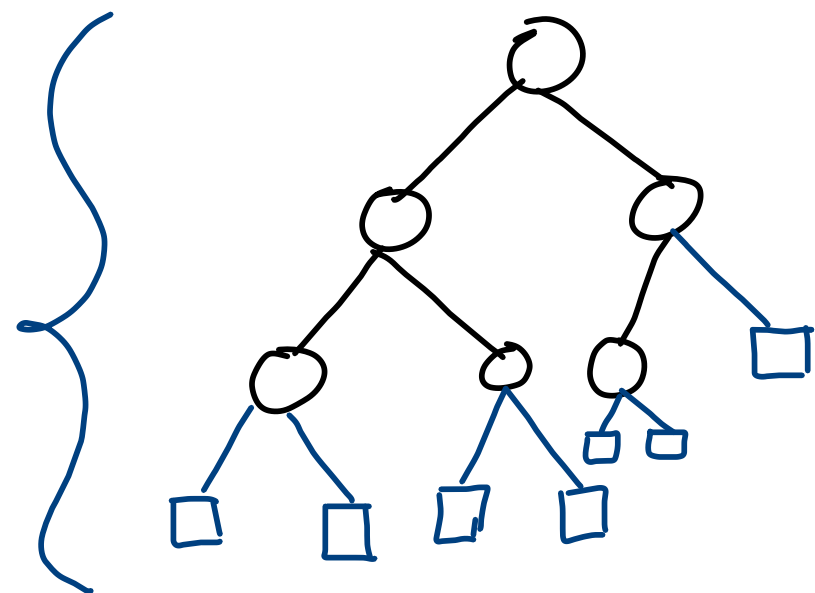


RED BLACK TREES

- Balanced Binary Search Tree

Properties

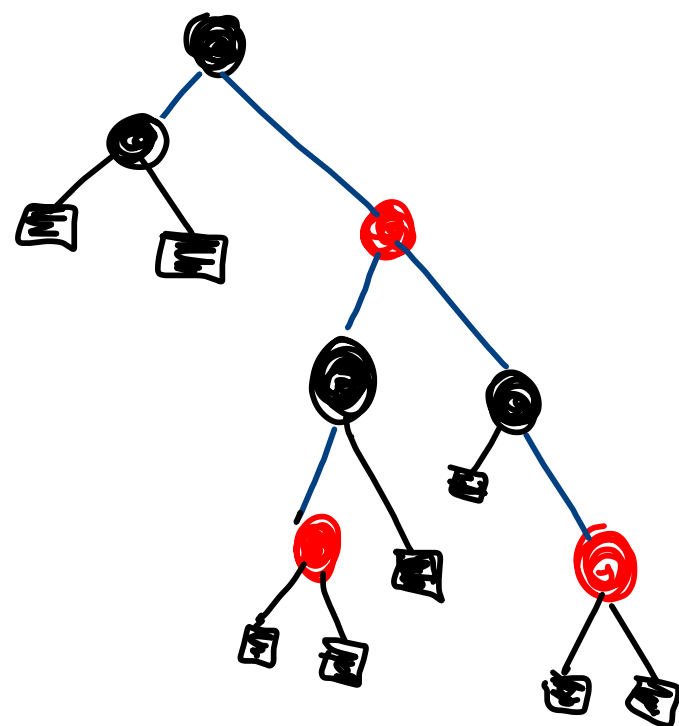
- 1> Each node is colored with Red or Black.
- 2> The root is black.
- 3> All leaf nodes (NIL) are black.
- ✓ 4> Black-height is same for all the leaves.
- ✓ 5> Both child of a red node are black.



- Every node has exactly 2 children
- All leaves are NIL.

