Cryptology: Problem Sheet 6

Topic: Block Cipher: Design and Cryptanalysis

1. Consider a 64-bit block cipher Light-AES composed of 12 rounds each having 4 operations:

   □ Sub-Cell: Uses a 4-bit S-Box GS and apply it to each cells. The S-Box operation is given in Fig.1, and the difference distribution table corresponding to the S-Box is given in Fig.3.

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$GS(x)$	1	a	4	c	6	f	3	9	2	d	b	7	5	0	8	e

   Figure 1: S-Box for Light-AES

   □ Shuffle-Cells: Shuffle the cells of the state as follows:

   $$\begin{pmatrix} s_0 & s_4 & s_8 & s_{12} \\ s_1 & s_5 & s_9 & s_{13} \\ s_2 & s_6 & s_{10} & s_{14} \\ s_3 & s_7 & s_{11} & s_{15} \end{pmatrix} \rightarrow \begin{pmatrix} s_0 & s_{14} & s_9 & s_7 \\ s_{10} & s_4 & s_3 & s_{13} \\ s_5 & s_{11} & s_{12} & s_2 \\ s_{15} & s_1 & s_6 & s_8 \end{pmatrix}.$$

   □ Mix-Columns: Multiply each column by the following matrix:

   $$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

   □ Add-Round-Key: XORs the round key with the state.

   The 64-bit message is divided into 16 cells of 4-bit, and loaded in the initial state. The key whitening is done in the beginning. Answer the following:

   (a) Show that Light-AES achieves full diffusion in 3 rounds.

   (b) Prove or Refute: The order of Sub-Cell and Shuffle-Cells is invariant in Light-AES.

   (c) Show that if there exists exactly two positions with a non-zero differences in a column, then the two positions will contain a non-zero difference even after the mix column operation.

   (d) What is the branch number of matrix M? Show that the matrix M is involutory (i.e., $M = M^{-1}$). State the main advantages and disadvantages of using this matrix as compared to the one used in AES.

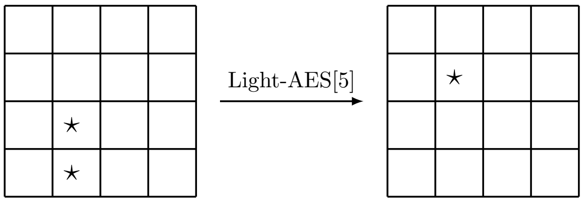


Figure 2: 5 round Impossible Differential of Light-AES.

(e) Using the above property, prove that 5-round Light-AES has the following impossible differential depicted in Fig.2.

(f) Assume that any r ($r \geq 2$) round differential characteristic of Light-AES has a minimum of $4r$ active S-Boxes. Find an estimate on the minimum number of rounds Light-AES should use to resist against differential cryptanalysis?

(g) Mount an Integral attack (distinguishing) on 3-round Light-AES. Compute the data, time and memory complexity of your attack.

Δ_I \ Δ_O	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	2	2	0	2	2	2	2	2	0	0	2
2	0	0	0	0	0	4	4	0	0	2	2	0	0	2	2	0
3	0	0	0	0	0	2	2	0	2	0	0	2	2	2	2	2
4	0	0	0	2	0	4	0	6	0	2	0	0	0	2	0	0
5	0	0	2	0	0	2	0	0	2	0	0	0	2	2	2	4
6	0	0	4	6	0	0	0	2	0	0	2	0	0	0	2	0
7	0	0	2	0	0	2	0	0	2	2	2	4	2	0	0	0
8	0	0	0	4	0	0	0	4	0	0	0	4	0	0	0	4
9	0	2	0	2	0	0	2	2	2	0	2	0	2	2	0	0
a	0	4	0	0	0	0	4	0	0	2	2	0	0	2	2	0
b	0	2	0	2	0	0	2	2	2	2	0	0	2	0	2	0
c	0	0	4	0	4	0	0	0	2	0	2	0	2	0	2	0
d	0	2	2	0	4	0	0	0	0	0	2	2	0	2	0	2
e	0	4	0	0	4	0	0	0	2	2	0	0	2	2	0	0
f	0	2	2	0	4	0	0	0	0	2	0	2	0	0	2	2

Figure 3: DDT for Light-AES

2. Consider a modified AES block cipher, dubbed mAES, which is identical to AES but uses the following matrix in the Mix-Column operation:

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

All the operations, S-Box description remain same as AES.

(a) Show that two round mAES does not achieve full diffusion. Use this fact to mount a differential attack (distinguishing) on two round mAES.

(b) Assume that the number of active S-Box in the linear as well as differential trail of mAES is given in the table below. As a designer how many rounds of mAES will you choose so that the cipher provides security against (i) differential cryptanalysis, (ii) linear cryptanalysis?

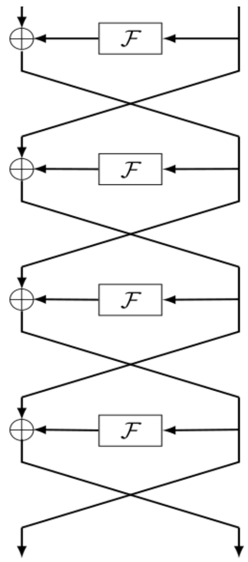
#Round	1	2	3	4	5	6	7	8
# Active S-Boxes	1	4	7	16	17	20	23	32

(c) Suppose the i^{th} round state (before Sub-byte) of mAES follows the following integral property:

$$\begin{pmatrix} C & C & C & C \\ A & C & C & C \\ A & A & C & C \\ A & A & A & C \end{pmatrix}.$$

What would be the integral property at the beginning of round $(i+1)$?

3. Consider a 4 round Feistel cipher (on 128-bits) depicted in the figure below. Show that $(\Delta, 0) \rightarrow (\Delta, \star)$ is an impossible differential for the cipher. How can you use this impossible differential to mount a distinguishing attack on 4 round Feistel? Note that $\mathcal{F}$ is a keyed permutation on 64-bits.



[Hint: If there is a non-zero input difference (say Δ_1) in the input of $\mathcal{F}$, then the output difference (say Δ_2) is also non-zero.]