[i,k-1] + D^{k-1}[k-1,j]\right].$$

$\underline{D^4}$ $(3,2)$