Bell Nonlocality in Networks

Nicolas Brunner

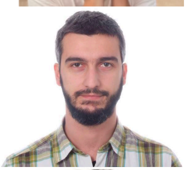
UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

ISQIC 1st April 2025

Sadra Boreiri

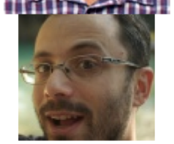
Tamas Krivachy (ICFO)

Bora Ulu



Ivan Supic (Paris)

Marc-Olivier Renou (Paris)

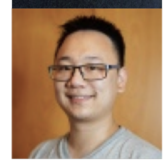


Jean-Daniel Bancal (Paris)

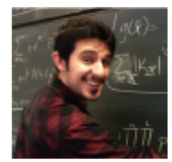
Elisa Bäumer (IBM)

Pavel Sekatski

Antoine Girardin



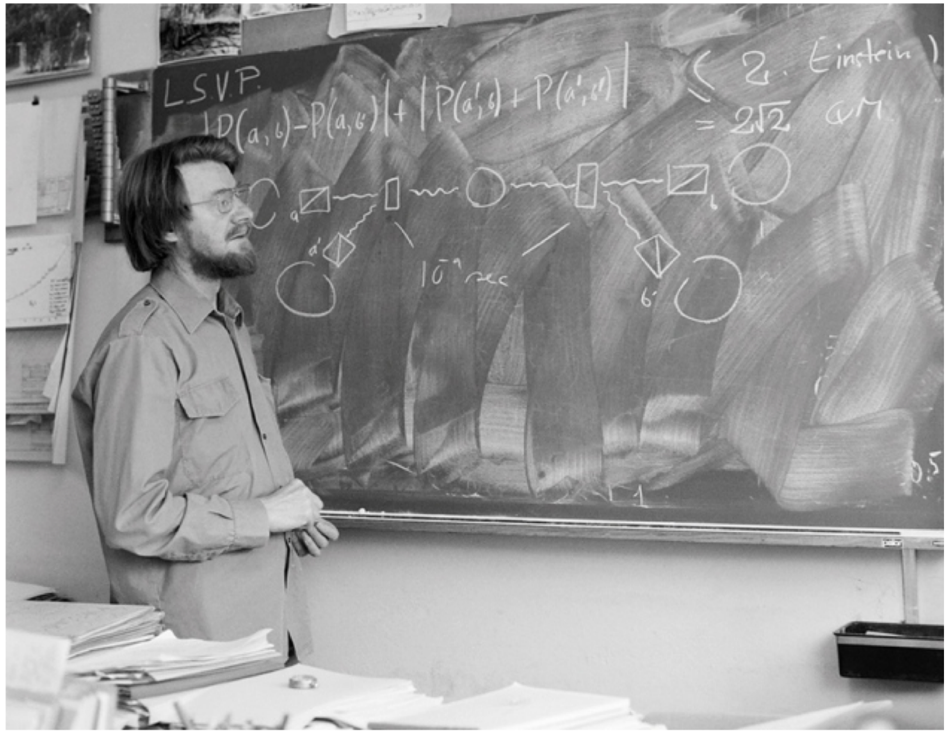
Yu Cai (Singapore)

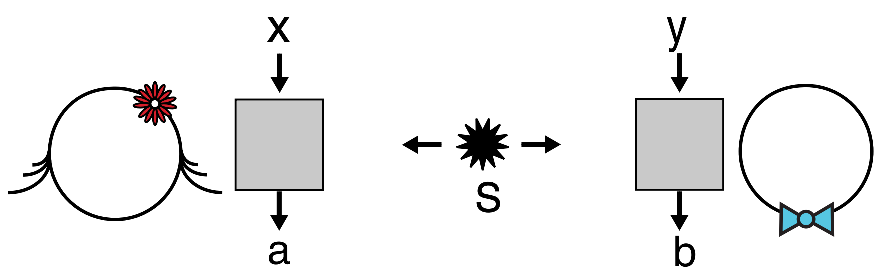


Armin Tavakoli (Lund)

Nicolas Gisin

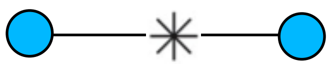
Salman Beigi (Teheran)

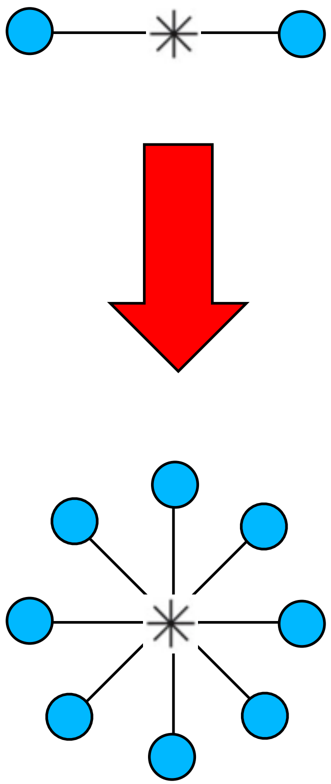




$$p(ab|xy) \overset{?}{=} \int_\Lambda d\lambda q(\lambda) p(a|x,\lambda) p(b|y,\lambda)$$

Bell 1964

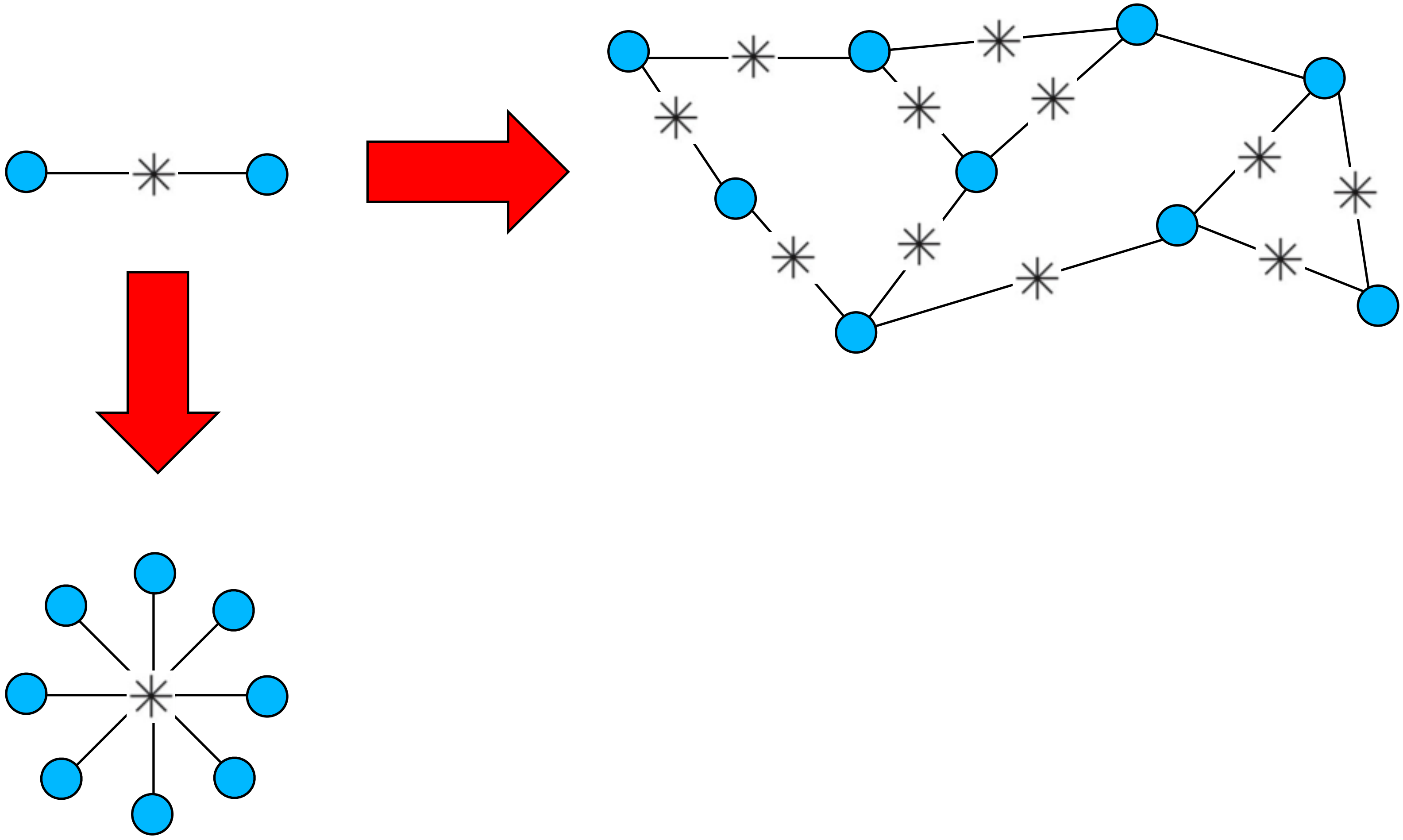




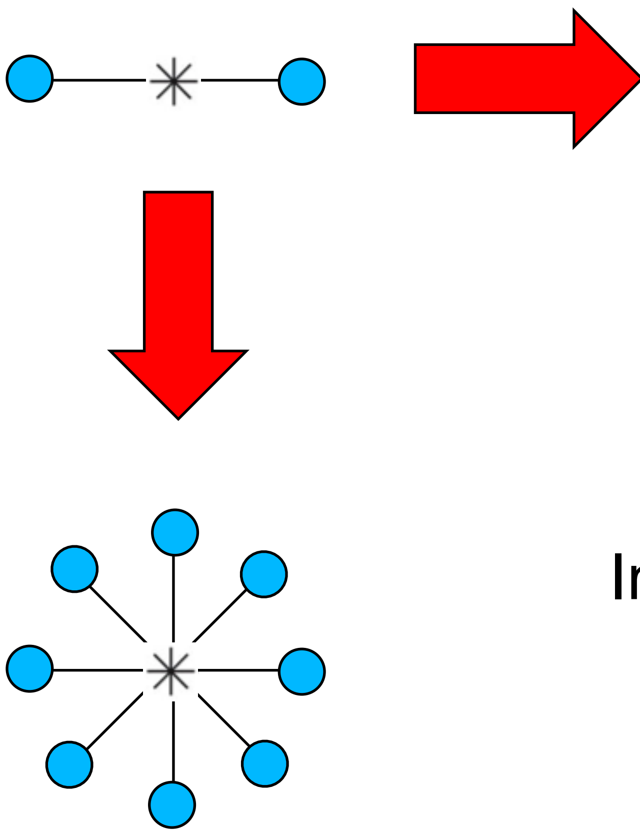
Multiparty nonlocality

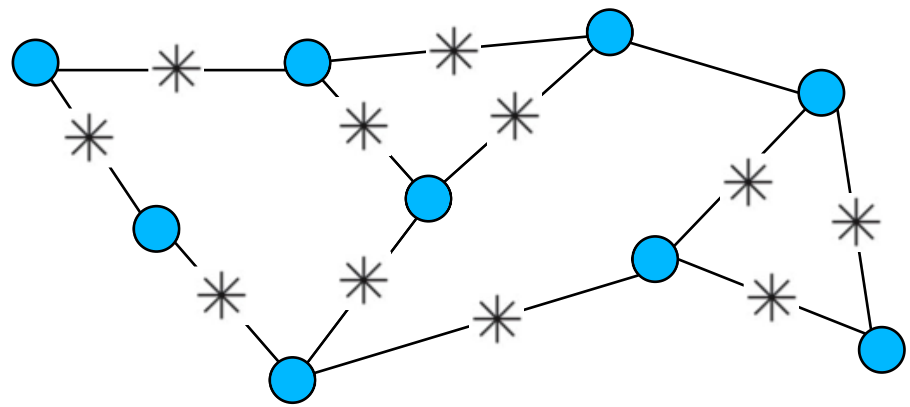
(à la Mermin, GHZ, Svetlichny,…)

