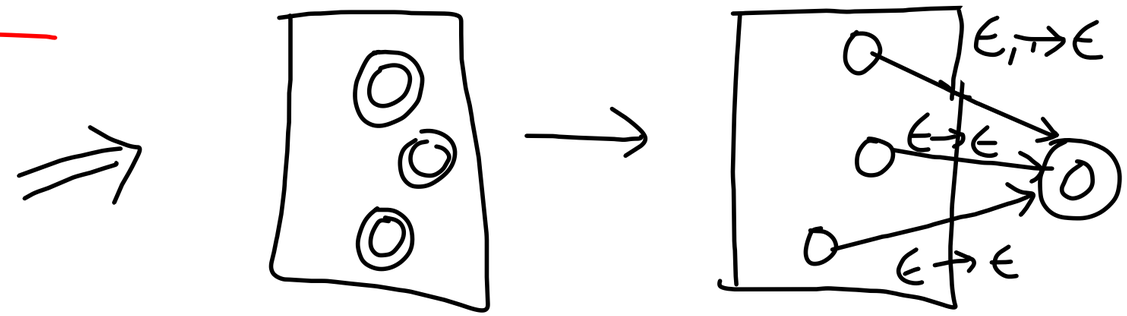
PDA to CFG Conversion

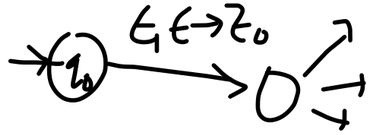
- Simplify the PDA — sPDA.
- Construct CFG from sPDA.

