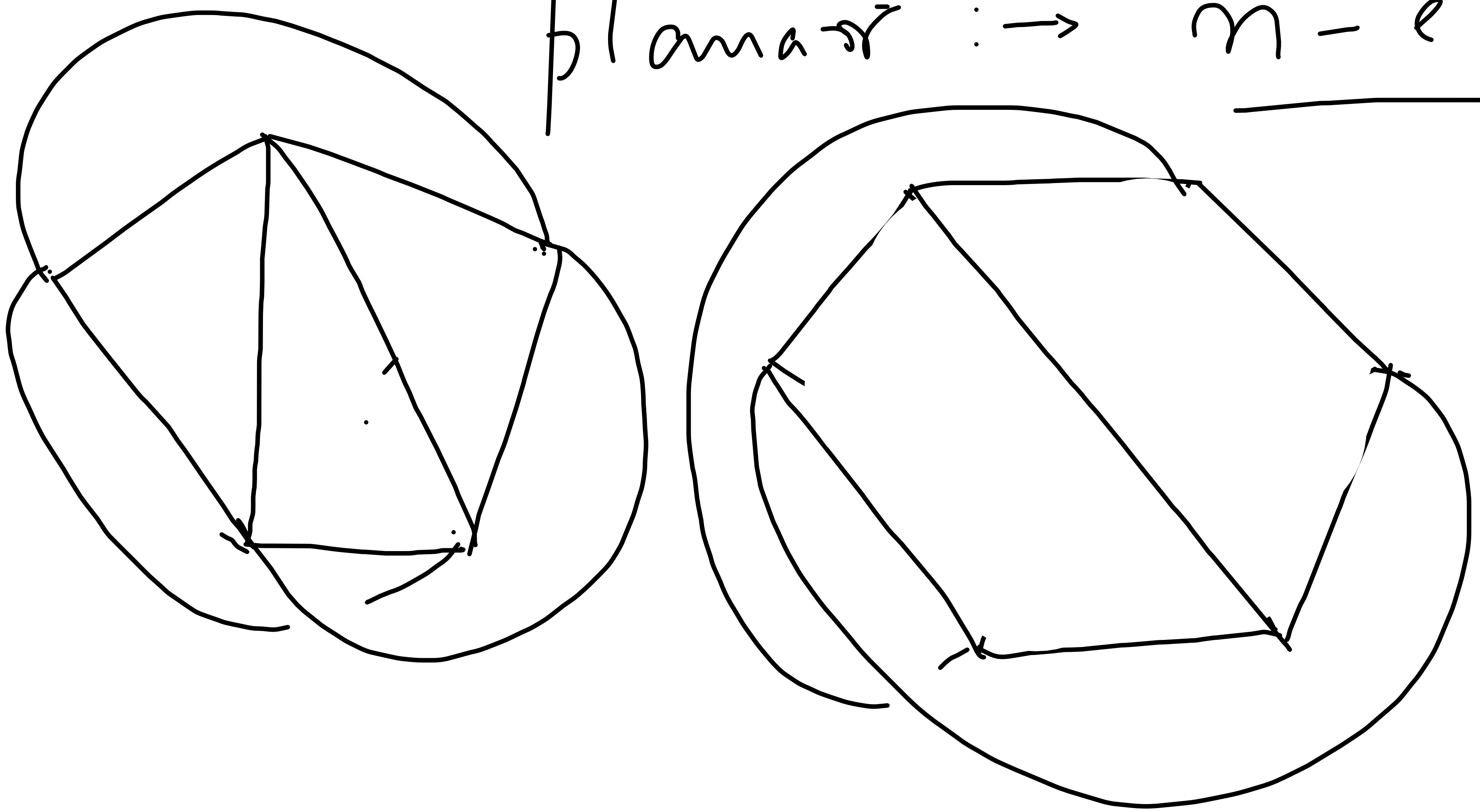
Planarity.