Nonlocality in networks



Nonlocality in networks



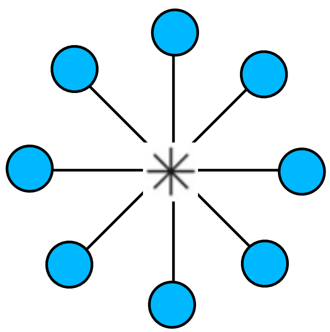


Main hope

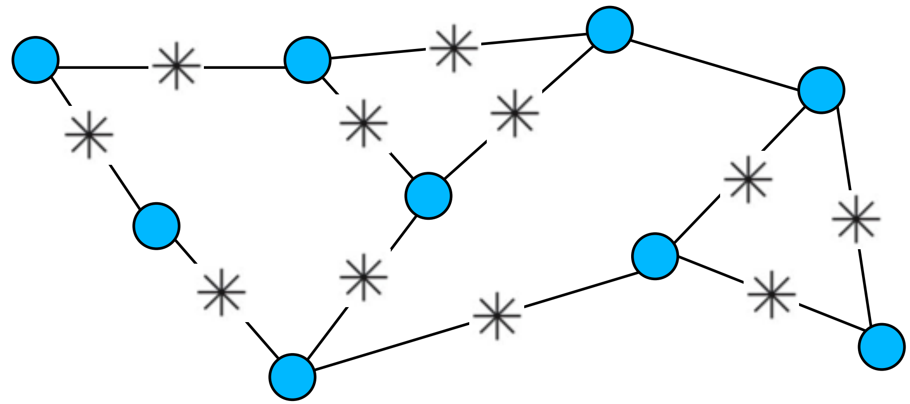
Interesting forms of Q correlations

Combine entangled states
& entangled measurements

Nonlocality in networks

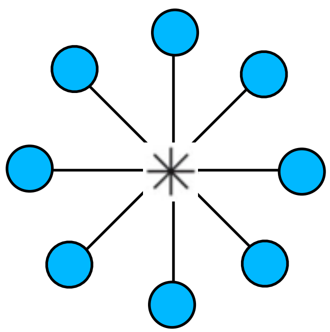


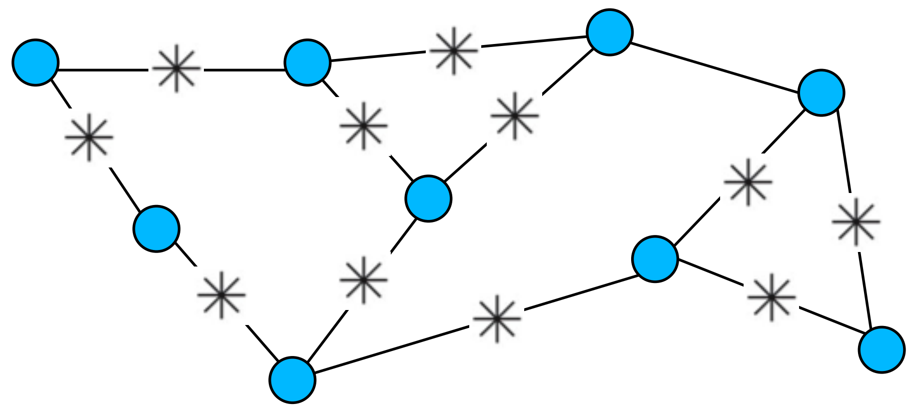
One common source



Independent sources

Nonlocality in networks





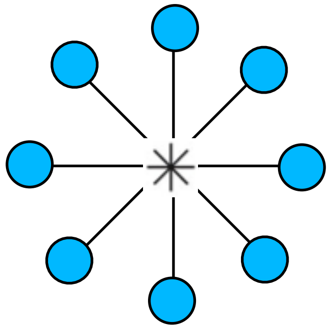
One common source

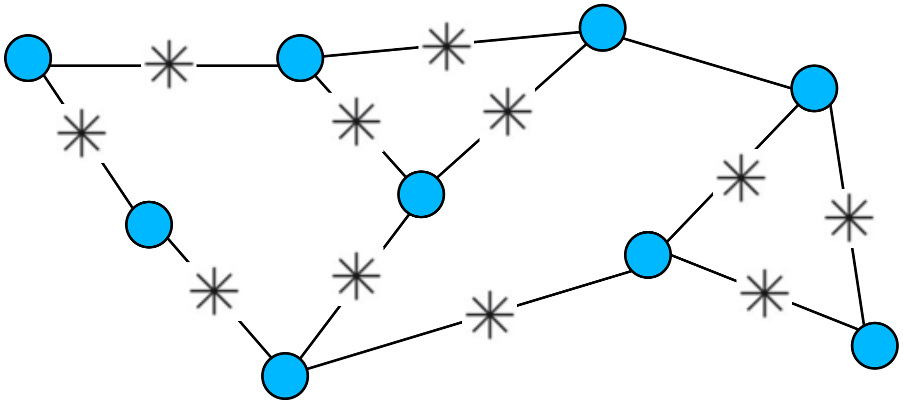
Independent sources

Convex sets
Linear Bell inequalities

→ Adapted methods
→ Good understanding

Nonlocality in networks





One common source

Independent sources

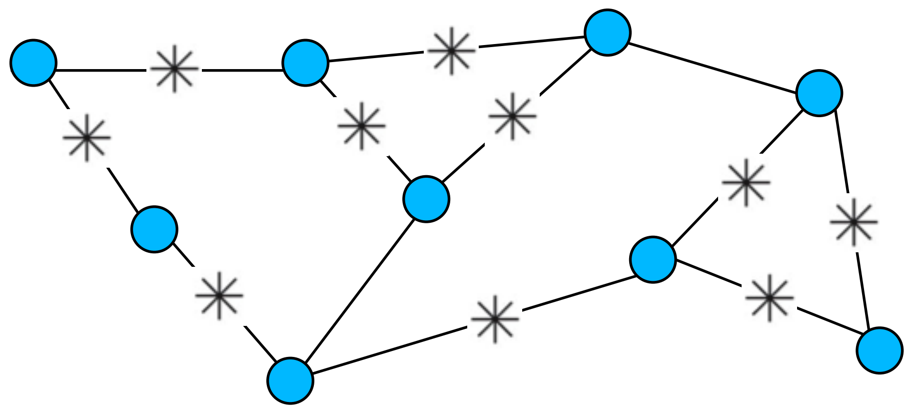
Convex sets
Linear Bell inequalities

→ Adapted methods
→ Good understanding

Non-convex sets
Non-linear Bell inequalities

???

This talk

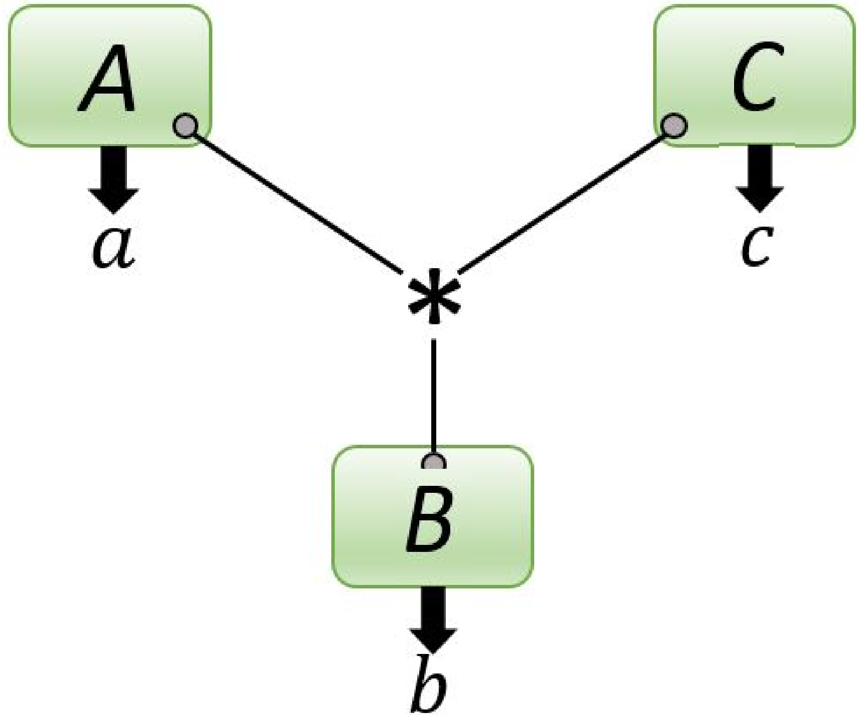


1. Local (classical) correlations in networks → N-locality

2. Genuine network quantum nonlocality

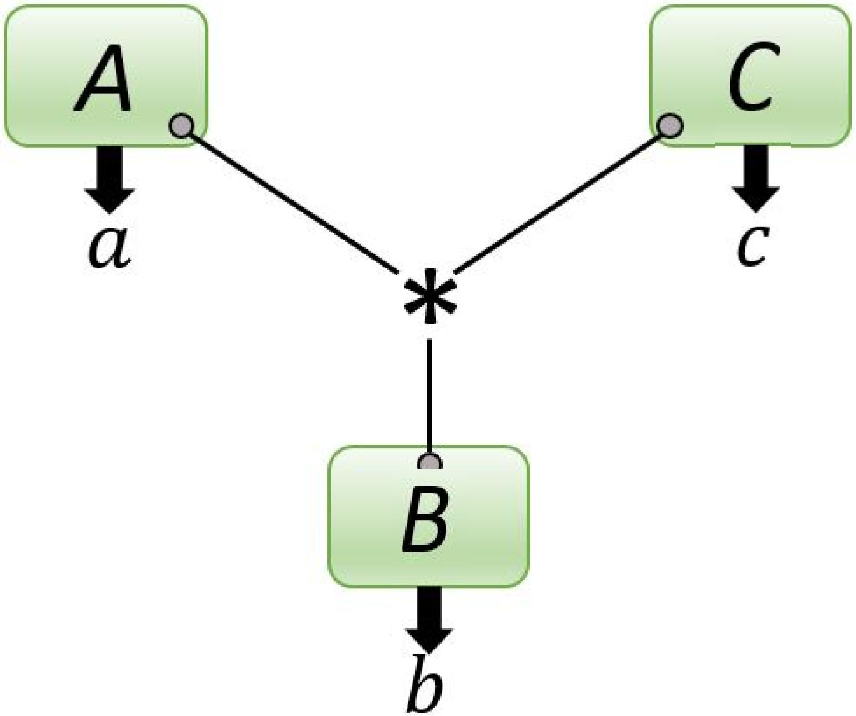
3. Topologically robust nonlocality

No inputs



No inputs → Joint output distribution P(a,b,c)

