Entanglement Monogamy in Indistinguishable Particle Systems

Contributed talk

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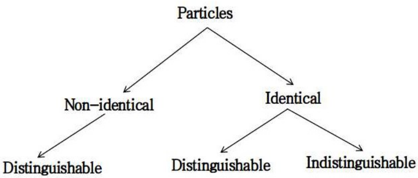
This talk is based on the following papers

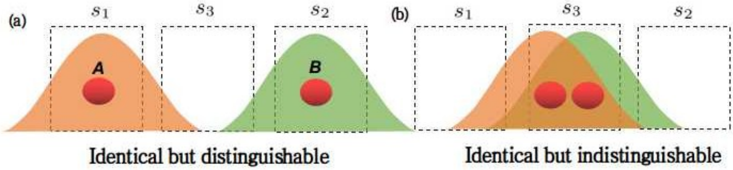
1. **Maximum violation of monogamy of entanglement for indistinguishable particles by measures that are monogamous for distinguishable particles**
   Goutam Paul, Soumya Das, & *Anindya Banerji*
   Physical Review A 104, L010402, Published 20 July 2021
2. **Entanglement monogamy in indistinguishable particle systems**
   Soumya Das, *Goutam Paul* & *Ritabrata Sengupta*
   Scientific Reports, 13, 21972 (2023).

Overview

Introduction to Distinguishable and Indistinguishable particles



• Two particles are said to be identical if all their intrinsic properties (e.g. mass, electrical charge, spin, colour, . . .) are exactly the same.
• Indistinguishable particles means identical particles like bosons or fermions where each particle cannot be addressed individually.



Related works and Open questions

	Distinguishable particles →	**→ Indistinguishable particles**
Quantum teleportation	PRL 70, 1895 (1993)	PRL 120, 240403 (2018)
EPR steering	Phys. Rev. 47, 777 (1935)	Science 360, 409 (2018)
Entanglement swapping	PRL 71, 4287 (1993)	PRA 99, 062322 (2019)

	Indistinguishable particles →	**→ Distinguishable particles**
Duality of entanglement	PRL 110, 140404 (2013)	PRA 94, 032124 (2016)

Open questions:

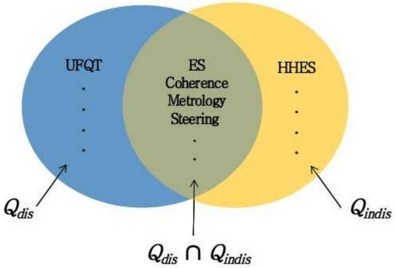
Is there any quantum correlations/application unique to

Q1 ONLY indistinguishable particles?

Q2 ONLY distinguishable particles?

Related Work

- Unit Fidelity Quantum teleportation possible for only distinguishable particles.
- Hyper-Hybrid Entangled State possible for only indistinguishable particles [1].



- Q_{dis}/ Q_{indis} consisting of quantum properties and applications of distinguishable/ indistinguishable particles

[1] **Hyper-hybrid entanglement, indistinguishability, and two-particle entanglement swapping**
Soumya Das, *Goutam Paul*, and *Anindya Banerji*, PRA 102, 052401, 2020

Motivation

Is there any Fundamental quantum properties that is unique to (In)-distinguishable particles?

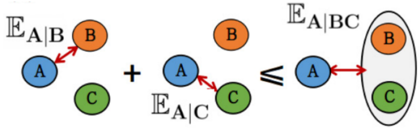
Monogamy of Marriage

Monogamy is a rule of marriage with only one partner.

Source: https://www.ozy.com/news-and-politics/is-monogamy-a-myth/31441/

Monogamy of Entanglement

If two parties are maximally entangled, then none of them can share entanglement with any part of the rest of the system.



$$\mathbb{E}_{A|B}(\rho_{AB}) + \mathbb{E}_{A|C}(\rho_{AC}) \leq \mathbb{E}_{A|BC}(\rho_{ABC}), \quad (1)$$

Applications: Quantum key distribution, classification of quantum states, condensed-matter physics, black-hole physics, etc.

Does Monogamy always hold?