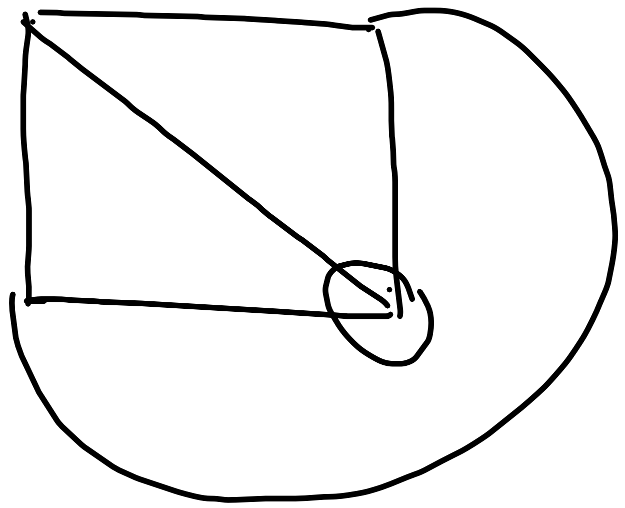
K_5 or $K_{3,3}$ are not planar. (why?)

planar :→ $\underline{n - e + f = 2}$.



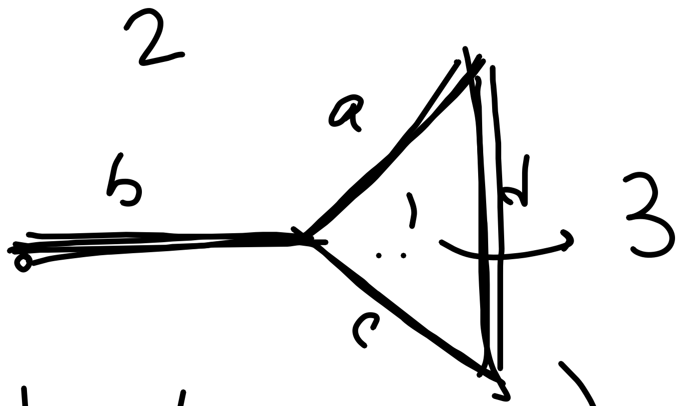
Outerplanar graph has a drawing s.t. all the vertices are adjacent to the unbounded face.

Example :- K_4, $K_{2,3}$ $\longrightarrow$ planar.
are these graphs outerplanar?



$K_{2,3}$ does not have a spanning cycle.

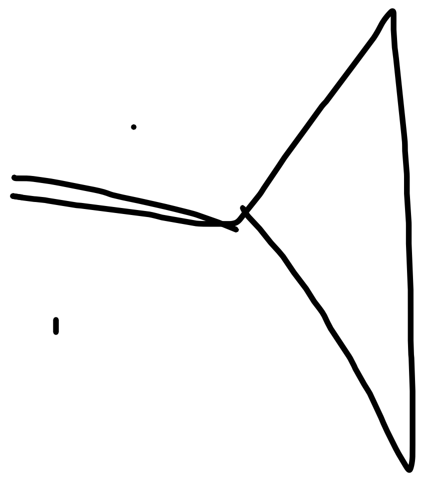
Length of a face :— total length of the closed walk(s) bounding the face.



$(a+c+d) + (a+2b+c+d)$

$= 2(a+b+c+d)$

$(a+b+b+c+d)$ length of the unbounded face.



$f_1, f_2, \ldots f_k, \ldots$

$\sum l(f_i) = 2e$

Application n, e, f

If a simple planar graph has at least 3 vertices then:

$$e \leq 3n - 6.$$

Proof.

$$l(f_i) \geq 3$$

$$2e = \sum l(f_i) \geq 3f.$$

$$n - e + f = 2.$$

$$e \leq 3n - 6.$$

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$$e \leq 3n - 6$$

If a simple planar graph with at least 3 vertices is triangle free then

$$e \leq 2n - 4.$$

$$2e = \sum e(f_i) \geq 4f \rightarrow n - e + f = 2.$$

K_5, $K_{3,3}$ are non planar?

$n = 5$, $e = 10$

If a simple planar graph with at least 3 vertices is triangle free. then

$$e \leq 2n - 4.$$

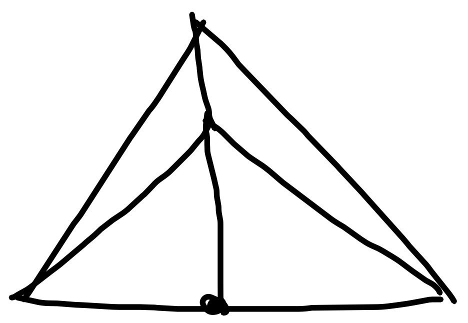
$$2e = \sum e(f_i) \geq 4f \quad \hookrightarrow \quad n - e + f = 2.$$

$\underline{K_5}$, $K_{3,3}$ are non planar?