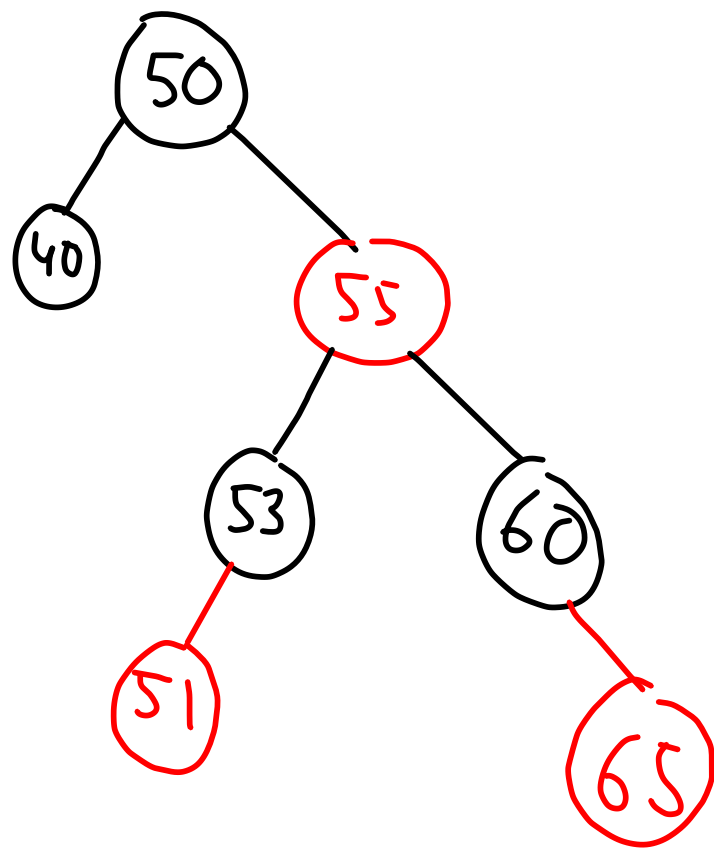
Black height (v) ^{→ leaf}

↳ # of black nodes in the path from root to v .