Monogamy depends on two properties

1. **Dimension of the particles**:
2. **Entanglement measure**:

Examples: For qutrits using squared concurrence its is shown that monogamy is violated [2].

$$|\Psi\rangle = \frac{1}{\sqrt{6}}(|123\rangle - |132\rangle + |231\rangle - |213\rangle + |312\rangle - |321\rangle).$$

$$C_{AB}^2 + C_{AC}^2 = 2 \geqslant \frac{4}{3} = C_{A(BC)}^2,$$

Monogamous entanglement measures in qubit: Squared concurrence, log-negativity, entanglement of formation, Tsallis-q entropy, etc.

[2] Yong-Cheng Ou, Phy. Rev. A **75**, 034305 (2007)

Monogamy Violation: Maximum vs Non-maximum

Definition: $\mathbb{E}_{max}$:= The maximum value of the entanglement measure $\mathbb{E}$.

Non-Maximum violation: If in the monogamy inequality of Eq (1), we get

$$\begin{aligned} \mathbb{E}_{A|B}(\rho_{AB}) &< \mathbb{E}_{max}, \\ \mathbb{E}_{A|C}(\rho_{AC}) &< \mathbb{E}_{max}, \\ \mathbb{E}_{A|B}(\rho_{AB}) + \mathbb{E}_{A|C}(\rho_{AC}) &> \mathbb{E}_{max}, \end{aligned} \tag{2}$$

Maximum violation If we have

$$\begin{aligned} \mathbb{E}_{A|B}(\rho_{AB}) &= \mathbb{E}_{max}, \\ \mathbb{E}_{A|C}(\rho_{AC}) &= \mathbb{E}_{max} \end{aligned} \tag{3}$$

Cloning: Animals vs particles

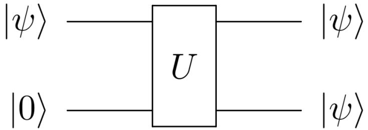
Dolly the sheep was successfully cloned in 1996.

Source: *https : / / www . reddit . com / r / physicsmemes / comments / jvdmn6 / we_went_through_the_nocloning_theorem_today /*

Monogamy vs Cloning

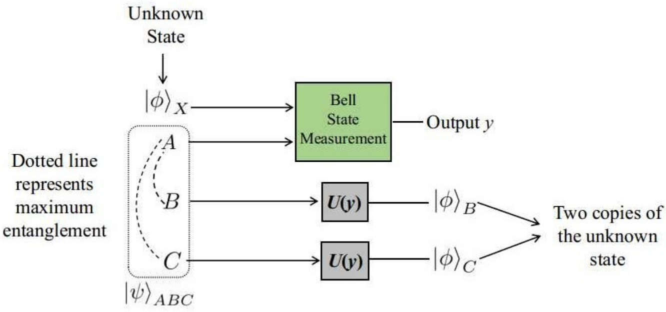
• **No Cloning theorem** forbids the creation of an independent and identical copy of an arbitrary unknown quantum state. It was stated by Wootters, Zurek, and Dieks in 1982.

• It follows from the fact that all quantum operations must be unitary linear transformation on the state.



• **Applications:** Quantum error correction, Quantum cryptography, Black-hole physics etc.

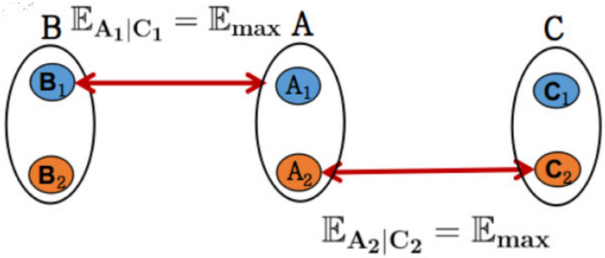
Maximum Violation of Monogamy implies cloning using distinguishable particles



$$\mathbb{E}_{A|B}(\rho_{AB}) = \mathbb{E}_{max} \text{ and } \mathbb{E}_{A|C}(\rho_{AC}) = \mathbb{E}_{max}, \quad (4)$$

Apparent maximum monogamy violation

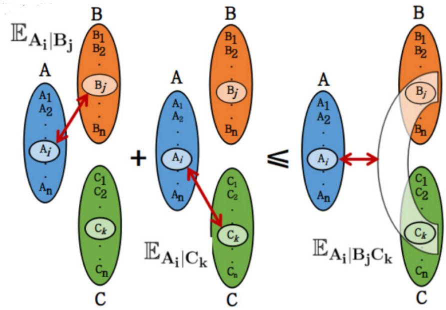
Consider Three particles, A, B, and C, each having two degrees of freedom (DoF)



This type of state is proposed in [3].