$n = 5$, $e = \underline{10}$

Maximal planar graph :— if a planar graph is not a spanning subgraph of another graph.

Defn : Triangulation :— is a simple graph s.t the boundary of every face is a 3-cycle



Characterization :-

Equivalent statements (G is planar)

① G has exactly $3n-6$ edges

② G is a triangulation.

③ G is a maximal planar graph.

Proof :- ① $e \leq 3n-6$ $\quad 1 \Rightarrow 3$.

$1 \Rightarrow 2$. $\quad 2 \Rightarrow 1$

Characterization :-

Equivalent statements (G is planar)

① G has exactly $3n-6$ edges

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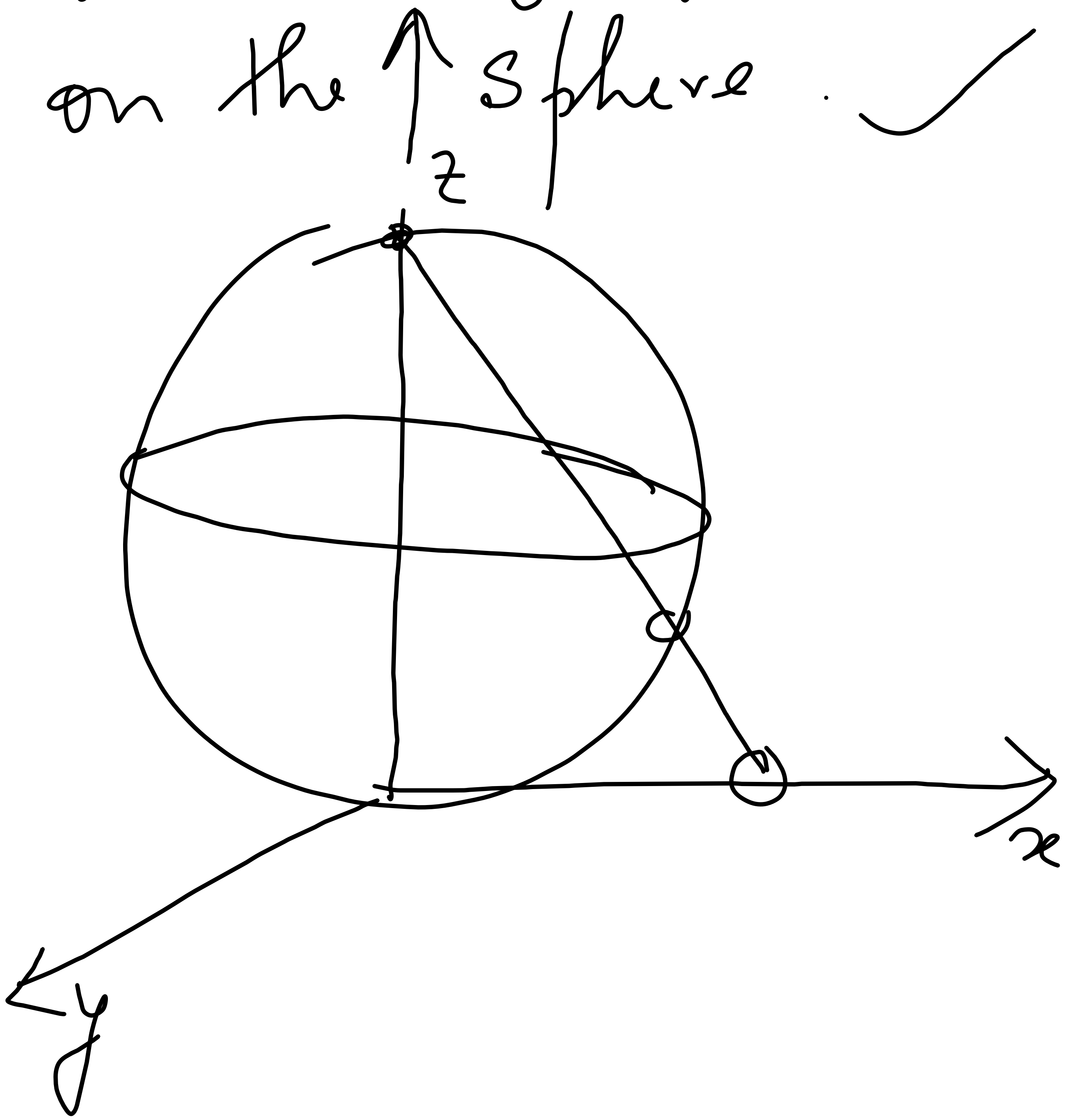
Proof :- ① $e \leq 3n-6$ $1 \Rightarrow 3$.

