Grammers to PDA

$G = (V, T, P, S)$

PDA: $P' = (\{q\}, T, V \cup T, \underline{\underline{\delta}}, q, S)$.

{Empty Stack}

① $\forall$ variable A,

$$\delta(q, \epsilon, A) = \{(q, \beta) \mid A \to \beta \text{ is a Production in } P\}$$

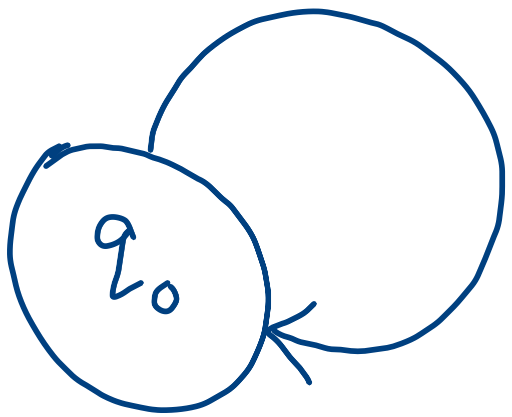
② $\forall$ terminal a,

$$\delta(q, a, a) = \{(q, \epsilon)\}$$

Set of Palindrome's over $\{0,1\}$

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

PDA



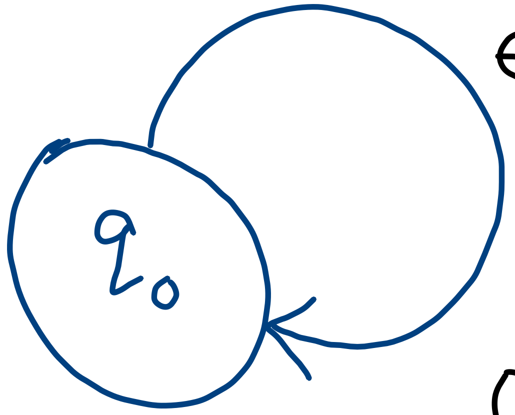
Set of Palindrome's over $\{0,1\}$

$$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

$\underline{01010}$ (w) $\Rightarrow$ left most derivation

$S \to 0\underline{S}0$
$\to 01S10$
$\to 01010$

PDA



$\epsilon, S \to 0S0$
$\epsilon, S \to 1S1$
$\epsilon, S \to 0$
$\epsilon, S \to 1$
$\epsilon, S \to \epsilon$
$0, 0 \to \epsilon$
$1, 1 \to \epsilon$
$\epsilon, \epsilon \to \epsilon$

y_1: $\boxed{01010}$, stack: S

(1) $\Downarrow$ $\epsilon, S \to 0S0$

$\boxed{01010}$, stack: 0, S, 0

(2) $\Downarrow$ $0, 0 \to \epsilon$

$\boxed{1010}$ y_2, stack: S, 0

(3) $\Downarrow$ $\epsilon, S \to 1S1$

$\boxed{1010}$, stack: 1, S, 1, 0

(4) $\Downarrow$ $1, 1 \to \epsilon$

$\boxed{010}$ y_3, stack: S, 1, 0

(5) $\Downarrow$ $\epsilon, S \to 0$

$\boxed{010}$, stack: 0, 1, 0

$\Downarrow$ $0,0 \to \epsilon$; $1,1 \to \epsilon$; $0,0 \to \epsilon$

$\boxed{\epsilon}$ y_4, stack: empty

$w \in L(G) \Rightarrow w \in N(P')$

$(q_0, w, S) \vdash^* (q_0, \epsilon, \epsilon)$

Sentential Form

$\Downarrow$

{leftmost derivation}

$\Downarrow$

$\gamma_i = \underline{x_i} \alpha_i$

$$\underbrace{S}_{\gamma_1} \rightarrow \underbrace{0S0}_{\gamma_2} \rightarrow \underbrace{01S10}_{\gamma_3} \rightarrow \underbrace{01010}_{\gamma_4}$$

$\gamma_2 = \underset{x_2}{\underline{0}} \; \underset{\alpha_2}{\underline{S0}}$