Height of a Red-Black Tree is $O(\log n)$

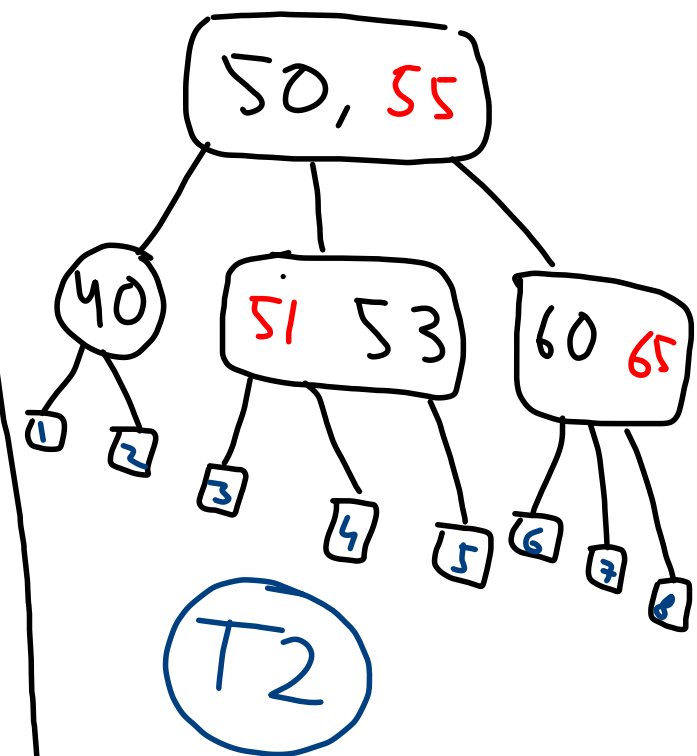