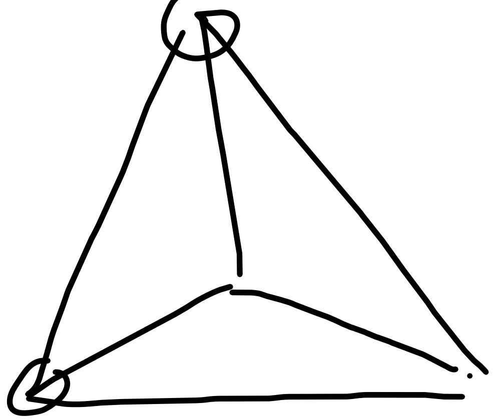
$1 \Rightarrow 2$ $2 \Rightarrow 1$

$$2e = 3(f-1)+4$$
$$= 3f-3+4$$
$$= 3f+1$$

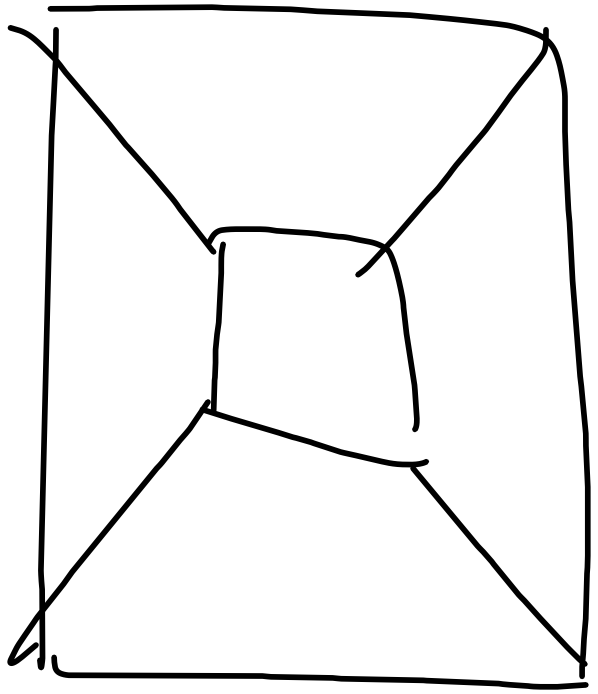
Any planar graph can also be embedded on the sphere. ✓ Stereographic projection.