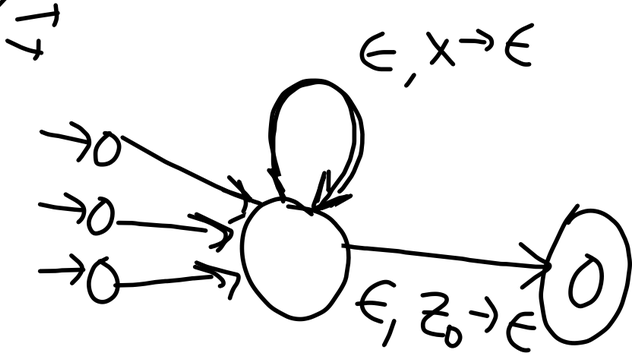
Simplifying the PDA

1. Exactly one accepting state. (final)

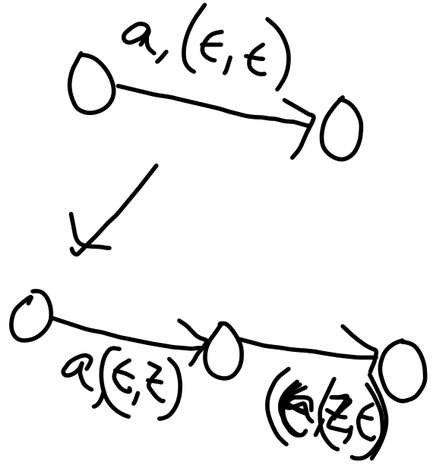


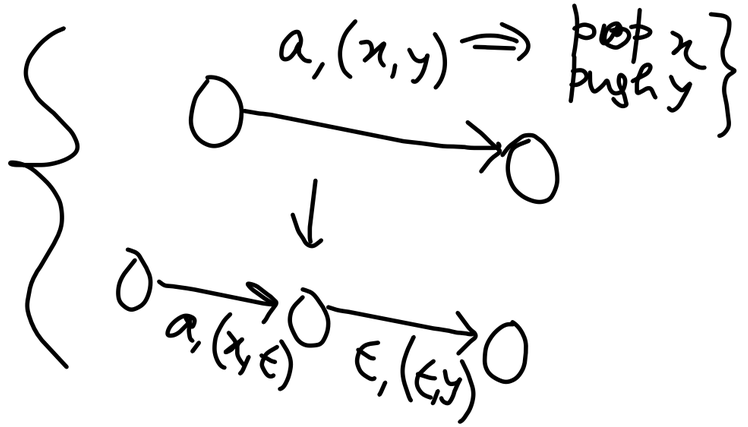
2. Stack should be empty when accepting



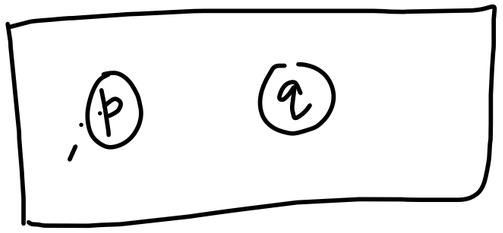


3. At each transition, either push or pop, exactly one of them.





Constructing CFG from sPDA



SPDA

Nonterminal/Variable

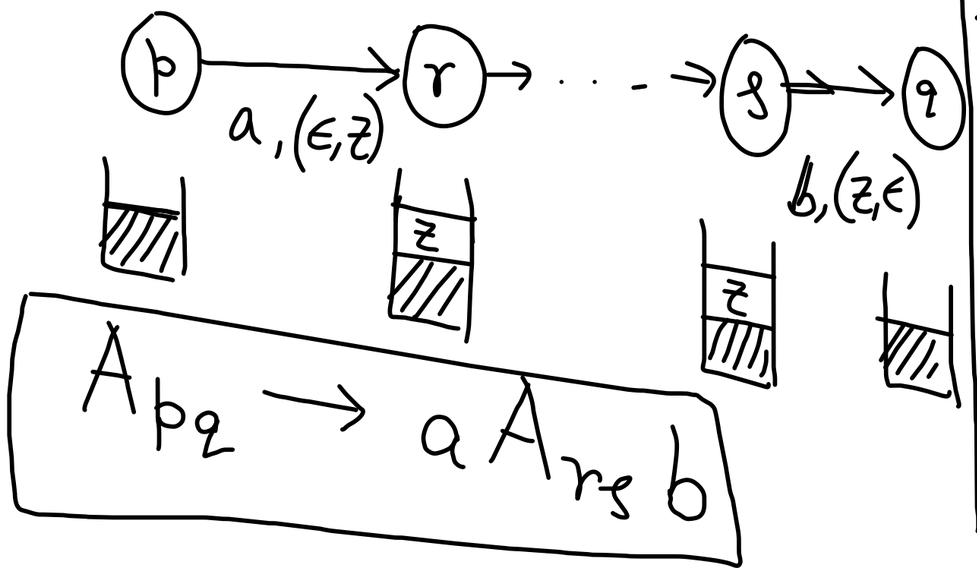
↳ A_{pq} $(\forall p, q \in \text{State})$

⇓

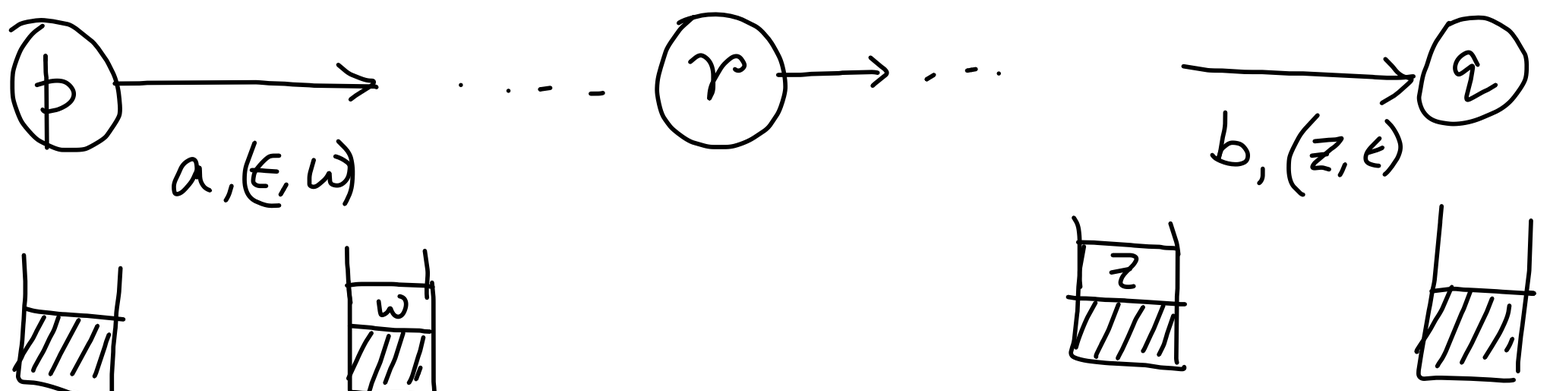
{ Should produce all the strings that can bring from state p to state q w/o changing the content of the stack.