$\gamma_3 = \underset{x_3}{\underline{01}} \; \underset{\alpha_3}{\underline{S10}}$

$\gamma_4 = \underset{x_4}{\underline{01010}} \; \underset{\alpha_4}{\underline{\epsilon}}$

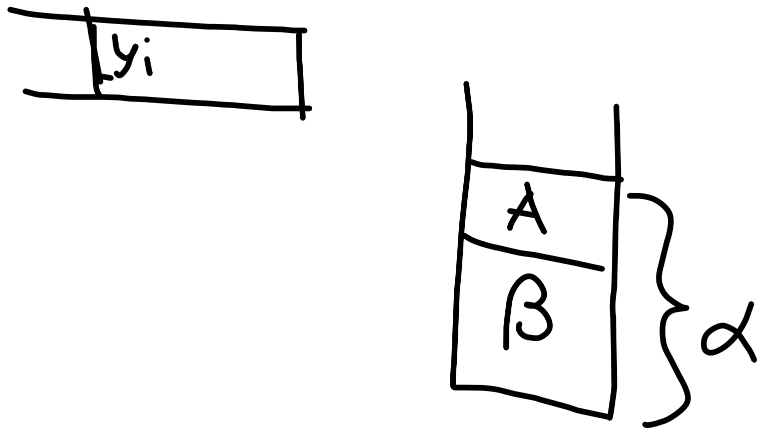
$\overleftrightarrow{x_i} \, | \, y_i = w$

$$\underbrace{(q_0, w, S)}_{\gamma_1} \rightarrow \underbrace{(q_0, y_1, \alpha_1)}_{\gamma_2} \rightarrow \cdots \rightarrow (q_0, \epsilon, \epsilon)$$

$$S \overset{*}{\Rightarrow} \gamma_1 \overset{*}{\Rightarrow} \gamma_2 \cdots \overset{*}{\Rightarrow} \gamma_m = w \Leftarrow \text{CFG}$$

$$(q_0, y_i, \alpha_i) \rightarrow (q_0, \underline{y_{i+1}}, \alpha_{i+1})$$

(Induction)