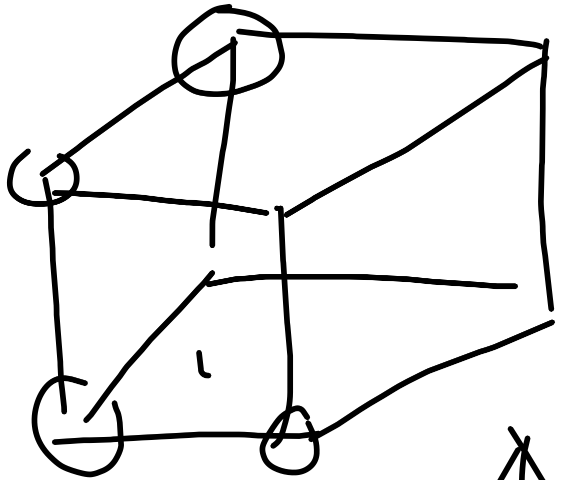


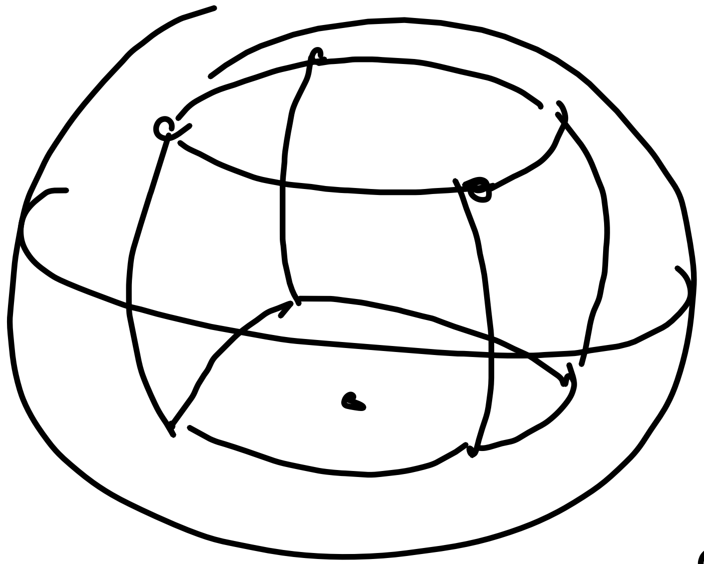
Regular polyhedra

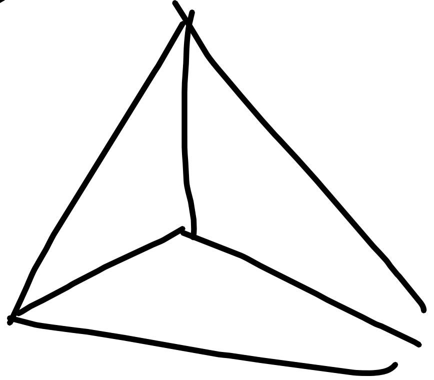
is a 3d-object (3d-graph) 

Such that every bounded face has the same length.









→ as a planar graph on $\mathbb{R}^2$.

Suppose G is a planar k-regular graph s.t all the faces have the same length. ($n \longrightarrow$ no of vertices).

$$2e = k \times n = \underline{l} \times f.$$

$$n - e + f = 2.$$

$$\Rightarrow e\left(\frac{n}{e} - 1 + \frac{f}{e}\right) = 2.$$

$$\Rightarrow \underline{e}\left(\frac{2}{k} - 1 + \frac{2}{l}\right) = \underline{2}.$$

$$\longrightarrow \left(\frac{2}{k} - 1 + \frac{2}{l}\right) > 0$$

$$\Longleftrightarrow \underline{(k-2)(l-2) > 0.}$$

Suppose G is a planar k-regular graph s.t all the faces have the same length. ($n \longrightarrow$ no of vertices).

$$2e = k \times n = \underline{l} \times f.$$

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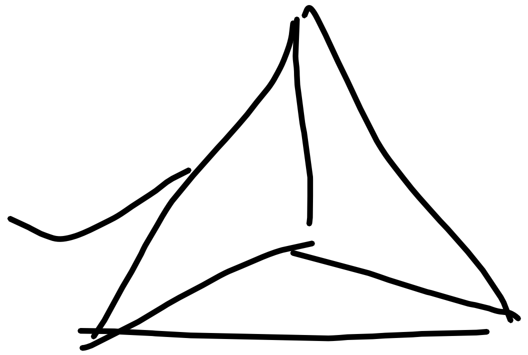
$$\Rightarrow \underline{e}\left(\frac{2}{k} - 1 + \frac{2}{l}\right) = \underline{2}.$$

$$\left(\frac{2}{k} - 1 + \frac{2}{l}\right) > 0$$

$$\Leftrightarrow \underline{(k-2)(\underline{l}-2) < 4}$$

$$(k-2)(l-2) < 4.$$



Suppose $k, l \geq 3$.

$k, l \leq 5$

(k, l) tetrahedron cube octahedron dodecahedron cube

$(3,3)$, $(3,4)$, $(3,5)$, $(4,3)$, $(5,3)$

icosahedron.

Platonic solids.

Colouring

$f : V \longrightarrow S$. proper colouring (vertex).