[3] P. Chithrabhanu *et al.*, Quant. Inf. Process., **14**, 10, (2015).

Particle paradigm to DoF paradigm

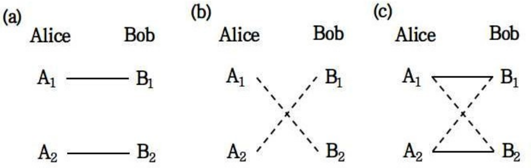


Monogamy inequality using DoFs

$$\mathbb{E}_{A_i|B_j}(\rho_{A_iB_j}) + \mathbb{E}_{A_i|C_k}(\rho_{A_iC_k}) \leq \mathbb{E}_{A_i|B_jC_k}(\rho_{A_iB_jC_k}), \quad (5)$$

Problem Statement

- $A_1 - -B_1$ are spin/polarization entangled state.
- $A_2 - -B_2$ are path entangled state.
- $A_1 - -B_2$ and $A_2 - -B_1$. are spin-path entangled states.



- **Prove that** $A_1 - -B_1$ and $A_1 - -B_2$ are maximally entangled.

DoF Trace-out rule for indistinguishable particles

- Trace-out rule for distinguishable particles each having a single DoF is known[4].
- This can be extented trivially for distinguishable particles having multiple DoFs.
- Trace-out rule for indistinguishable particles each having a single DoF is proposed in[5].

Motivation: The trace-out rule for indistinguishable particles having multiple DoFs are non-trivial.

[4] M. A. Nielsen and I. L. Chuang. Quantum computation and quantum information

[5] R. Lo Franco & G. Compagno. Sci. Rep., 6, 1, (2016).

Representation of the state of indistinguishable particles

- Two particles each having two DoFs

$$|\Psi^{(2,2)}\rangle := \sum_{\alpha^1,\alpha^2,a_1^1,a_2^1,a_1^2,a_2^2} \eta^u \kappa^{\alpha^1,\alpha^2}_{a_1^1,a_2^1,a_1^2,a_2^2} |\alpha^1 a_1^1 a_2^1, \alpha^2 a_1^2 a_2^2\rangle , \tag{6}$$

$$\rho^{(2,2)} := \sum_{\alpha^i,\beta^i,a_j^i,b_j^i} \eta^{(u+\bar{u})} \kappa^{\alpha^1,\alpha^2}_{a_1^1 a_2^1,a_1^2 a_2^2} \kappa^{\beta^1,\beta^2 *}_{b_1^1 b_2^1,b_1^2 b_2^2} |\alpha^1 a_1^1 a_2^1, \alpha^2 a_1^2 a_2^2\rangle \langle \beta^1 b_1^1 b_2^1, \beta^2 b_1^2 b_2^2| , \tag{7}$$

- Two particles each having n DoFs

$$|\Psi^{(2,n)}\rangle := \sum_{\alpha^i,a_j^i} \eta^u \kappa^{\alpha^1,\alpha^2}_{a_1^1 a_2^1\cdots a_n^1,a_1^2 a_2^2\cdots a_n^2} |\alpha^1 a_1^1 a_2^1 \cdots a_n^1, \alpha^2 a_1^2 a_2^2 \cdots a_n^2\rangle , \tag{8}$$