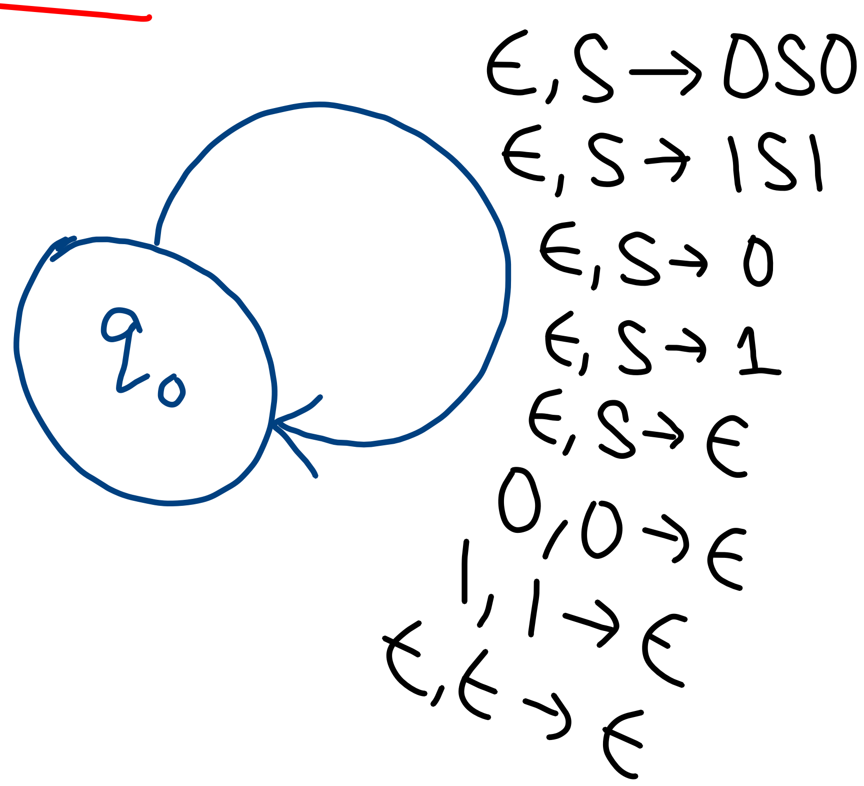
Set of Palindrome's over {0,1}

$$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

01010

$$S \to 0S0 \to 01S10 \to 01010$$

PDA



$E \to E * E \mid E + E \mid a \mid b$

$a * a + a$

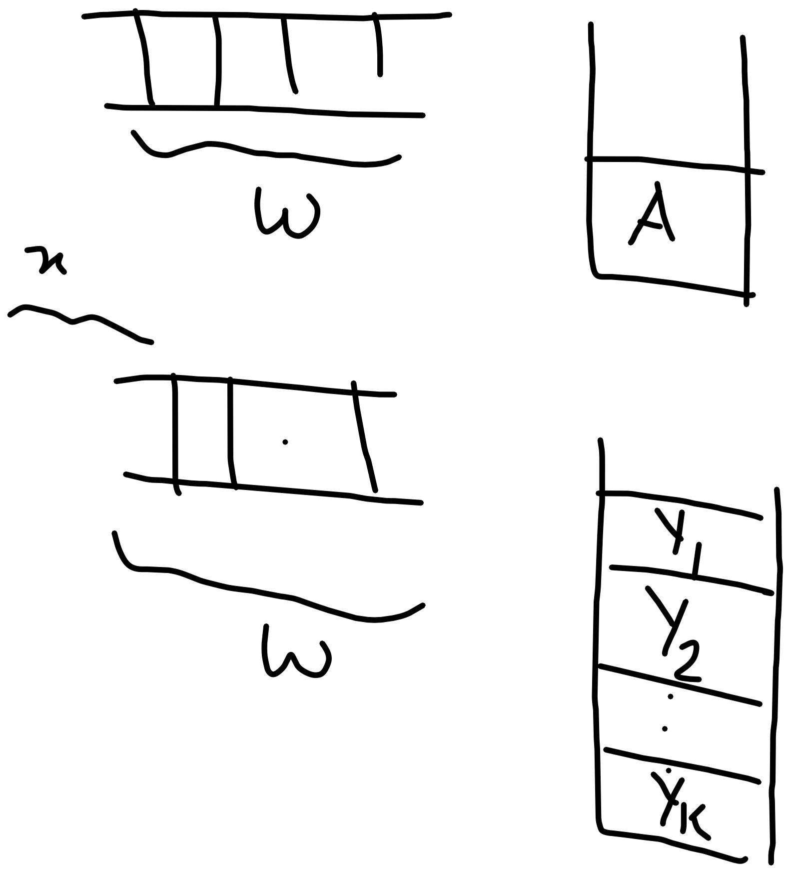
$$\begin{aligned} E &\to E * E \\ &\to a * E + E \\ &\to a * a + a \end{aligned}$$

(leftmost derivation)

$$\begin{aligned} E &\to E + E \\ &\to E * E + a \\ &\to a * a + a \end{aligned}$$

$w \in N(P') \Longrightarrow w \in L(G)$

$A \to \underbrace{Y_1 Y_2 \cdots Y_k}$



Induction on k

$Y_i \xRightarrow{*} x_i$

$A \Rightarrow Y_1 Y_2 \cdots Y_k \xRightarrow{*} x_1 x_2 \cdots x_k = \underline{\underline{w}}$

Closure Properties

$L_1, L_2 \rightarrow$ CFL

$L_1 \circ L_2 \rightarrow$ CFL ?