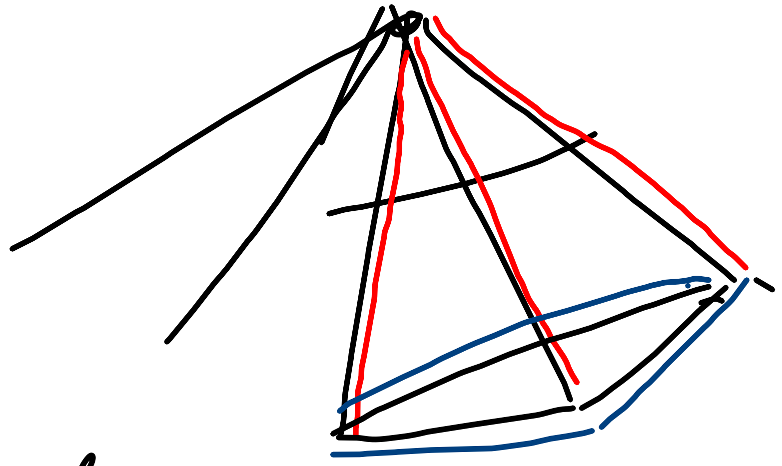
$\chi(G)$ is the minimum no of colours required to properly colour a graph.

corresponding edge ch... no.

Any edge colouring of K_6 with <u>two colours</u>. must contain a mono-chromatic triangle.

(Ramsey theory)

(Extremal combinatorics)



<u>Exc</u> $\rightarrow$ try to generalize this statement

Evasive Boolean functions / graph properties.

Example : Emptiness.

Players A (no of vertices n)

Adversary B.

no

Question that A can ask is "(u, v) is an edge?"

(Evasive)

Example: Connectedness

A (n vertices
u,v is
(u,v) an edge?) $\longrightarrow$ B (Starts with a complete graph G on n vertices)

1. if removing (u,v) from G disconnects G then leave (u,v) in G and answer YES
2. else remove (u,v) from G and answer NO