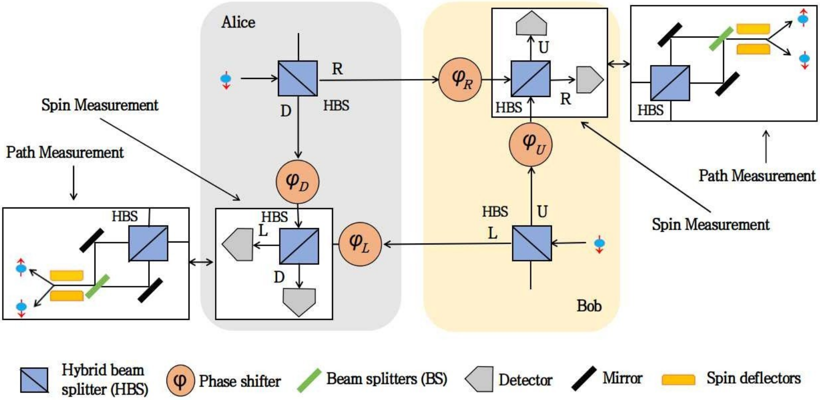
- p particles each having two DoFs

$$|\Psi^{(p,n)}\rangle := \sum_{\alpha^i,a_j^i} \eta^u \kappa^{\alpha^1,\alpha^2,\cdots,\alpha^p}_{a_1^1 a_2^1\cdots a_n^1,a_1^2 a_2^2\cdots a_n^2,\cdots,a_1^p a_2^p\cdots a_n^p} |\alpha^1 a_1^1 a_2^1 \cdots a_n^1, \alpha^2 a_1^2 a_2^2 \cdots a_n^2, \cdots, \alpha^p a_1^p a_2^p \cdots a_n^p\rangle . \tag{9}$$

DoF Trace-out Rule

$$
\begin{aligned}
\rho_{s_{x_{\bar{i}}}} \equiv \mathrm{Tr}_{s_{x_i}}\left(\rho^{(2,2)}\right) \equiv \sum_{m_i \in \mathcal{D}_i} \langle s_x m_i | \rho^{(2,2)} | s_x m_i \rangle \\
:= \sum_{m_i} \Bigg\{ \sum_{\substack{\alpha,\beta,a_i,a_{\bar{i}},b_1,b_2,\\ \gamma,\delta,c_i,c_{\bar{i}},d_1,d_2}} \kappa^{\alpha a_i a_{\bar{i}}}_{\beta b_1 b_2} \kappa^{\gamma c_i c_{\bar{i}} *}_{\delta d_1 d_2} \langle s_x m_i | \alpha a_i \rangle \langle \gamma c_i | s_x m_i \rangle | \alpha a_{\bar{i}}, \beta b_1 b_2 \rangle \langle \gamma c_{\bar{i}}, \delta d_1 d_2 | \\
+ \eta \sum_{\substack{\alpha,\beta,a_1,a_2,b_i,b_{\bar{i}},\\ \gamma,\delta,c_i,c_{\bar{i}},d_1,d_2}} \kappa^{\alpha a_1 a_2}_{\beta b_i b_{\bar{i}}} \kappa^{\gamma c_i c_{\bar{i}} *}_{\delta d_1 d_2} \langle s_x m_i | \beta b_i \rangle \langle \gamma c_i | s_x m_i \rangle | \alpha a_1 a_2, \beta b_{\bar{i}} \rangle \langle \gamma c_{\bar{i}}, \delta d_1 d_2 | \\
+ \eta \sum_{\substack{\alpha,\beta,a_i,a_{\bar{i}},b_1,b_2,\\ \gamma,\delta,c_1,c_2,d_i,d_{\bar{i}}}} \kappa^{\alpha a_i a_{\bar{i}}}_{\beta b_1 b_2} \kappa^{\gamma c_1 c_2 *}_{\delta d_i d_{\bar{i}}} \langle s_x m_i | \alpha a_i \rangle \langle \delta d_i | s_x m_i \rangle | \alpha a_{\bar{i}}, \beta b_1 b_2 \rangle \langle \gamma c_1 c_2, \delta d_{\bar{i}} | \\
+ \sum_{\substack{\alpha,\beta,a_1,a_2,b_i,b_{\bar{i}},\\ \gamma,\delta,c_1,c_2,d_i,d_{\bar{i}}}} \kappa^{\alpha a_1 a_2}_{\beta b_i b_{\bar{i}}} \kappa^{\gamma c_1 c_2 *}_{\delta d_i d_{\bar{i}}} \langle s_x m_i | \beta b_i \rangle \langle \delta d_i | s_x m_i \rangle | \alpha a_1 a_2, \beta b_{\bar{i}} \rangle \langle \gamma c_1 c_2, \delta d_{\bar{i}} | \Bigg\},
\end{aligned}
\tag{10}
$$