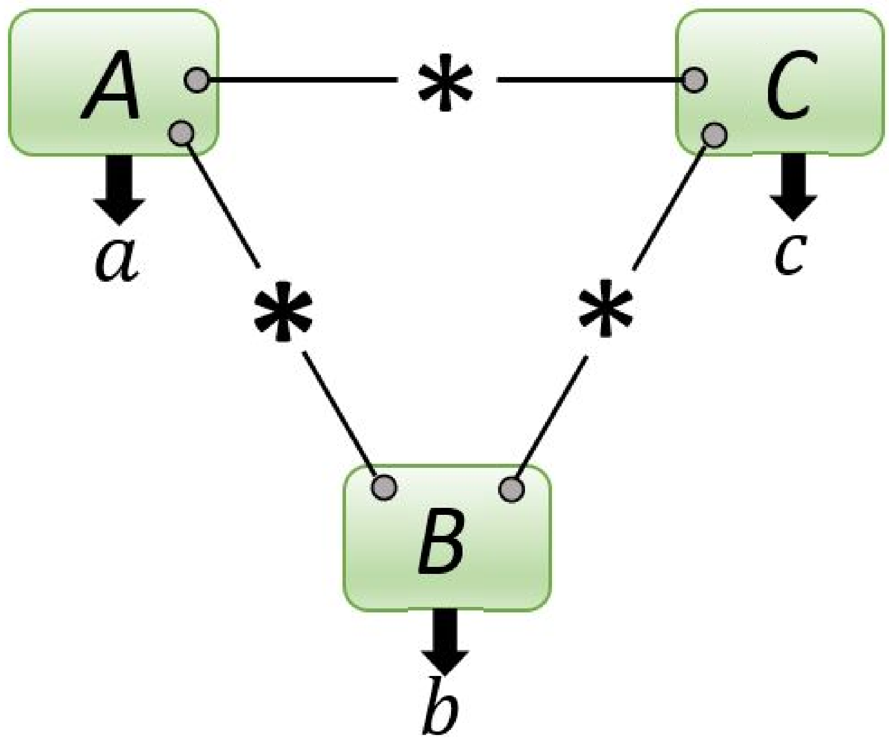
No inputs



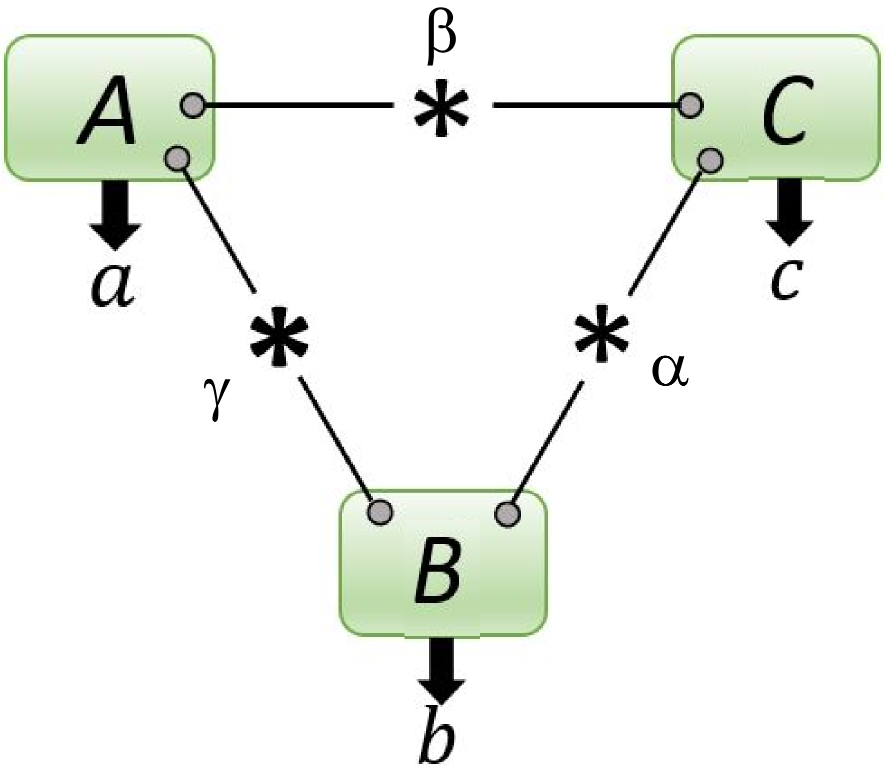
No inputs → Joint output distribution P(a,b,c)

One common source → any P(a,b,c) is possible classically

Triangle network



Triangle network

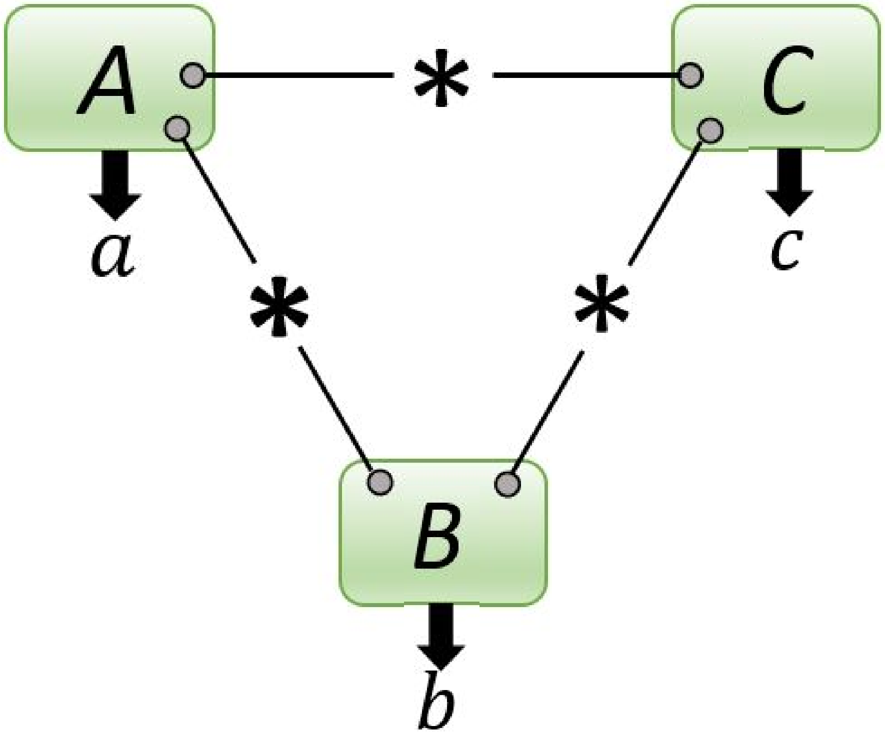


Definition: trilocal (i.e. classical) correlations

$$p(a,b,c) = \sum_{\alpha,\beta,\gamma} p(\alpha)p(\beta)p(\gamma)p(a|\beta,\gamma)p(b|\alpha,\gamma)p(c|\alpha,\beta)$$

Branciard, Gisin, Pironio 2010; Fritz 2012

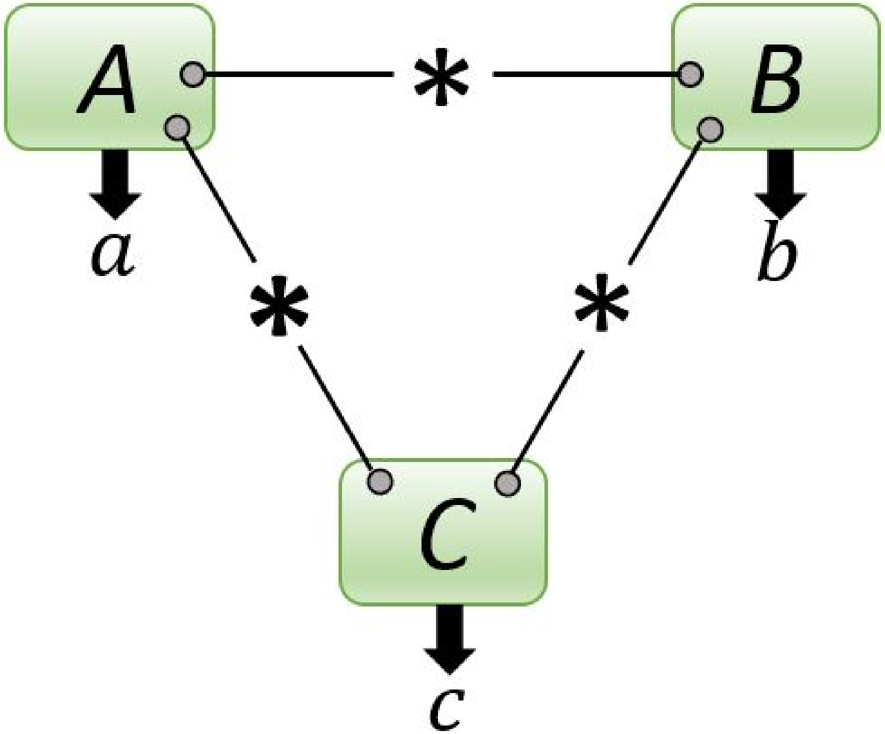
Triangle network



There exist Q distributions P(a,b,c) that are non-trilocal !!