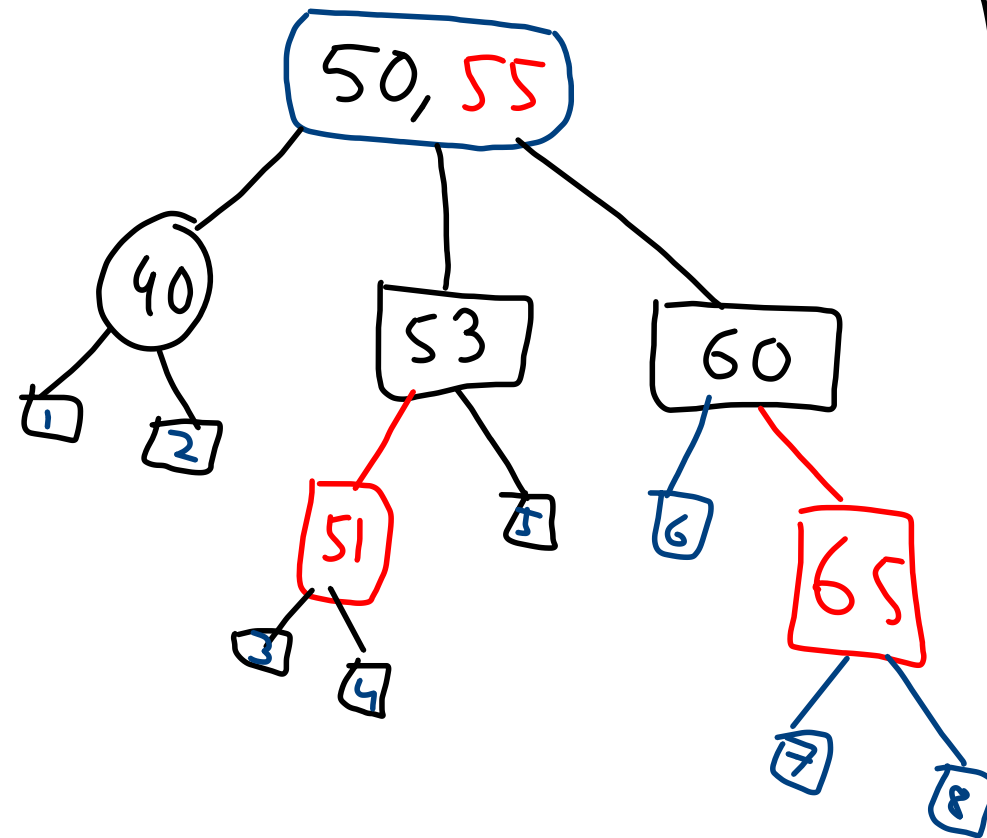
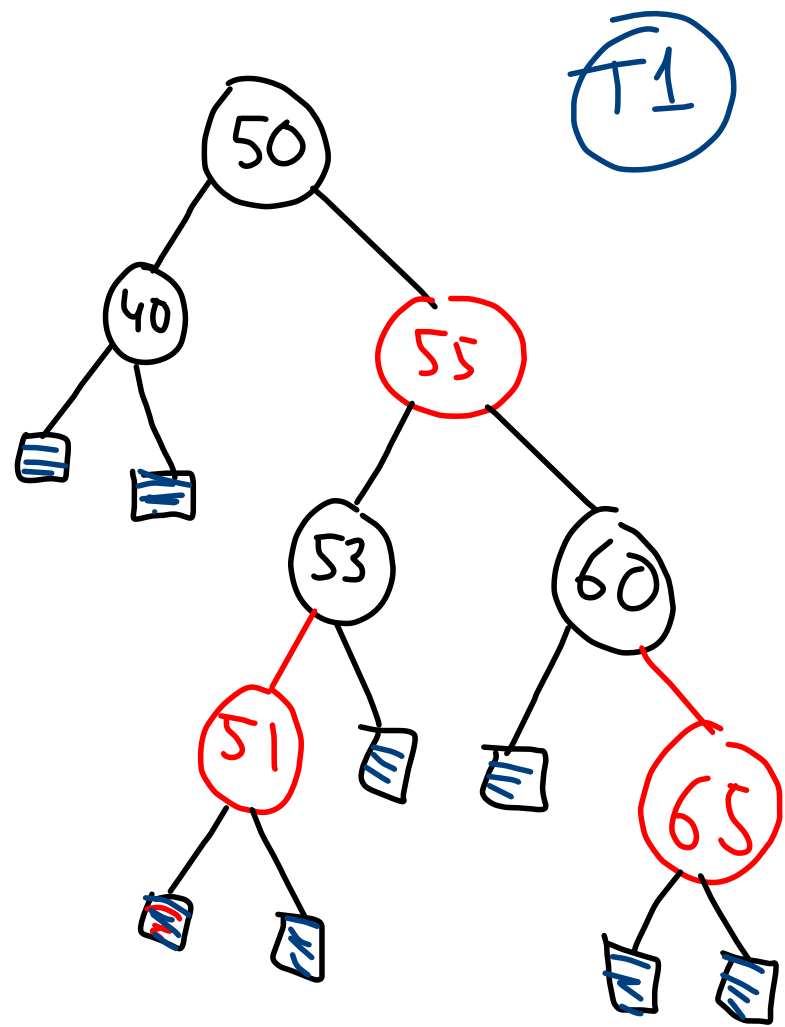
- Induction on black height of a tree.