Circuit for Maximum violation of Monogamy [6]



[6] Li *et al*, Phys. Rev. Lett. 120, 050404 (2018)

Proof Sketch

We consider the state

$$|\Psi^{(2)}\rangle = \sum_{\alpha,\beta\in\{s_1,s_2\},a_1,b_1\in\{L,D,R,U\},a_2,b_2\{\uparrow,\downarrow\}} \kappa^{\alpha a_1 a_2}_{\beta b_1 b_2} |\alpha a_1 a_2, \beta b_1 b_2\rangle$$

where

$$\kappa^{s_1 L\downarrow}_{s_2 R\downarrow} = -\kappa^{s_1 D\uparrow}_{s_2 U\uparrow} = \frac{1}{4}(\kappa_1+\kappa_2),\quad \kappa^{s_1 D\uparrow}_{s_2 R\downarrow} = \kappa^{s_1 L\downarrow}_{s_2 U\uparrow} = \frac{i}{4}(\kappa_1-\kappa_2),\quad \kappa^{s_2 R\downarrow}_{s_2 R\downarrow} = \kappa^{s_2 U\uparrow}_{s_2 U\uparrow} = \frac{i\kappa_1}{4},\quad \kappa^{s_1 D\uparrow}_{s_1 D\uparrow} = \kappa^{s_1 L\downarrow}_{s_1 L\downarrow} = \frac{i\kappa_2}{4}$$

$$\kappa_1 = e^{i(\phi_R+\phi_L)} \text{ and } \kappa_2 = e^{i(\phi_D+\phi_U)}$$

We show [7], using concurrence entanglement measure

$$\mathcal{C}_{s_{1_{a_2}}|s_{2_{b_2}}}\left(\rho_{s_{1_{a_2}} s_{2_{b_2}}}\right) = 1.$$

$$\mathcal{C}_{s_{1_{a_2}}|s_{2_{b_1}}}\left(\rho_{s_{1_{a_2}} s_{2_{b_1}}}\right) = 1.$$

[7] Supplemental material of G. Paul, S. Das, and A. Banerji, Phys. Rev. A **104**, L010402 (2021)

Main Theorem

Theorem

In qubit systems, indistinguishability is a necessary criterion for maximum violation of monogamy of entanglement by the same measures that are monogamous for distinguishable particles[a].

[a] G. Paul, S.Das, and A. Banerji, Phy. Rev. A 104, L010402 (2021).

Open Questions?

1. Which one is more fundamental? No cloning theorem or Monogamy of entanglement?
2. What is the application this maximum violation of monogamy?
3. Is this violation possible for other quantum correlations like steering, coherence, discord, etc?

Open Question and motivation

Table: Summary of the results related to monogamy of entanglement for distinguishable and indistinguishable particles.

	Distinguishable	**Indistinguishable**
2 particles	Holds	Can violate maximally
$\geq$ **3 particles**	Holds	????

Main Result

Result

Three or more indistinguishable particles, each having an arbitrary number of degrees of freedom, obey the monogamy of entanglement using squared concurrence.

Interesting Result

Corollary

If monogamy is calculated using three (or more) indistinguishable particles, then for all pure states we can write it as

$$\mathcal{C}^2_{\alpha_i|\beta_j}(\rho_{\alpha_i\beta_j}) + \mathcal{C}^2_{\alpha_i|\gamma_k}(\rho_{\alpha_i\gamma_k}) = \mathcal{C}^2_{\alpha_i|\beta_j\gamma_k}(\rho_{\alpha_i\beta_j\gamma_k}), \tag{11}$$

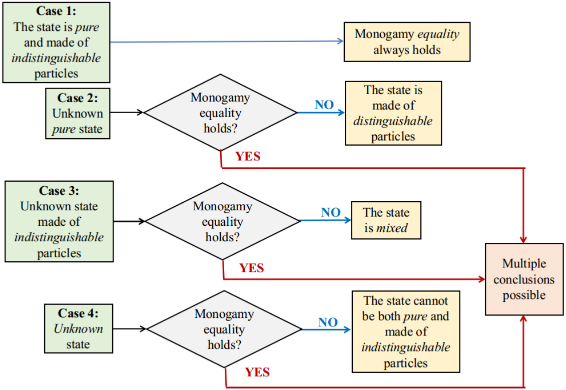
where α, β, γ are spatial locations and i, j, k denote the DoF indices $\in \mathbb{N}_n$.