→ **Q nonlocality without inputs**

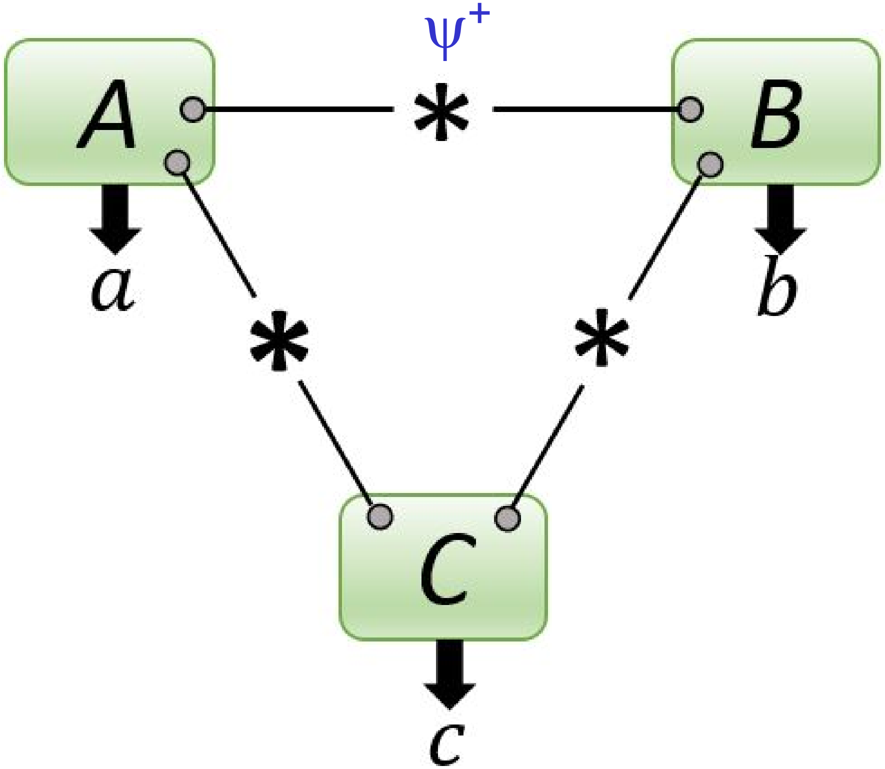
Fritz 2012



IDEA: embed CHSH test in triangle network

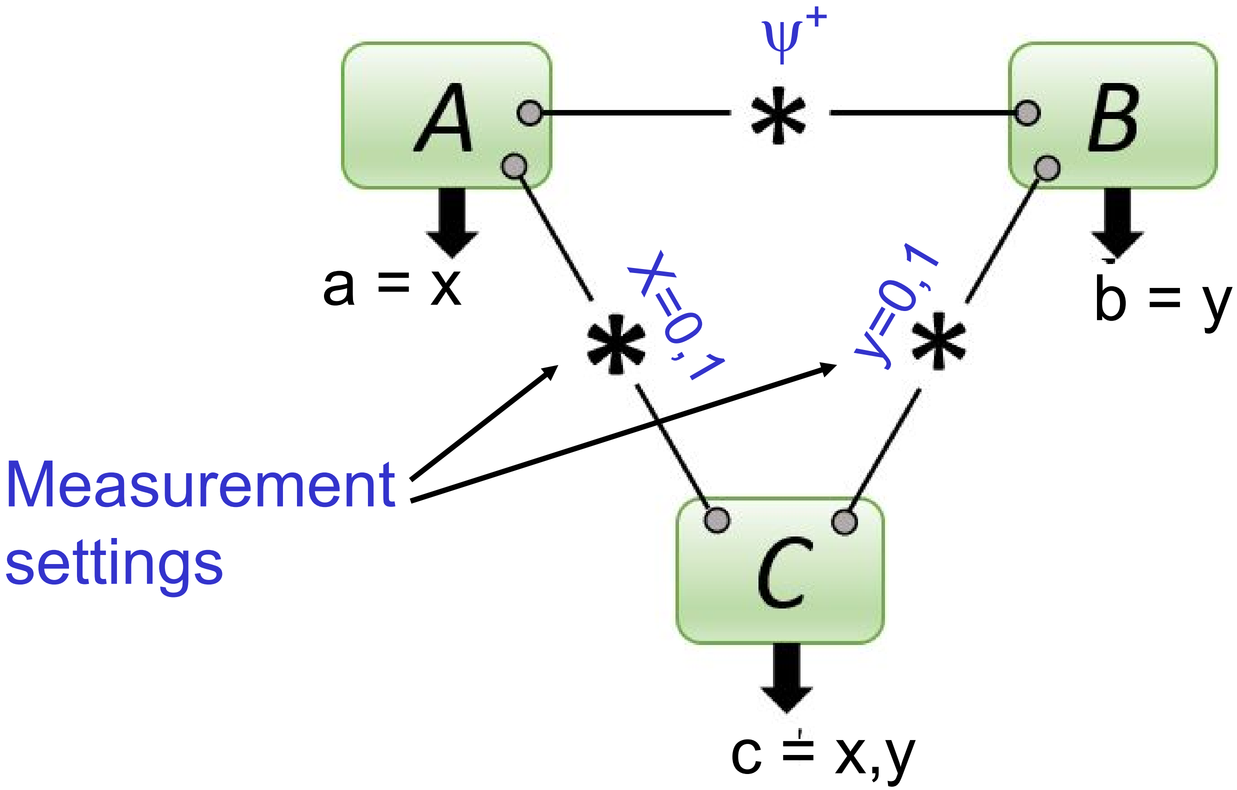
Fritz NJP 2012

Fritz 2012



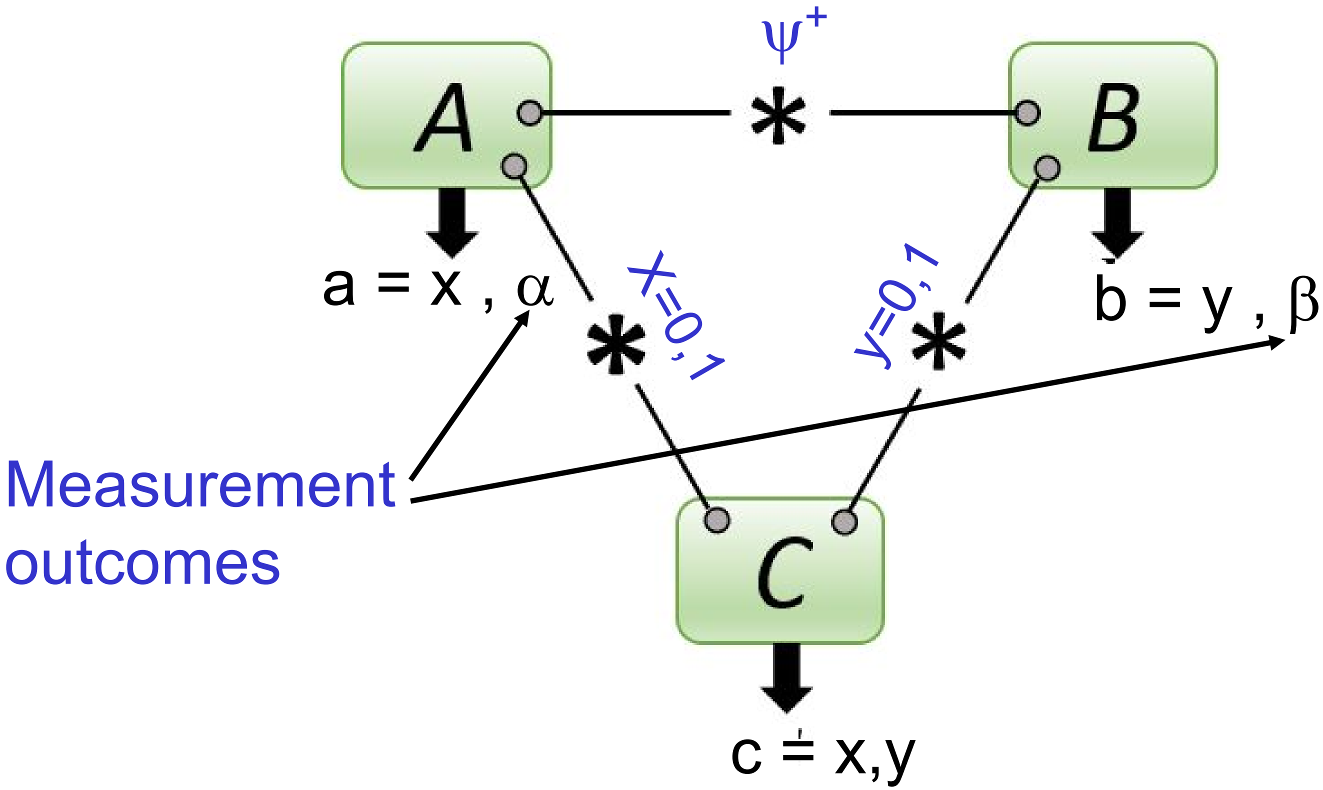
A and B make CHSH test

Fritz 2012



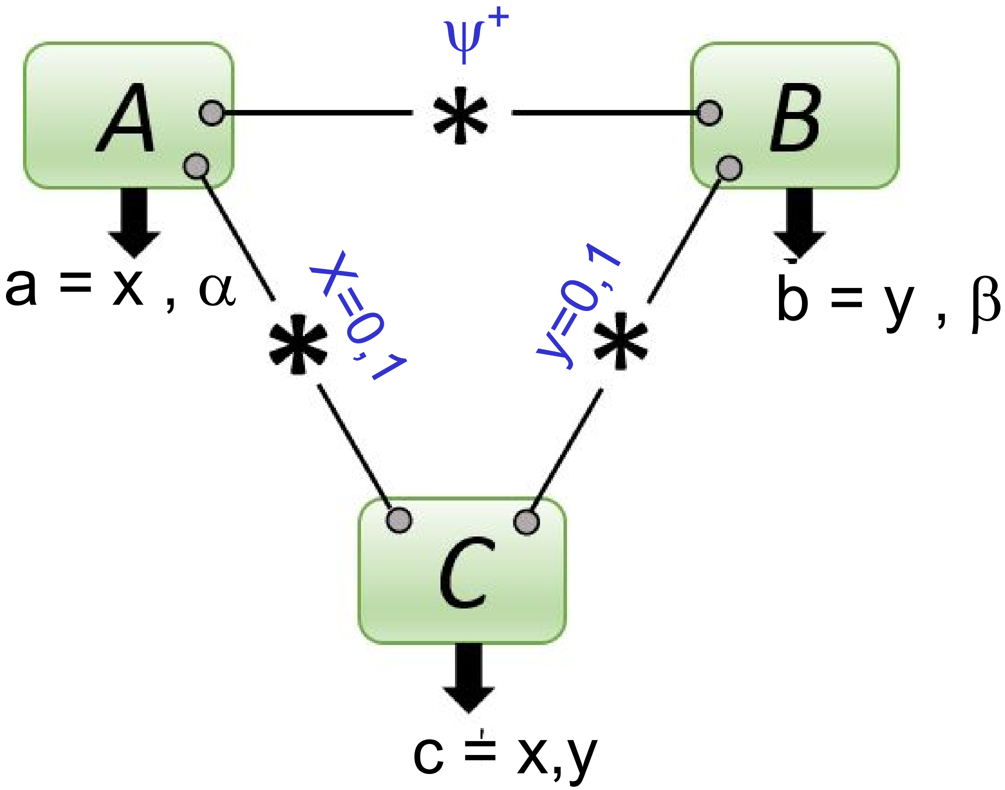
A and B make CHSH test

Fritz 2012



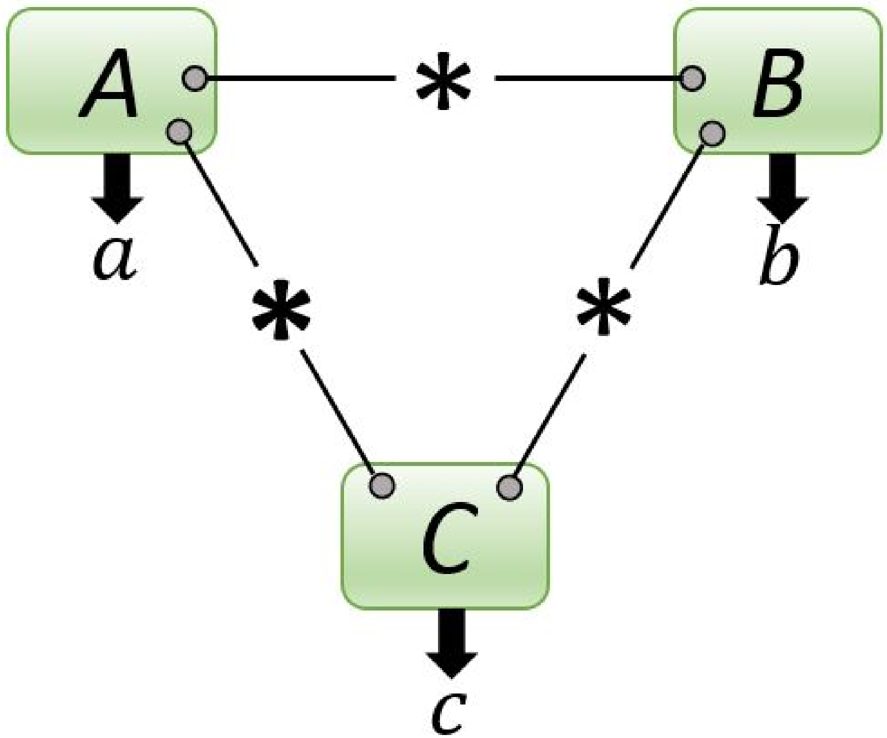
A and B make CHSH test

Fritz 2012



If p(α,β|x,y) violates CHSH, then p(a,b,c) is non trilocal

Genuine network nonlocality

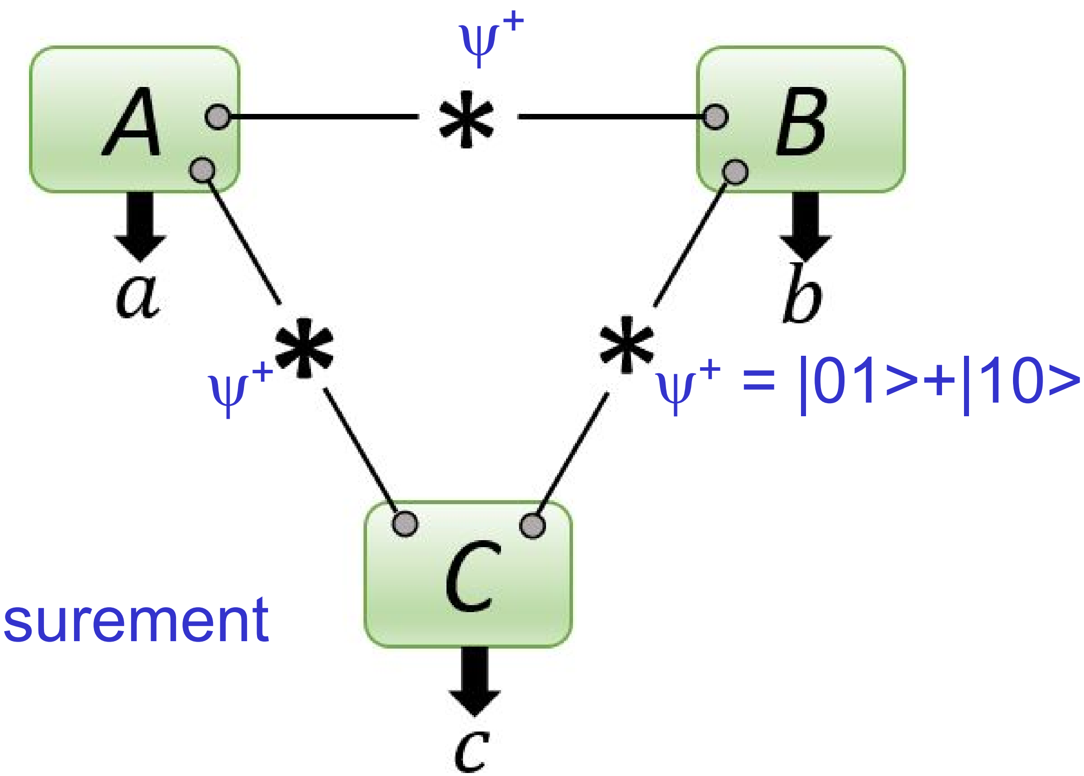


Very different from Fritz

All sources entangled / Symmetry / Entangled measurements