Height of a Red-Black Tree is $O(\log n)$

- Induction on black height of a tree.

2-3-4 Tree
↓



Height of a Red-Black Tree is $O(\log n)$

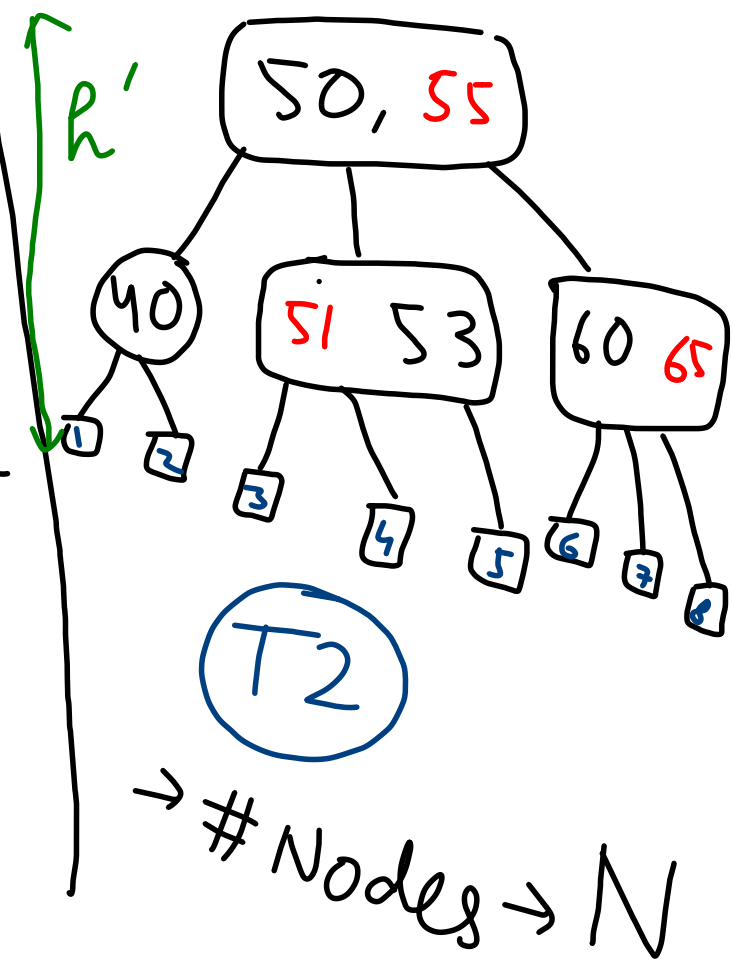
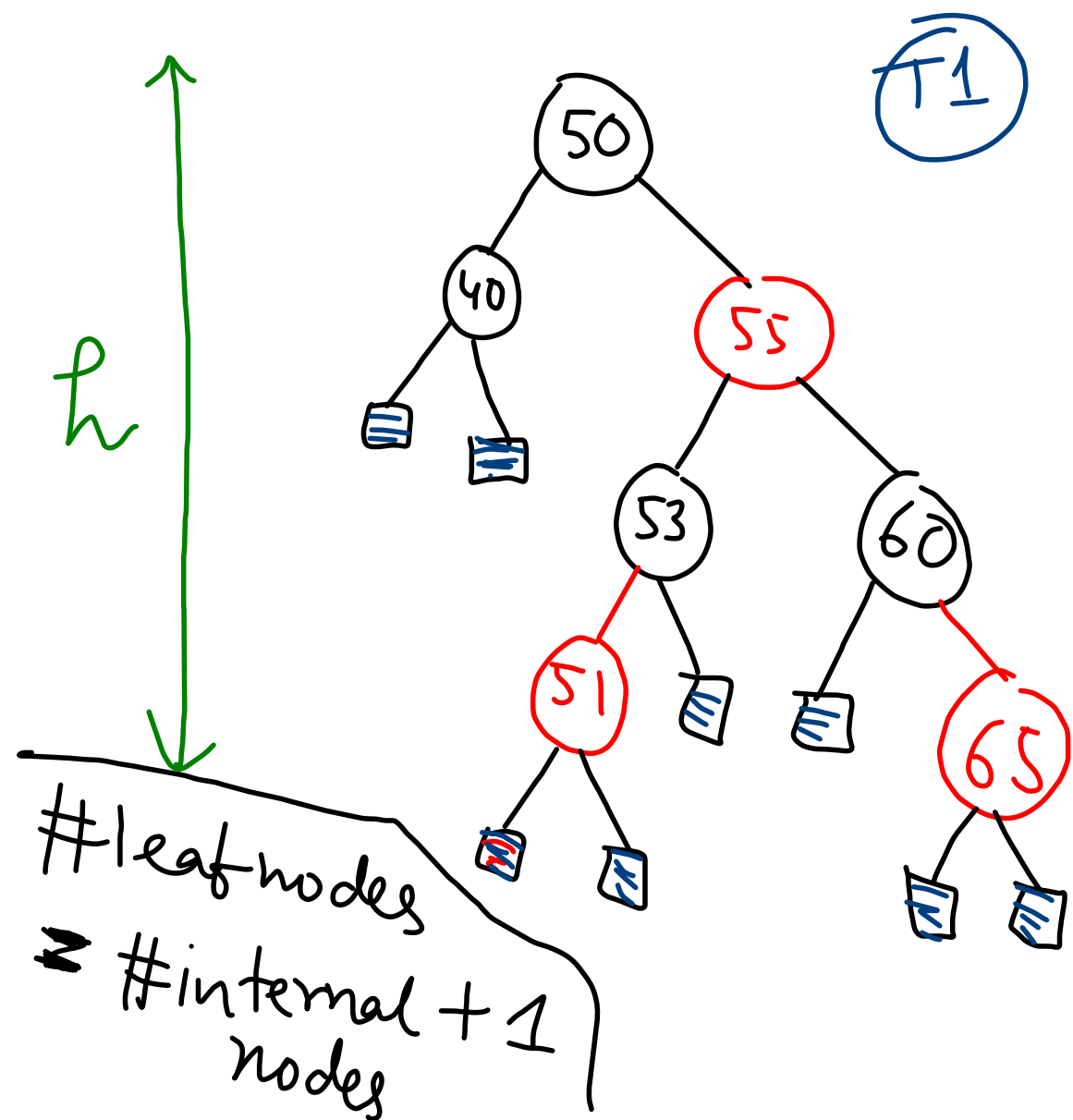
- Induction on black height of a tree.