Two cases:

Case 1

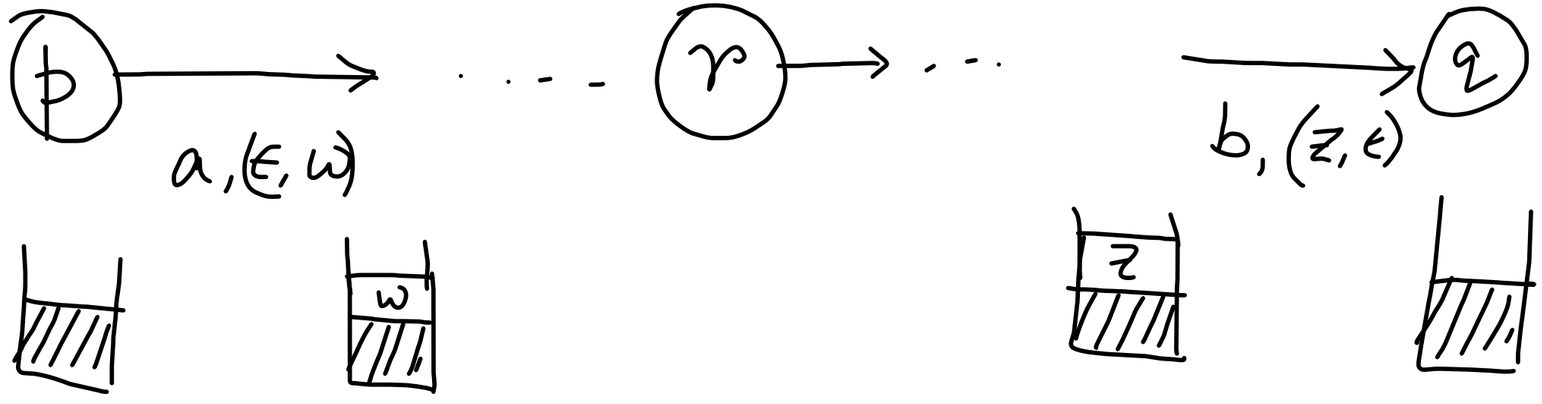


Case 2



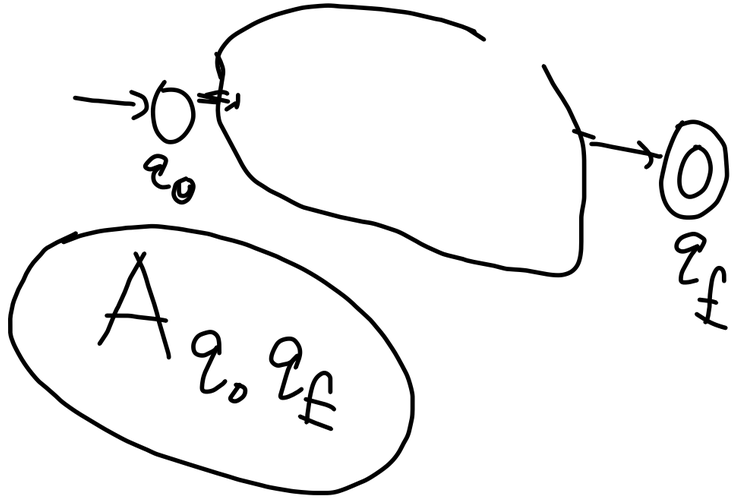
$$A_{pq} \rightarrow A_{pr} A_{rq}$$

Case 2



$$A_{pq} \rightarrow A_{pr} A_{rq}$$

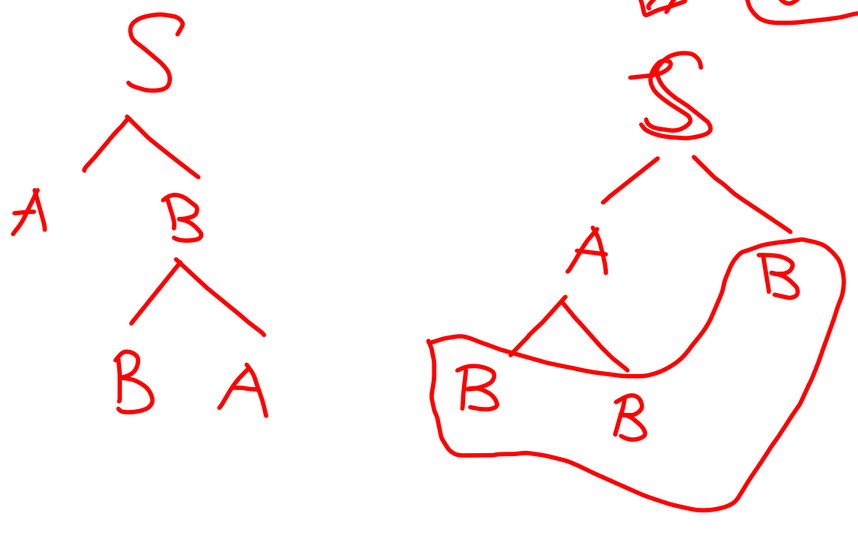
$$\begin{cases} A_{pp} \rightarrow \epsilon \end{cases}$$



Q. Whether CFG generate w? (Membership Testing)

$|w| = n$

$(n-1)$ many transitions



G

$S \to AB \mid BC$

$A \to BB \mid 0$

$B \to BA \mid 1$

$C \to AC \mid AA \mid 0$

Test for membership of "11010"

Membership Testing

(CYK Algorithm)

$w =$

Membership Testing

(CYK Algorithm)

(Dynamic Programming)

$$X_{ij} = \{ A \mid A \overset{*}{\Rightarrow} x_i \cdots x_j \}$$

$$X_{11} = \{ A \mid A \overset{*}{\Rightarrow} 1 \} = \{ B \}$$

$$X_{33} = \{ A, C \}$$

$w = x_1 \cdots x_n$

x_1	x_2	x_3	x_4	x_5	x_6
1	1	0	1	0	0

X_{16}					
X_{15}	X_{26}				
X_{14}	X_{25}	X_{36}			
X_{13}	X_{24}	X_{35}	X_{46}		
X_{12}	X_{23}	X_{34}	X_{45}	X_{56}	
X_{11}	X_{22}	X_{33}	X_{44}	X_{55}	X_{66}