Renou, Boreiri, Bäumer, Brunner, Gisin, Beigi, PRL 2019

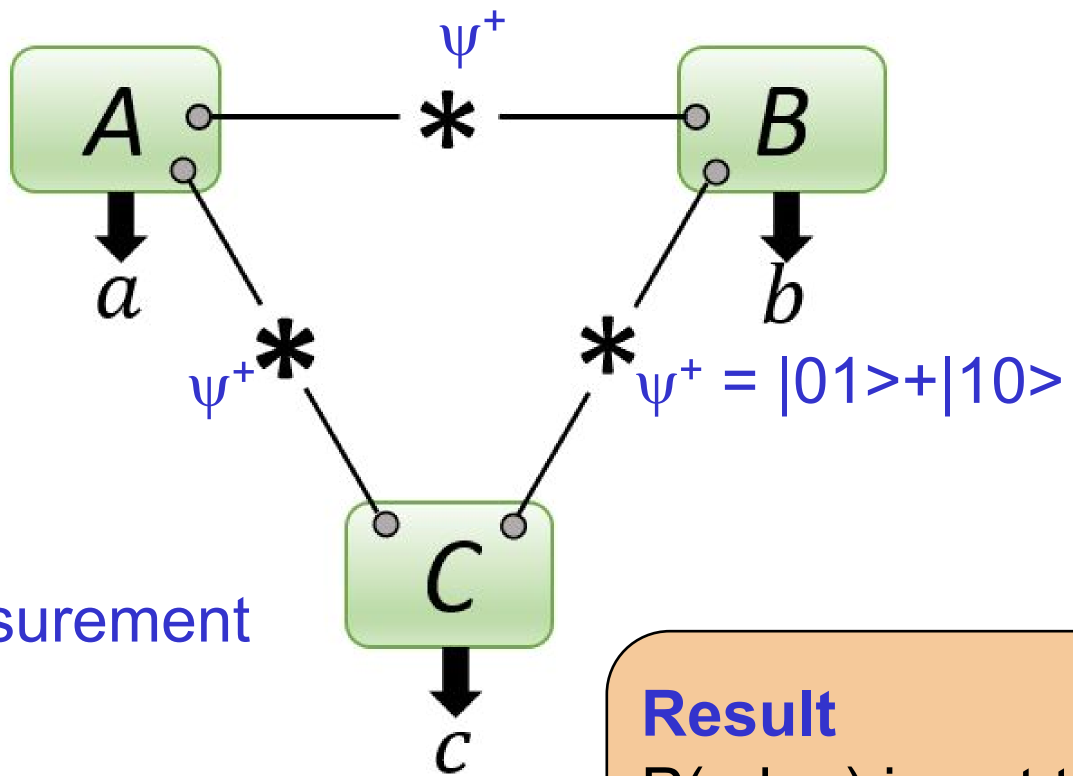
Genuine network nonlocality



Entangled measurement

$$
\begin{cases}
|\bar{0}\rangle = |00\rangle \\
|\bar{1}_0\rangle = u\,|01\rangle + v\,|10\rangle \\
|\bar{1}_1\rangle = v\,|01\rangle - u\,|10\rangle \\
|\bar{2}\rangle = |11\rangle
\end{cases}
$$

Genuine network nonlocality



Entangled measurement

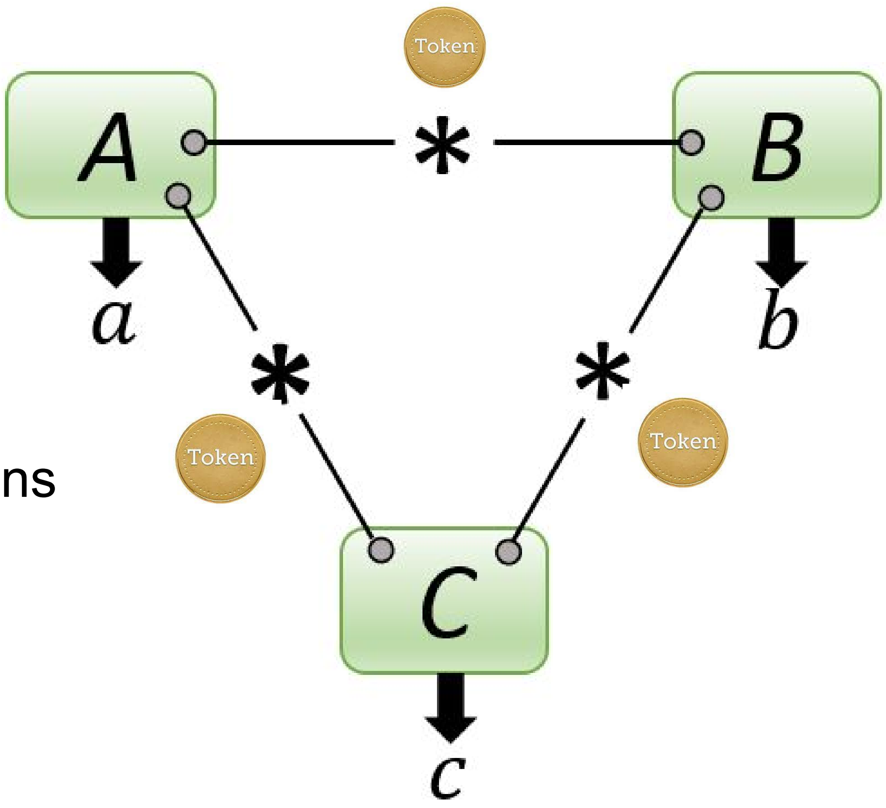
$$\begin{cases} |\bar{0}\rangle = |00\rangle \\ |\bar{1}_0\rangle = u\,|01\rangle + v\,|10\rangle \\ |\bar{1}_1\rangle = v\,|01\rangle - u\,|10\rangle \\ |\bar{2}\rangle = |11\rangle \end{cases}$$