2-3-4 Tree
↓

Properties

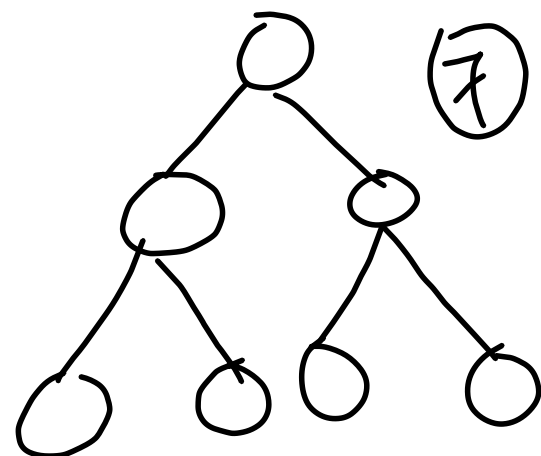
- ① Black height of both trees are same.
- ② $T2 \Rightarrow$ All leaves are at same height.

$$\left. \begin{array}{l} \textcircled{3} \quad h \leq 2h' \\ \textcircled{4} \quad 2^{h'} \leq N \leq 4^{h'} \end{array} \right\}$$
$$h \leq 2h' \leq 2\log_2(N+1)$$

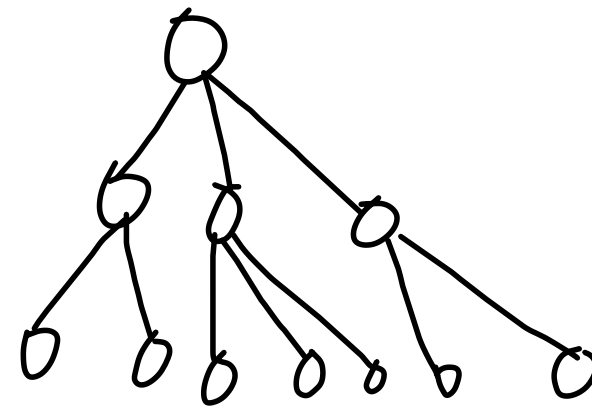


$$\frac{(2^3 - 1)}{}$$

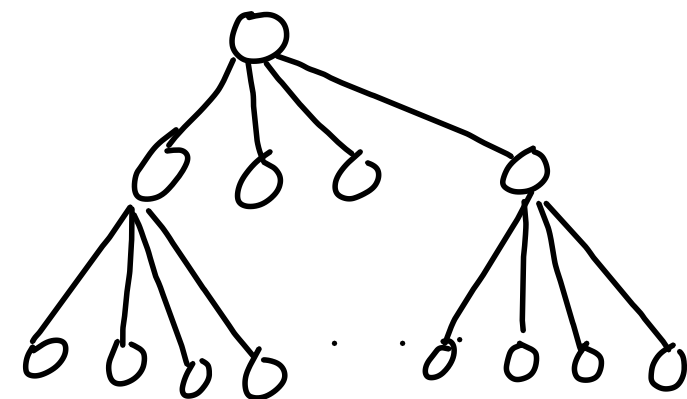
3



↓
min

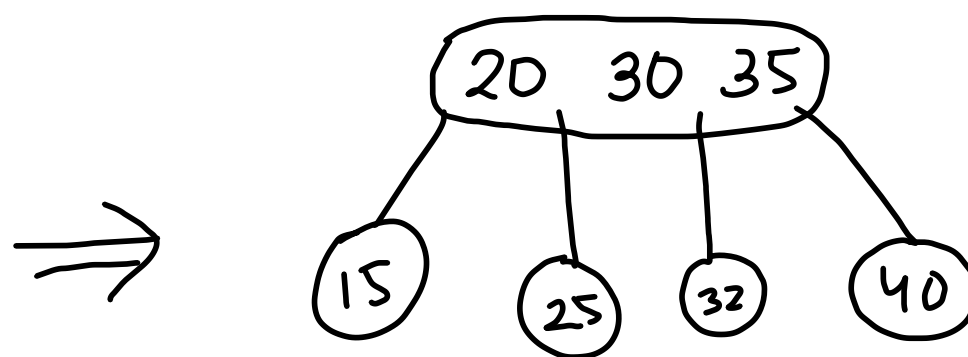
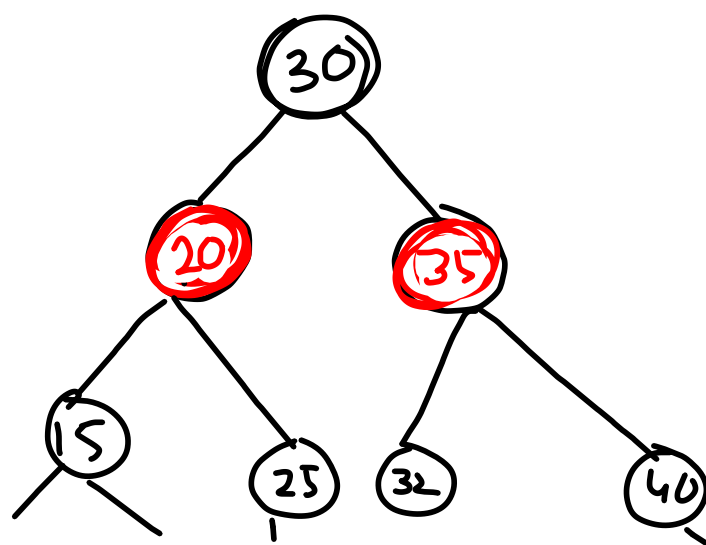


3



$$\leq \frac{(4^3 - 1)}{4 - 1}$$

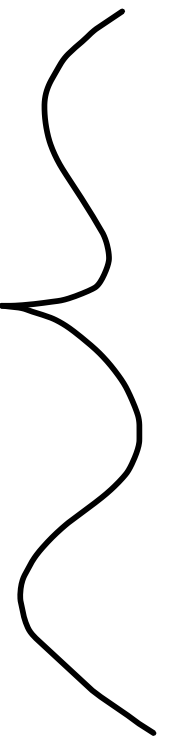
$$1 + 4 + 4^2$$



2-3-4 trees

- At most 4
child
- At least 2
child

AVL Tree vs Red-Black Tree

- 
- # insertions, deletions are frequent $\Rightarrow$ Red-Black Tree
 - # Search are frequent $\Rightarrow$ AVL