Example

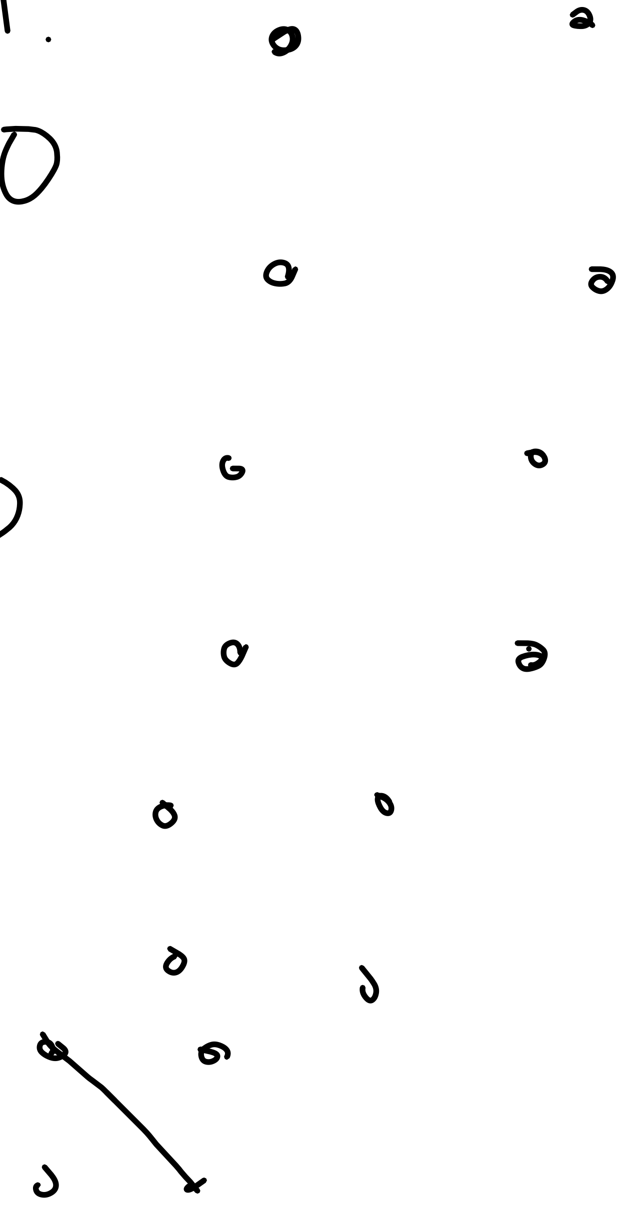
$n = 4$

$A \xrightarrow{6} \text{NO}$

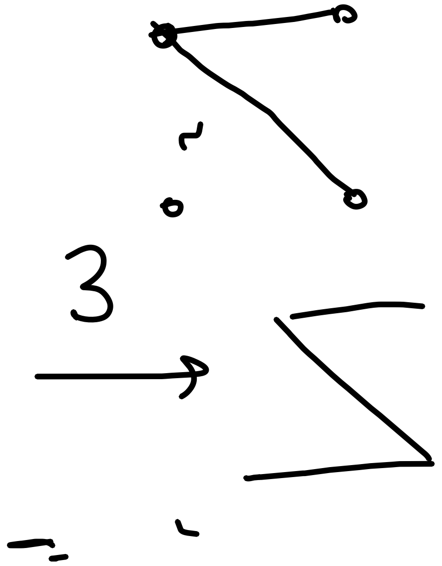
$A \xrightarrow{4} \text{NO}$

$A \xrightarrow{2} \text{NO}$

$A \xrightarrow{5} \text{YES}$

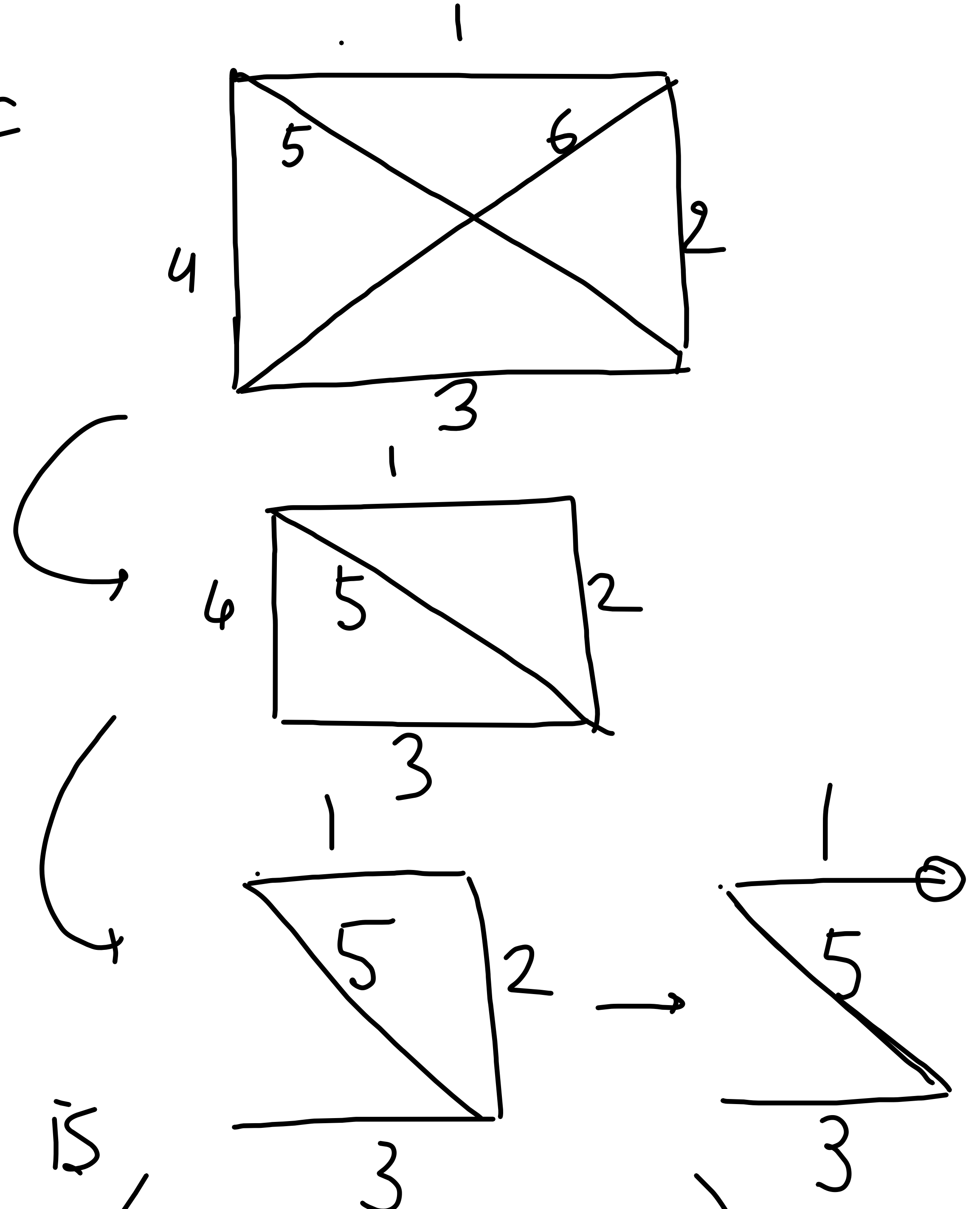


$A \xrightarrow{1} \text{YES}$



$A \xrightarrow{3}$

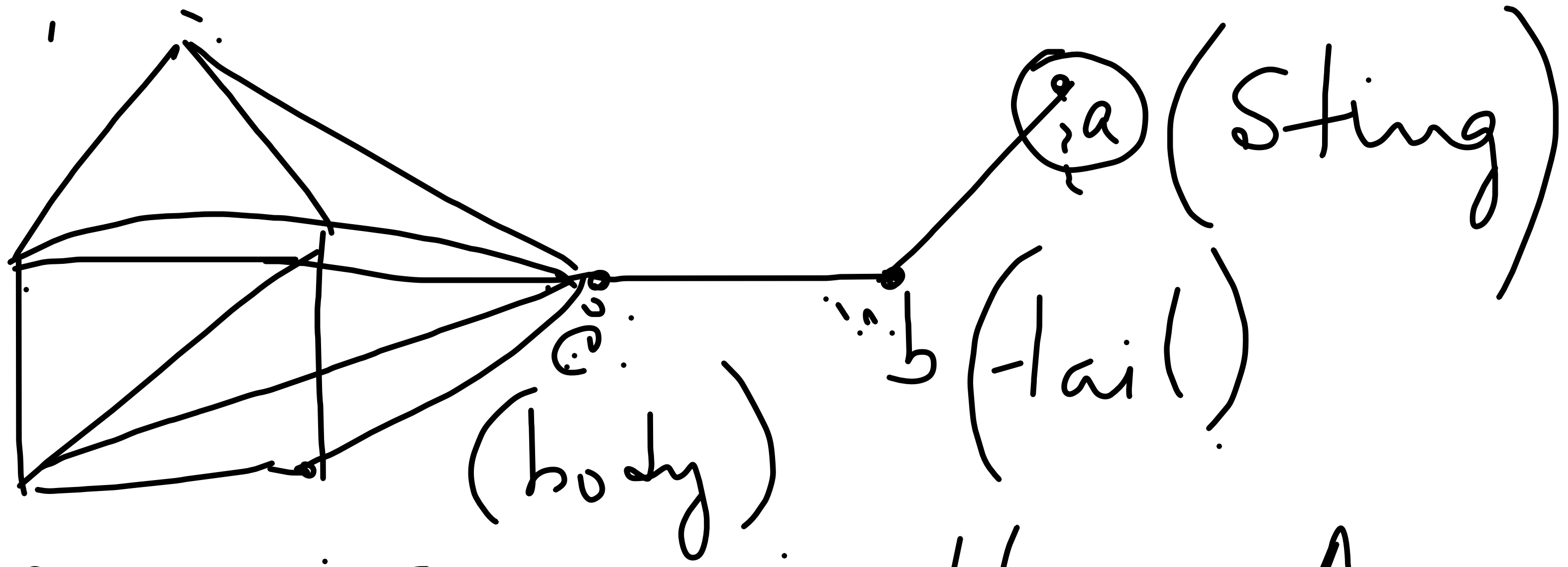
$B =$



Connectedness is erasable (Assignment)

Example of a non-evasive graph property.

<u>Scorpion graph property</u>



to determine whether a graph is a scorpion graph is non-evasive

<u>Proof</u>: Suppose the player A selects a vertex <u>p</u>. → p is connected to every other vertex except one vertex (a) <u>(n-1)</u>, $\frac{n-2}{3n-6}$.

Suppose p is a.

$n-1$

$O(n)$

$\frac{n(n-1)}{2} \approx O(n^2)$

$\Omega(n^2)$

$8n-26$ not $O(n^2)$.

Complete the proof. (Exc)