Result

P(a,b,c) is not trilocal for

$$0.785 < u^2 < 1$$

Token Counting



TC strategy

Each source distributes one token

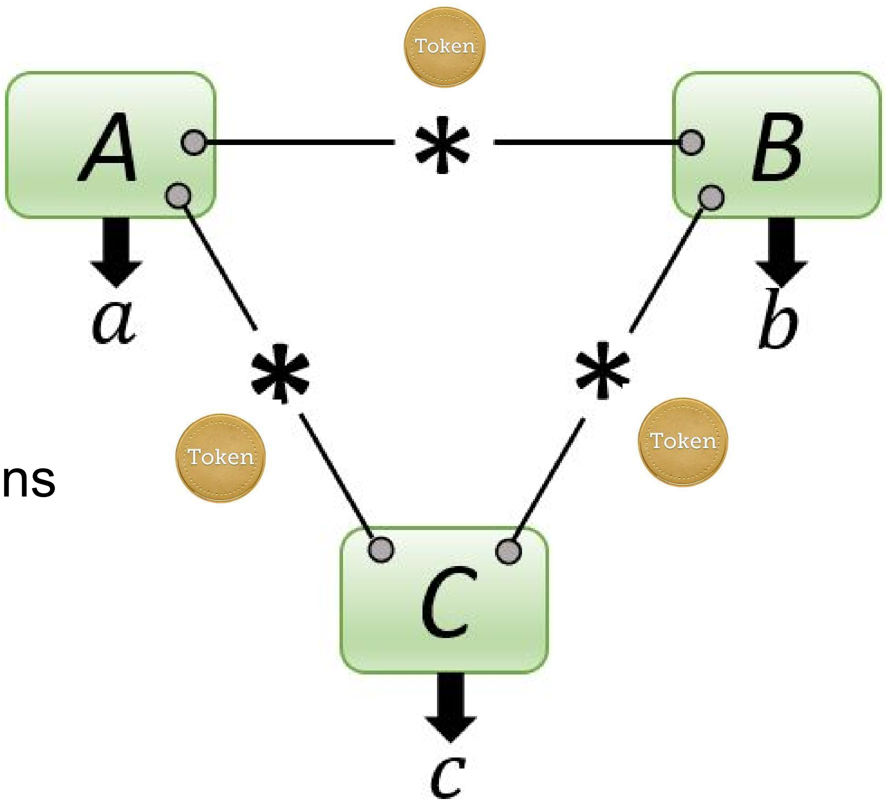
Each party outputs # of received tokens

TC distribution

P(a,b,c) s.t. P(a+b+c=3)=1

Renou & Beigi, PRL 2022

Token Counting



TC strategy

Each source distributes one token

Each party outputs # of received tokens

TC distribution

P(a,b,c) s.t. P(a+b+c=3)=1

TC rigidity or "classical self-testing"

TC strategy ⟺ TC distribution

Renou & Beigi, PRL 2022

Sketch of proof

Local measurement

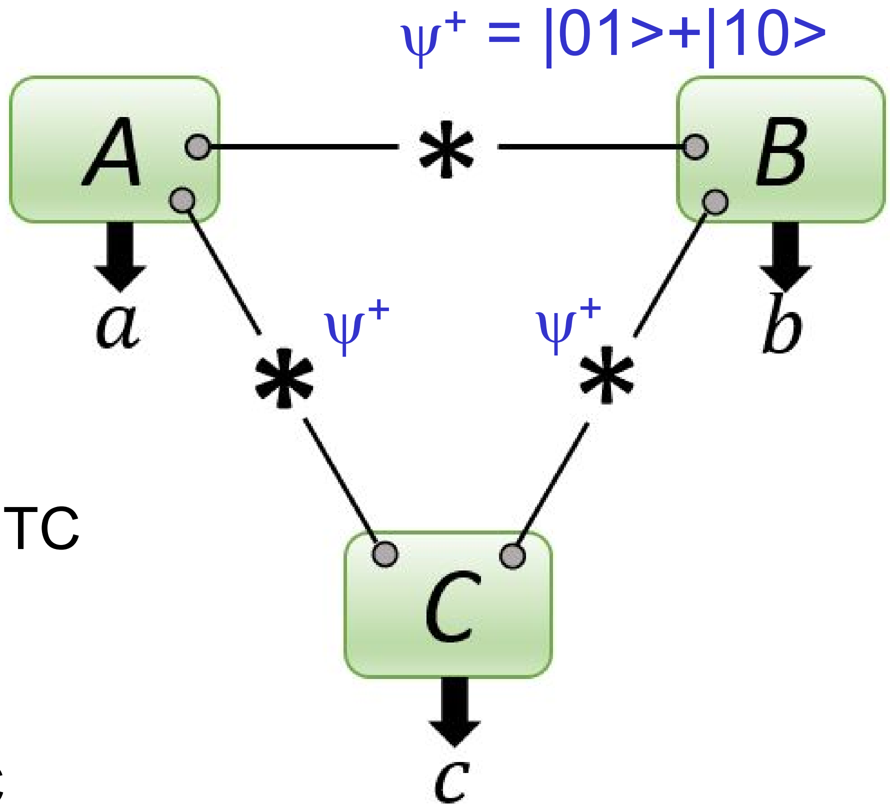
$$|\bar{0}\rangle = |00\rangle \qquad \begin{cases} |\bar{1}_0\rangle = u\,|01\rangle + v\,|10\rangle \\ |\bar{1}_1\rangle = v\,|01\rangle - u\,|10\rangle \end{cases}$$

$$|\bar{2}\rangle = |11\rangle$$

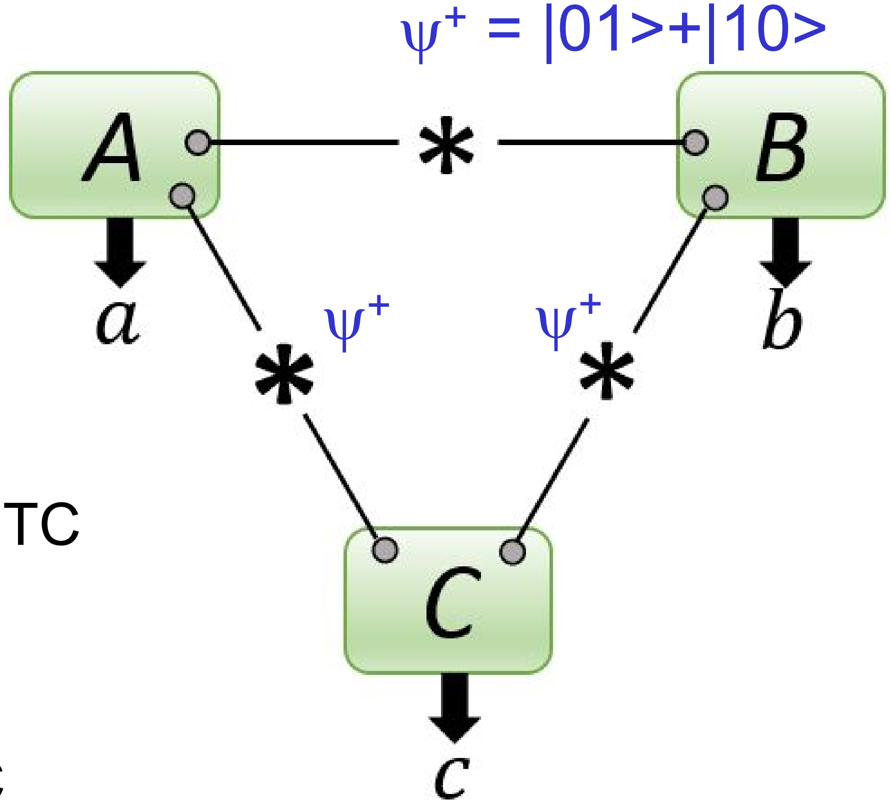
1. Coarse-grained distribution is TC

⇩

Local model must be TC



Sketch of proof



Local measurement

$$|\bar{0}\rangle = |00\rangle \qquad \begin{cases} |\bar{1}_0\rangle = u\,|01\rangle + v\,|10\rangle \\ |\bar{1}_1\rangle = v\,|01\rangle - u\,|10\rangle \end{cases}$$

$$|\bar{2}\rangle = |11\rangle$$

1. Coarse-grained distribution is TC

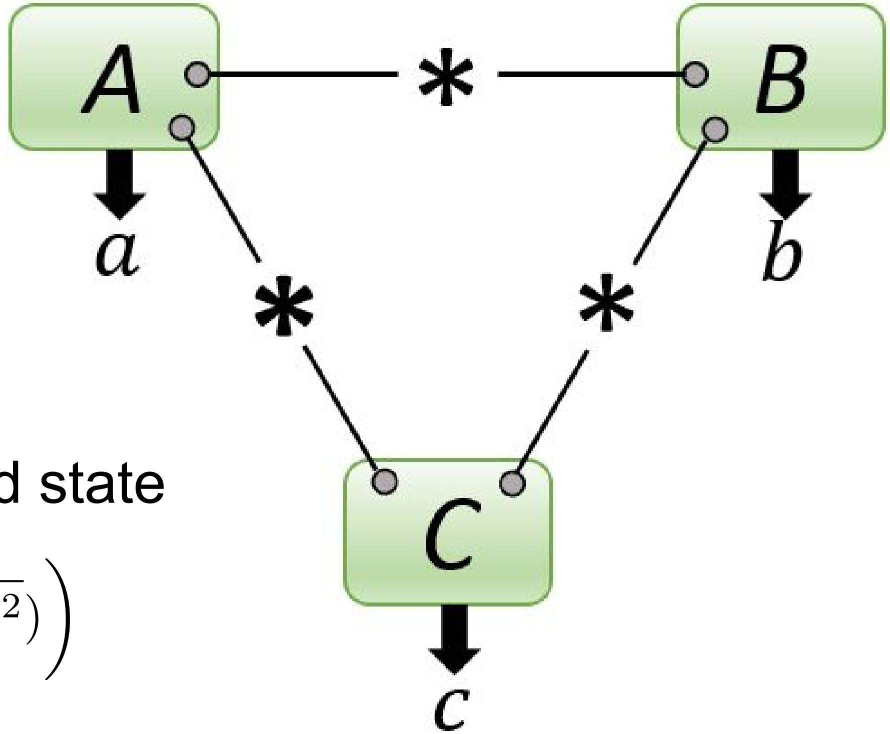
↓

Local model must be TC

2. Original (fine-grained) Q distribution cannot be achieved

→ P(a,b,c) is incompatible with any trilocal model

Partial self-testing of RGB4



RGB4 distribution

Q rigidity of TC

1. All sources produce entangled state

$$\mathcal{E}_F(\rho^{(\alpha)}) \geq h_{\text{bin}}\left(\frac{1}{2}(1-\sqrt{1-16\,r^2})\right)$$