Full Picture

Table: Operational meaning of our result. Here we see that MoE equality holds for only pure indistinguishable particles using three or more particles and taking concurrence as an entanglement measure. For the rest of the cases, the MoE inequality holds.

	Distinguishable	**Indistinguishable**
Pure	Inequality ($\leq$) Holds	Equality ($=$) Holds
Mixed	Inequality ($\leq$) Holds	Inequality ($\leq$) Holds

Relationship b/w Purity, Indistinguishability and Monogamy



MoE for three or more indistinguishable particles each having n DoFs

	DoF	Eigenstate	1st particle	2nd particle	3rd particle	Relations	s^1	s^2	s^3	$\mathscr{C}^2_{s^1\|s^2}$	$\mathscr{C}^2_{s^1\|s^3}$	$\mathscr{C}^2_{s^1\|s^2s^3}$
							Measures in the DoF					
1	Same	Same	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_k}$	Nil	j	j	j	0	0	0
2	Same	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_{k'}}$	$\mathscr{D}_{j_k}, \mathscr{D}_{j_{k'}} \in \mathbb{D}_j$, $\|\mathscr{D}\rangle_{j_{k'}} = \|\mathscr{D}\rangle^{\perp}_{j_k}$	j	j	j	≥ 0	≥ 0	≥ 0
3	Different	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j'_l}$	$j \neq j', \mathscr{D}_{j_k} \in \mathbb{D}_j$, $\mathscr{D}_{j'_l} \in \mathbb{D}_{j'}$	j	j	j'	0	0	0
4	Different	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_{k'}}$	$\|\mathscr{D}\rangle_{j'_l}$	$\|\mathscr{D}\rangle_{j_{k'}} = \|\mathscr{D}\rangle^{\perp}_{j_k}$, $\mathscr{D}_{j'_l} \in \mathbb{D}_{j'}$	j	j	j'	≥ 0	0	≥ 0
5	Different	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j''_h}$	$\|\mathscr{D}\rangle_{j'_l}$	$j \neq j' \neq j''$, $\mathscr{D}_{j''_h} \in \mathbb{D}_{j''}$	j	j''	j'	0	0	0
6	Same	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_k}$	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$	$\kappa^2_{j_k} + \kappa^2_{j_{k'}} = 1$	j	j	j	≥ 0	≥ 0	≥ 0
7	Same	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_{k'}}$	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$	$\|\mathscr{D}\rangle_{j_{k'}} = \|\mathscr{D}\rangle^{\perp}_{j_k}$, $\kappa^2_{j_k} + \kappa^2_{j_{k'}} = 1$	j	j	j	≥ 0	≥ 0	≥ 0
8	Same	Same superposition	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$	$\kappa^2_{j_k} + \kappa^2_{j_{k'}} = 1$	j	j	j	0	0	0
9	Same	Different superposition	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi_1}\|\mathscr{D}\rangle_{j_{k'}}$ where $\kappa^2_{j_k} + \kappa^2_{j_{k'}} = 1$	$\kappa'_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa'_{j_{k'}} e^{i\phi_2}\|\mathscr{D}\rangle_{j_{k'}}$, where $\kappa'^2_{j_k} + \kappa'^2_{j_{k'}} = 1$	$\kappa''_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa''_{j_{k'}} e^{i\phi_3}\|\mathscr{D}\rangle_{j_{k'}}$, where $\kappa''^2_{j_k} + \kappa''^2_{j_{k'}} = 1$	$\phi_1 \neq \phi_2 \neq \phi_3$ $\kappa_{j_k} \neq \kappa'_{j_k} \neq \kappa''_{j_k}$ $\kappa_{j_{k'}} \neq \kappa'_{j_{k'}} \neq \kappa''_{j_{k'}}$	j	j	j	≥ 0	≥ 0	≥ 0