$S \to AB \mid BC$

$A \to BB \mid 0$

$B \to BA \mid 1$

$C \to AC \mid AA \mid 0$

$w \overset{?}{\in} 110100$

Membership Testing

(CYK Algorithm)

(Dynamic Programming)

$$X_{ij} = \{ A \mid A \overset{*}{\Rightarrow} x_i \cdots x_j \}$$

$$X_{12} = \{ A \mid A \overset{*}{\Rightarrow} 11 \} = A$$

$A \to BC$

$x_1 \quad x_2$

$B \quad B$

$X_{23} = X_{22} \cdot X_{33}$

$= \{B\}\{A,C\}$

$= \{BA, BC\}$

$= \{S, B\}$

$w = x_1 \cdots x_n$

x_1	x_2	x_3	x_4	x_5	x_6
1	1	0	1	0	0

X_{16}					
X_{15}	X_{26}				
X_{14}	X_{25}	X_{36}			
X_{13}	X_{24}	X_{35}	X_{46}		
X_{12}	X_{23}	X_{34}	X_{45}	X_{56}	
$\{B\}$	$\{B\}$	$\{A,C\}$	$\{B\}$	$\{A,C\}$	$\{A,C\}$

$S \in X_{16} \Rightarrow$ Accept

$S \to AB \mid BC$

$A \to BB \mid 0$

$B \to BA \mid 1$

$C \to AC \mid AA \mid 0$

$w \overset{?}{\in} 110100$

$X_{23} = X_{22} \cdot X_{33}$

$X_{24} = X_{23} \cdot X_{44}$

$X_{22} \cdot X_{34}$

$X_{ij} = X_{ik} \cdot X_{k+1\,j}$

Membership Testing

(CYK Algorithm)

(Dynamic Programming)