2. All measurements are entangled

3. Certified randomness

$$H_{\min}(\bar{A}|E) \geq -\log_2\left(\frac{1}{2}(1+\sqrt{1-4r})\right)$$

Best case

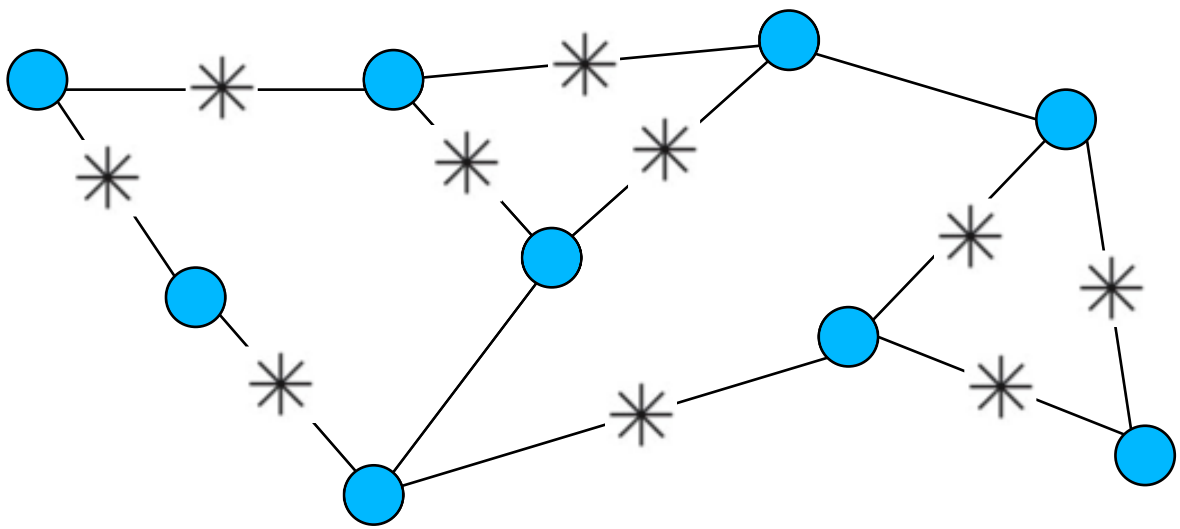
$\mathcal{E}_F > 2.5\%$

$H_{min} > 3.8\%$

Sekatski, Boreiri, NB, PRL 2023

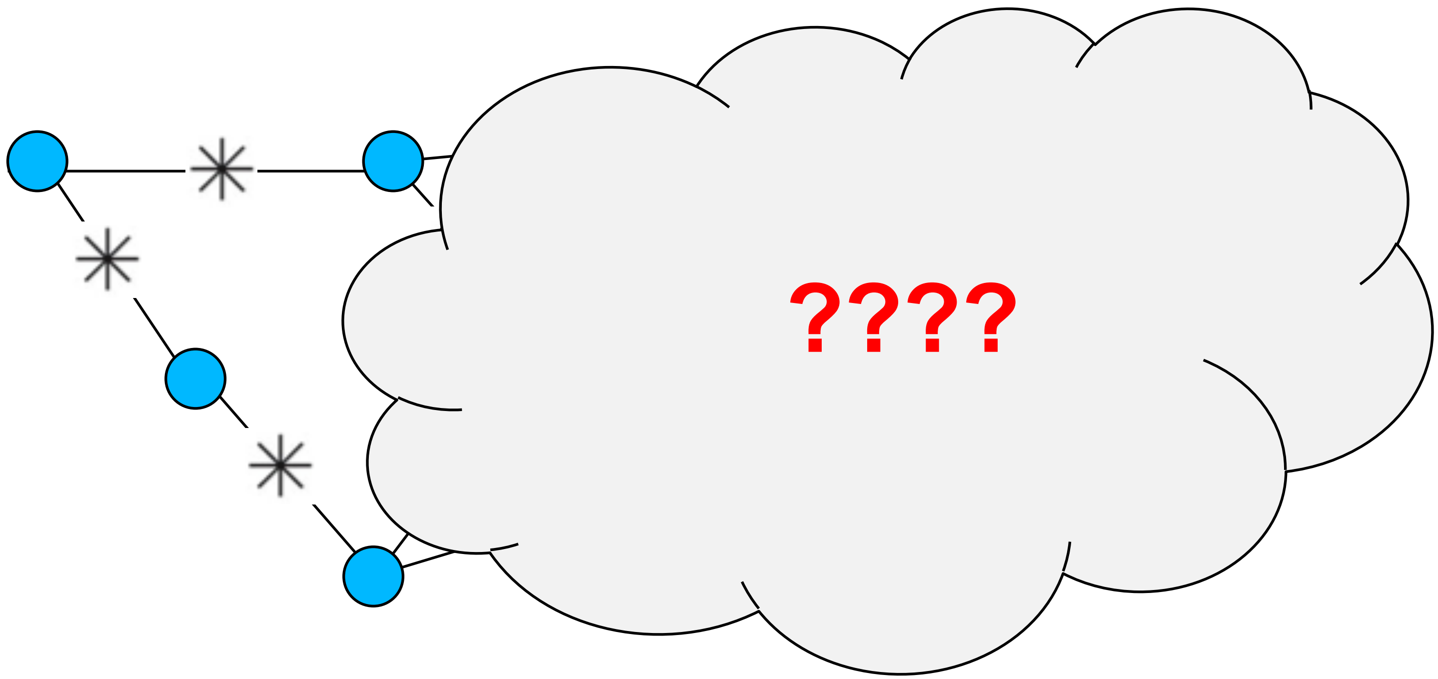
Topologically robust nonlocality

Certify nonlocality without knowing full network structure?



Topologically robust nonlocality

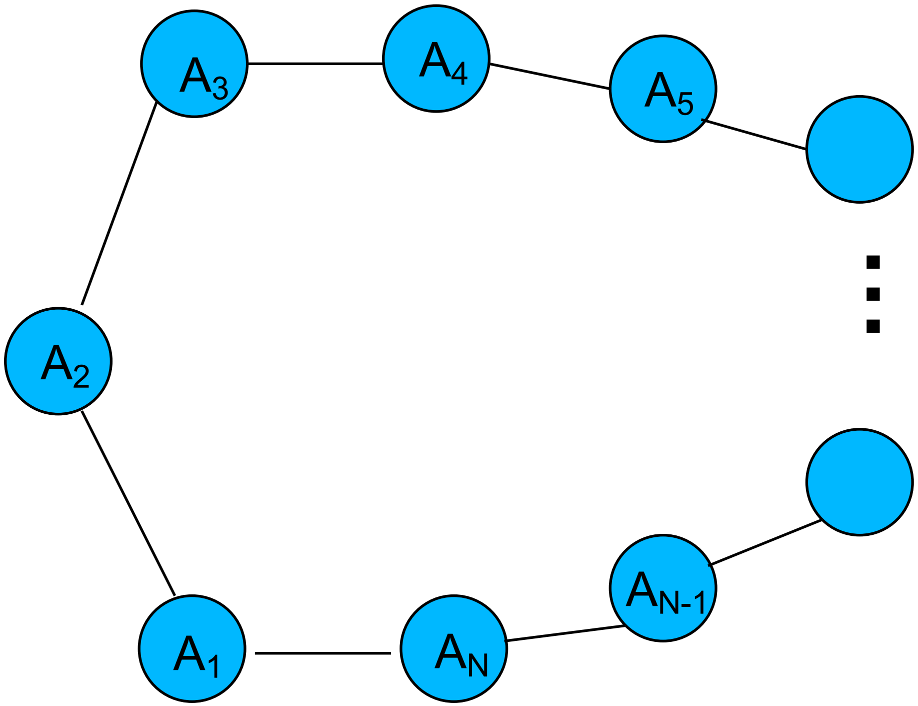
Certify nonlocality without knowing full network structure?



