Assignment 3
Design and Analysis of Algorithms

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1. You are given an array of n distinct integers. What is the minimum number of comparisons you require to report an element that is **not** the i^{th} $(1 \leq i \leq n)$ minimum?

2. Consider an array $A = [a_1, \ldots, a_n]$ of n real numbers sorted in ascending order as input. Another array $B = [b_1, \ldots, b_n]$ is created such that $b_i = a_i^2$.

   (a) Write a constant time algorithm to report the maximum element in B.

   (b) Write an efficient (logarithm time) algorithm to report the minimum element in B.

   (c) Write an efficient (linear time) algorithm to sort B.

3. To determine which of your Facebook followers were early adopters, you decide to sort them by their Facebook account ids, which are 64-bit integers. Which sorting algorithm you will use?

4. Prove or Refute: If we only assume that all buckets have the same size, Bucket Sort runs in $O(n)$-time on average independent of the input distribution.

5. Consider the following recursive algorithm for finding the 2-nd smallest element in an array of n elements:

```
Find_2nd_Minimum(A[1..n])
{
    if((n==2) and (A[1]<A[2]))
        return A[2]
    for(i=1; i<=n/2; i++)
        if(A[i] > A[n/2+i])
            Swap(A[i], A[n/2+i]);
    Find_2nd_Minimum(A[1..n/2]);
}
```

   Justify the correctness of the algorithm.

6. Consider an n integer array containing $\lceil n/\lg n \rceil$ even integers. Assuming the odd integers in A appear in sorted order, design an efficient algorithm to sort the array A in $O(n)$ time. You may use $O(n)$ extra space.