$\#elements = \binom{n}{2}$

$Time = O(n) \cdot \binom{n}{2}$

$= O(n^3)$

$w = x_1 \cdots x_n$

$x_1\ x_2\ x_3\ x_4\ x_5\ x_6$

$1\ 1\ 0\ 1\ 0\ 0$

$S \to AB \mid BC$

$A \to BB \mid 0$

$B \to BA \mid 1$

$C \to AC \mid AA \mid 0$

$w \in ? \ 11\,0100$

$X_{ij} = \{ A \mid A \overset{*}{\Rightarrow} x_i \cdots x_j \}$

$X_{12} = \{ A \mid A \overset{*}{\Rightarrow} 11 \} = A$

$A \to BC$

$x_1 \quad x_2$

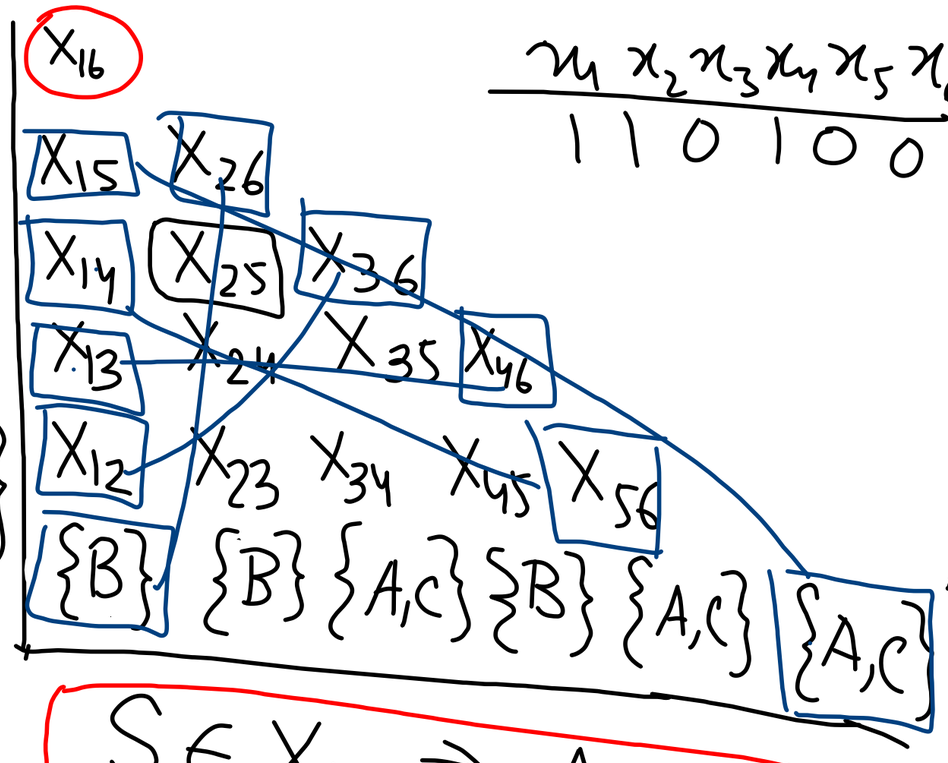
$B \quad B$

$X_{23} = X_{22} \cdot X_{33}$

$= \{B\}\{A,C\}$

$= \{BA, BC\}$

$= \{S, B\}$



$S \in X_{16} \Rightarrow$ Accept

$X_{23} = X_{22} \cdot X_{33}$

$X_{24} = X_{23} \cdot X_{44}$

$X_{22} \cdot X_{34}$

$X_{ij} = X_{ik} \cdot X_{k+1\,j}$

Shuffle

$$\begin{cases} x = x_1 x_2 \\ y = y_1 y_2 \end{cases}$$

$$\text{Shuffle}(x, y) = \{ x_1 x_2 y_1 y_2 \,,\; x_1 y_1 y_2 x_2 \,,\; x_1 y_1 x_2 y_2 \,,\; y_1 x_1 x_2 y_2 \,,\; y_1 y_2 x_1 x_2 \,,\; y_1 x_1 y_2 x_2 \}$$

$$\text{Shuffle}(\mathcal{L}_1, \mathcal{L}_2) = \bigcup_{\substack{x \in \mathcal{L}_1 \\ y \in \mathcal{L}_2}} \text{Shuffle}(x, y)$$

$$\text{Perfect shuffle}(x, y) = \{ x_1 y_1 x_2 y_2 \}$$

$$\mathcal{L}_1 = \{01, 011, 0111\}$$

$$\mathcal{L}_2 = \{00, 111, 0\}$$

1. (*) Shuffle $(L_1, L_2) \rightarrow$ regular $\Leftarrow$ if $L_1, L_2 \rightarrow$ regular
(Perfect Shuffle ?)