① Union: $(L_1 \cup L_2)$

$$\begin{cases} G_1 = (V_1, \Sigma, P_1, S_1) \\ G_2 = (V_2, \Sigma, P_2, S_2) \end{cases}$$

$$G = (V, \Sigma, P, S)$$

$$P = P_1 \cup P_2 \cup \{S \rightarrow S_1 \mid S_2\}$$

$$V = V_1 \cup V_2 \cup S$$

② Concat

$$P = P_1 \cup P_2 \cup \{ S \to S_1 S_2 \}$$

$(S_1) \to a_1 b_1 c_1 d_1$

$(S_2) \to a_2 d_2 c_2 e_2$

$a_1 b_1 c_1 d_1 a_2 d_2 c_2 e_2$

③ Star

(

② Concat

$$P = P_1 \cup P_2 \cup \{ S \to S_1 S_2 \}$$

$(S_1) \to a_1 b_1 c_1 d_1$

$(S_2) \to a_2 d_2 c_2 e_2$

$\Rightarrow a_1 b_1 c_1 d_1 a_2 d_2 c_2 e_2$

$\alpha 0 b$, $\alpha 1 b$, $\alpha 01 b$ $\to$ $b 0 \alpha$, $b 1 \alpha$, $b 10 \alpha$

③ Star

(V_1, Σ, P_1, S_1)

$$P = P_1 \cup \{ S \to S_1 S \mid \epsilon \}$$

④ Reversal

$$A \to A_1 A_2 \cdots A_k$$

$$\Rightarrow A \to A_k \cdots A_2 A_1$$

$$\left\{ \begin{array}{l} A \to \alpha B b \\ B \to 0 \mid 1 \mid 01 \end{array} \right.$$

$$\Downarrow$$

$$\begin{array}{l} A \to b B a \\ B \to 0 \mid 1 \mid 10 \end{array}$$

5 Intersection

$$L_1 = \{a^m b^m c^n \mid m, n \geq 0\} \text{ — CFL}$$

$$L_2 = \{a^m b^n c^n \mid m, n \geq 0\} \text{ — CFL}$$

$S \to S_1 S_2$

$S_1 \to aS_1b \mid \epsilon$

$S_2 \to cS_2 \mid \epsilon$

$$L_1 \cap L_2 = \{a^m b^m c^m \mid m, n \geq 0\} \text{ — (Not CFL)}$$

(5) Intersection

$$L_1 = \{ a^m b^m c^n \mid m, n \geq 0 \} - \text{CFL}$$

$$L_2 = \{ a^m b^n c^n \mid m, n \geq 0 \} - \text{CFL}$$

$S \to S_1 S_2$
$S_1 \to aS_1b \mid \epsilon$
$S_2 \to cS_2 \mid \epsilon$

$$L_1 \cap L_2 = \{ a^m b^m c^m \mid m, n \geq 0 \} - (\text{Not CFL})$$

5 Intersection

$L_1 = \{a^m b^m c^n \mid m, n \geq 0\}$ — CFL

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$ — CFL

$S \rightarrow S_1 S_2$
$S_1 \rightarrow aS_1b \mid \epsilon$
$S_2 \rightarrow cS_2 \mid \epsilon$

$L_1 \cap L_2 = \{a^m b^m c^m \mid m, n \geq 0\}$ — (Not CFL)

(not closure)

(6) Complement

$$L_1 \cap L_2 = \overline{\left(\overline{L_1} \cup \overline{L_2}\right)}$$

Assume, $L \rightarrow CFL \Rightarrow \overline{L} \rightarrow CFL$

$L_1, L_2 \rightarrow CFL \Rightarrow \left(L_1 \cap L_2 \Rightarrow CFL\right)$

(7) Set Difference

$$L_1 - L_2$$
$$= L_1 \cap \overline{L_2}$$

$$\underset{\substack{\downarrow \\ CFL}}{\overline{L}} = \underset{\substack{\downarrow \\ CFL}}{\Sigma^*} - L$$

$L = \{a^n b^n c^n \mid n \geq 0\}$

$\overline{L} = ?$

Th^m

$L_1 \to CFL$

$L_2 \to$ Regular

$$\left\{ L_1 \cap L_2 \to CFL \right.$$

$\mathcal{L} = \{ \omega \mid \#a = \#b = \#c \}$

is not CFL.

$\mathcal{L}_1 = a^* b^* c^* \Rightarrow$ Regular

$\mathcal{L} \cap \mathcal{L}_1 = \{ a^n b^n c^n \mid n \geq 0 \}$

$\Rightarrow \mathcal{L}$ is not CFL