10	Different	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_k}$	$\kappa_{j'_l}\|\mathscr{D}\rangle_{j'_l} + \kappa_{j'_{l'}} e^{i\phi}\|\mathscr{D}\rangle_{j'_{l'}}$	$\kappa^2_{j'_l} + \kappa^2_{j'_{l'}} = 1$	j	j	j'	0	0	0
11	Different	Different	$\|\mathscr{D}\rangle_{j_k}$	$\|\mathscr{D}\rangle_{j_{k'}}$	$\kappa_{j'_l}\|\mathscr{D}\rangle_{j'_l} + \kappa_{j'_{l'}} e^{i\phi}\|\mathscr{D}\rangle_{j'_{l'}}$	$\kappa^2_{j'_l} + \kappa^2_{j'_{l'}} = 1$	j	j	j'	≥ 0	0	≥ 0
12	Different	Different superposition	$\|\mathscr{D}\rangle_{j_k}$	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$, where $\kappa^2_{j_k} + \kappa^2_{j_{k'}} = 1$	$\kappa_{j'_l}\|\mathscr{D}\rangle_{j'_l} + \kappa_{j'_{l'}} e^{i\phi}\|\mathscr{D}\rangle_{j'_{l'}}$, where $\kappa^2_{j'_l} + \kappa^2_{j'_{l'}} = 1$	$j \neq j'$	j	j	j'	≥ 0	≥ 0	≥ 0
13	Different	Different superposition	$\kappa_{j_k}\|\mathscr{D}\rangle_{j_k} + \kappa_{j_{k'}} e^{i\phi}\|\mathscr{D}\rangle_{j_{k'}}$ where $\kappa^2_{j_k} + \kappa^2_{j_{k'}} = 1$	$\kappa_{j''_h}\|\mathscr{D}\rangle_{j''_h} + \kappa_{j''_{h'}} e^{i\phi''}\|\mathscr{D}\rangle_{j''_{h'}}$, where $\kappa^2_{j''_h} + \kappa^2_{j''_{h'}} = 1$	$\kappa_{j'_l}\|\mathscr{D}\rangle_{j'_l} + \kappa_{j'_{l'}} e^{i\phi'}\|\mathscr{D}\rangle_{j'_{l'}}$, where $\kappa^2_{j'_l} + \kappa^2_{j'_{l'}} = 1$	$j \neq j' \neq j''$ $\mathscr{D}_{j_{k'}}, \in \mathbb{D}_j$, $\mathscr{D}_{j''_{h'}} \in \mathbb{D}_{j''}$ $\mathscr{D}_{j'_{l'}} \in \mathbb{D}_{j'}$	j	j''	j'	0	0	0

Discussion

- Monogamy holds for three or more indistinguishable particles and the inequality becomes equality for all pure indistinguishable states.
- A connection between the three properties, say monogamy, purity, and distinguishability of some specific quantum states
- The full characterization of all the states based on monogamy, purity, and distinguishability an interesting future works.

My current works

- Security Analysis of Continuous Variable Quantum Key Distribution Systems:
  - Gaussian Modulation: Solved
  - Discrete Modulation: Open
- A Critical Analysis of Deployed Use Cases for Quantum Key Distribution and Comparison with Post-Quantum Cryptography[8]
- Security analysis of Hybrid (QKD+PQC) Eindhoven Quantum Communication Testbed
- Evaluating Quantumness, Efficiency and Cost of Quantum Random Number Generators[9]

[8] Aquina, Nick, et al. "A Critical Analysis of Deployed Use Cases for Quantum Key Distribution and Comparison with Post-Quantum Cryptography." arXiv preprint arXiv:2502.04009 (2025).

[9] Paul, Goutam, Nirupam Basak, and Soumya Das. "Evaluating Quantumness, Efficiency and Cost of Quantum Random Number Generators via Photon Statistics." arXiv preprint arXiv:2405.14085 (2024).

THANK YOU

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