2. (*) Shuffle $(L, R) \rightarrow$ context-free, if $L \rightarrow$ CFL,
$R \rightarrow$ Regular

3. Shuffle $(L_1, L_2) \rightarrow$ not necessarily context-free, if $L_1 \rightarrow$ CFL
$L_2 \rightarrow$ CFL

4. Perfect Shuffle $(L_1, L_2) \rightarrow$ ---

5. Perfect Shuffle $(L, R) \rightarrow$ not context-free

1. (*) Shuffle(L_1, L_2) → regular ⇐ if L_1, L_2 → regular
(Perfect Shuffle ?)

2. (*) Shuffle(L, R) → context-free, if L → CFL,
R → Regular

3. Shuffle(L_1, L_2) → not necessarily context-free, if L_1 → CFL, L_2 → CFL

4. Perfect Shuffle (L_1, L_2) → ⁻⁻

5. Perfect Shuffle (L, R) → not context-free

$L_1 \rightarrow$ CFL

$L_2 \rightarrow$ CFL

Shuffle (L_1, L_2) $\cap$ ——— regular

$\Downarrow$

Not CFL

① $Half(L) = \{ x \mid xw \in L \ \& \ |x| = |w| \}$

is not context-free

$L = \{010, 1100, 1010\}$

$Half(L) = \{00, 10\}$

Is $\mathcal{L} = \{ a^n b^n c^j \mid j, n \geq 0 \ \ j < n \}$

Context-free?

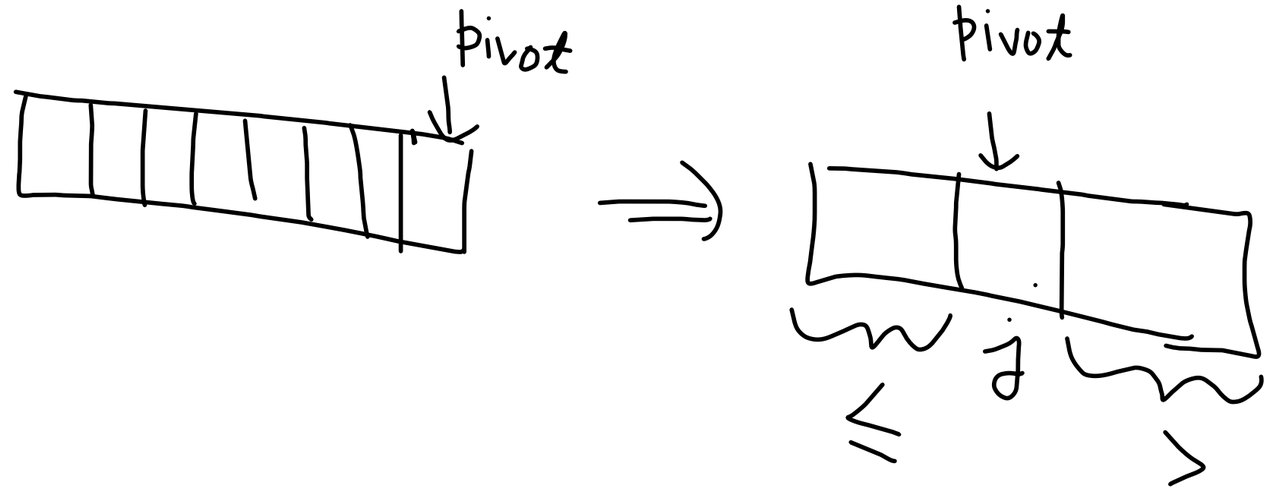
Some Interesting Problems in Algo

1. How to find the i^{th} smallest element in an array?

Find - Sort & find A[i]. ($O(n \log n)$)

i_Smallest (A, p, q, i) - Can you do better?

```
j ← $Partition(A, p, q)
if (i = j)
        return A[i]
else if (i < j)
        i_Smallest(A, p, j, i)
else
        i_Smallest(A, j+1, q, i-j)
```



Membership Testing

(CYK Algorithm)

(Dynamic Programming)

X_{16}

$w =$

$S \to AB \mid BC$

$A \to BB \mid 0$

$B \to BA \mid 1$

$C \to AC \mid AA \mid 0$

$w \in ?$ 10100