Boreiri, Krivachy, Girardin, Sekatski, Brunner PRL 2025

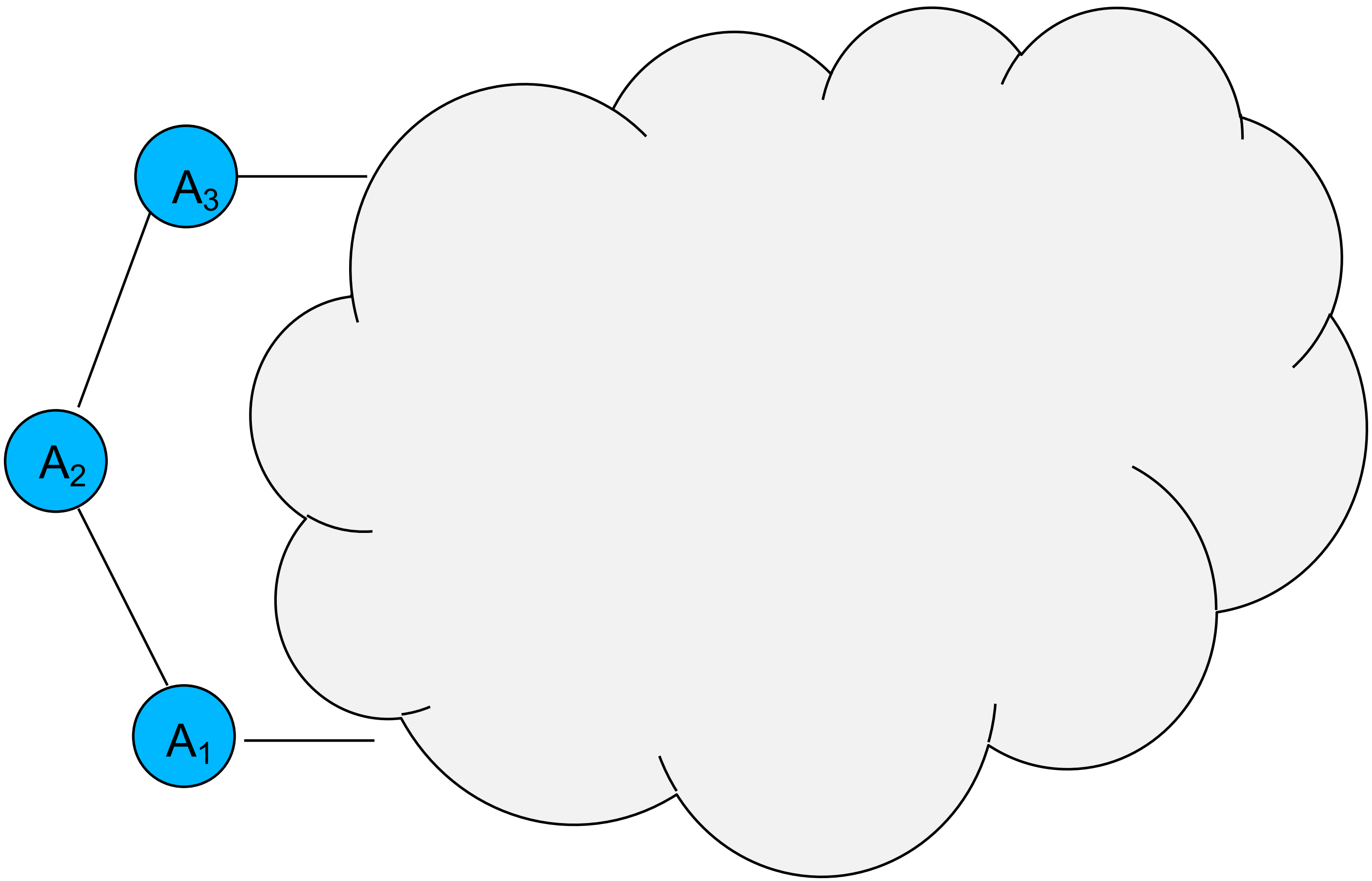
Large networks

Q Token Counting model on a ring



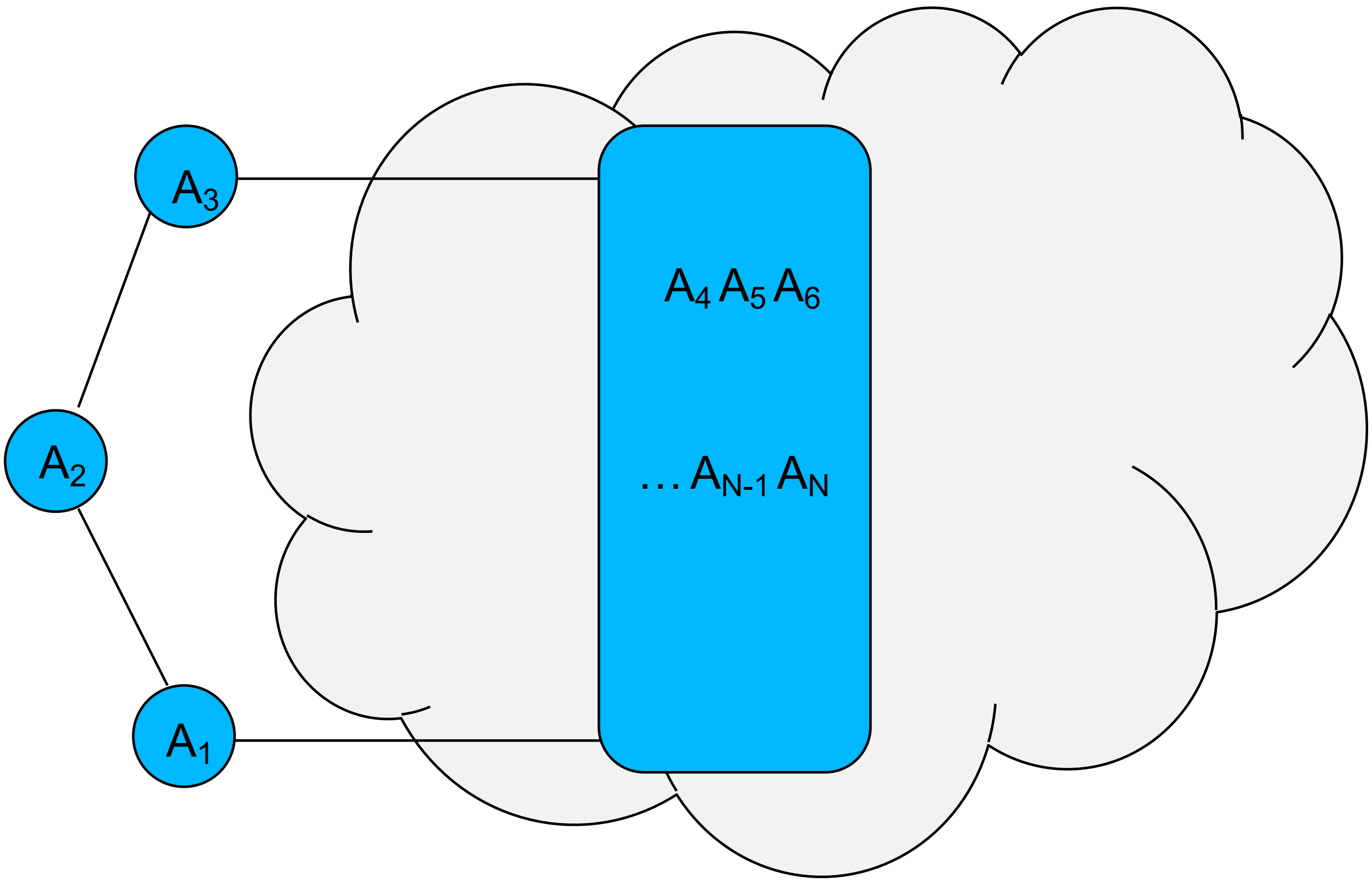
$P(a_1, a_2, \ldots , a_N)$ ⇨ Bell Nonlocality

Large networks



$P(a_1, a_2, \ldots, a_N)$ ➡ ????

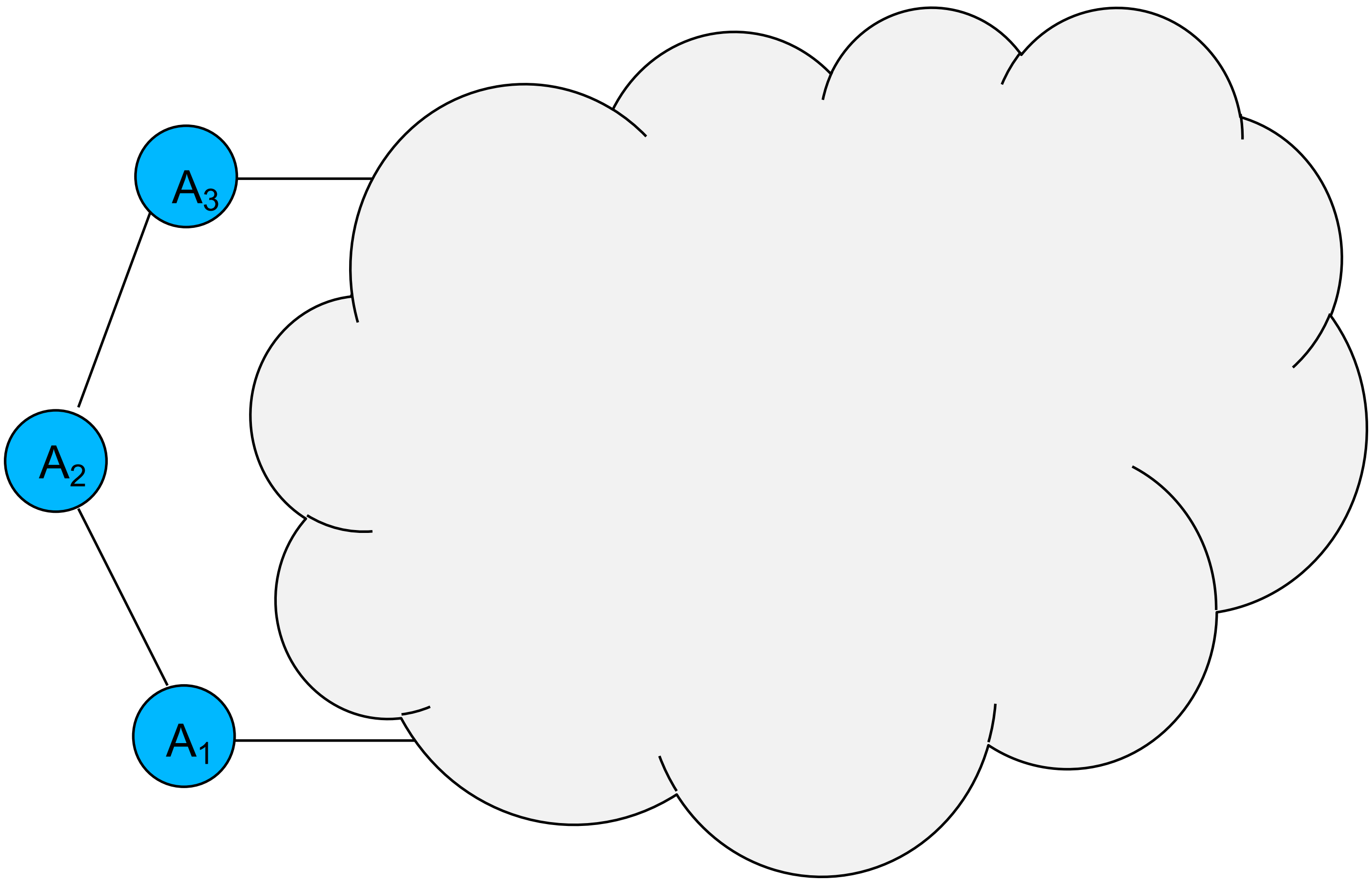
Large networks



$P(a_1, a_2, a_3 | a_4, \ldots, a_N)$ is NL (triangle) → $P(a_1, a_2, \ldots, a_N)$ is NL

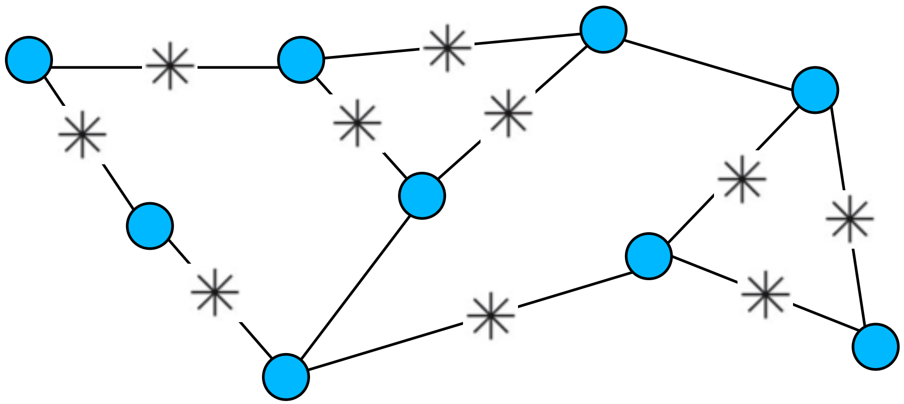
Certify randomness in a_2 (independent of $a_4, \ldots, a_N$)

Large networks



Knowledge of small part of network is enough to certify NL

Summary



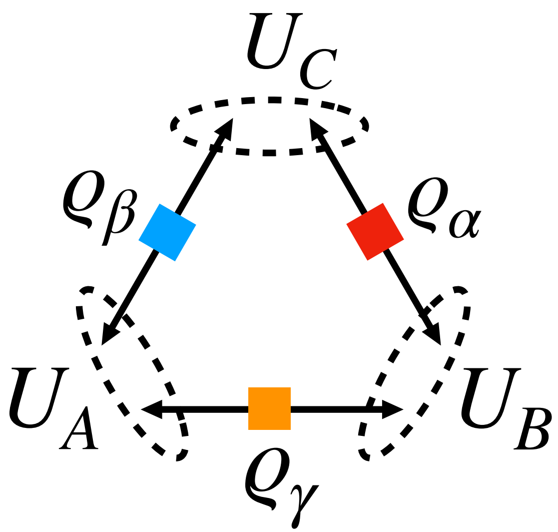
1. Local (classical) correlations in networks → N-locality

2. Genuine network quantum nonlocality

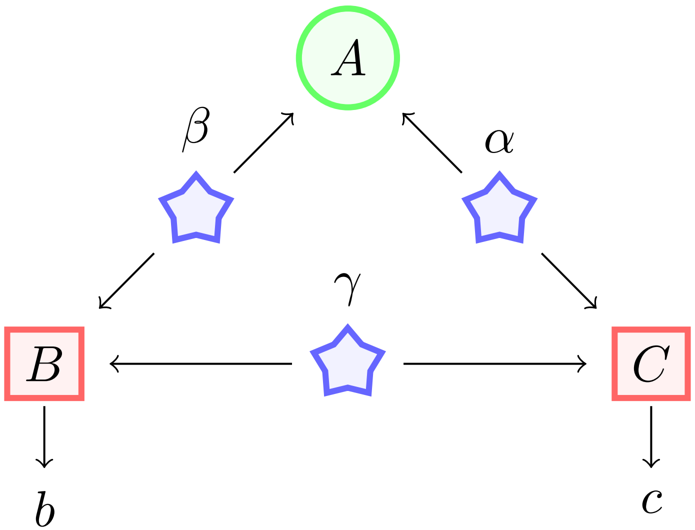
3. Topologically robust nonlocality

Outlook 1: Beyond Bell

Network entanglement



Network steering



Kraft, Designolle, Ritz, Brunner,
Gühne, Huber, PRA 2021
Navascues, Pozas, Rosset, Wolfe, PRL 2021

Jones, Uola, Supic, Brunner,
Skrzypczyk PRL 2020

Thank you!