International Symposium on Quantum Information and Communication (ISQIC), 2025
CQuERE, TCG CREST
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Quantum time and q spacetime

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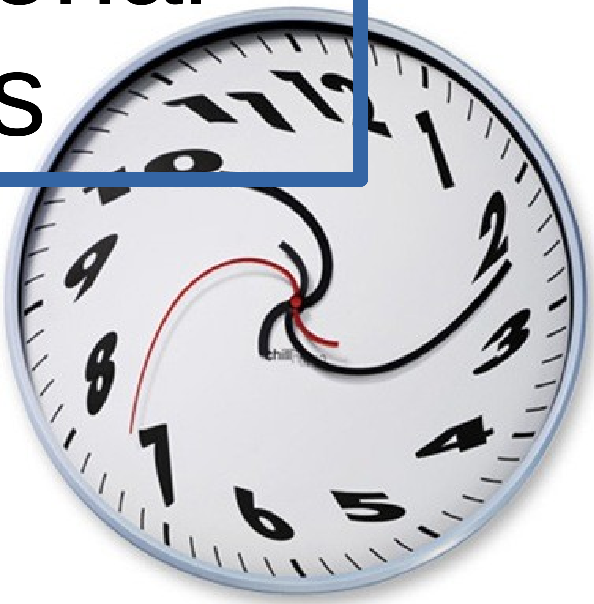
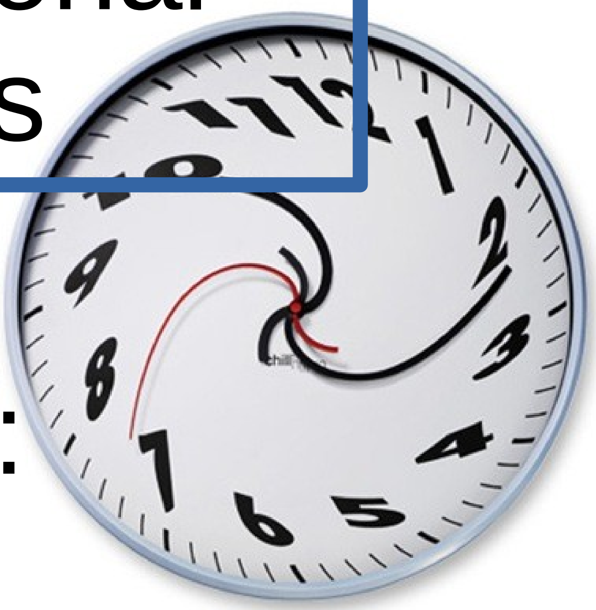


What I'm going to talk about

Time: its strange aspects and how they reflect on quantum mechanics

I'll give a quantum description of time based on conditional probability amplitudes

... and an idea for a relativistic generalization: q spacetime



WHAT is time?

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something that “measures” the distance between events

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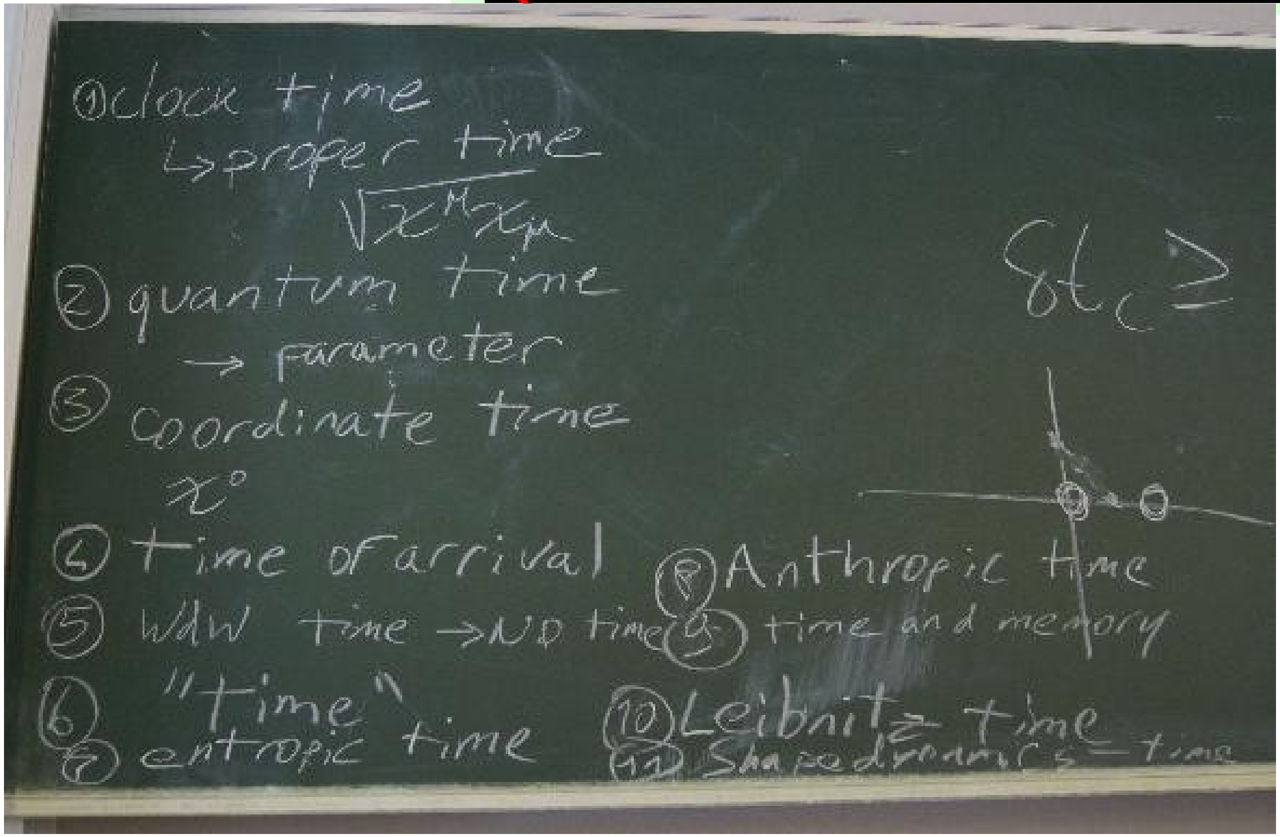
the two **main** meanings of “time” in physics

other meanings?!

Table 2.1: Times.

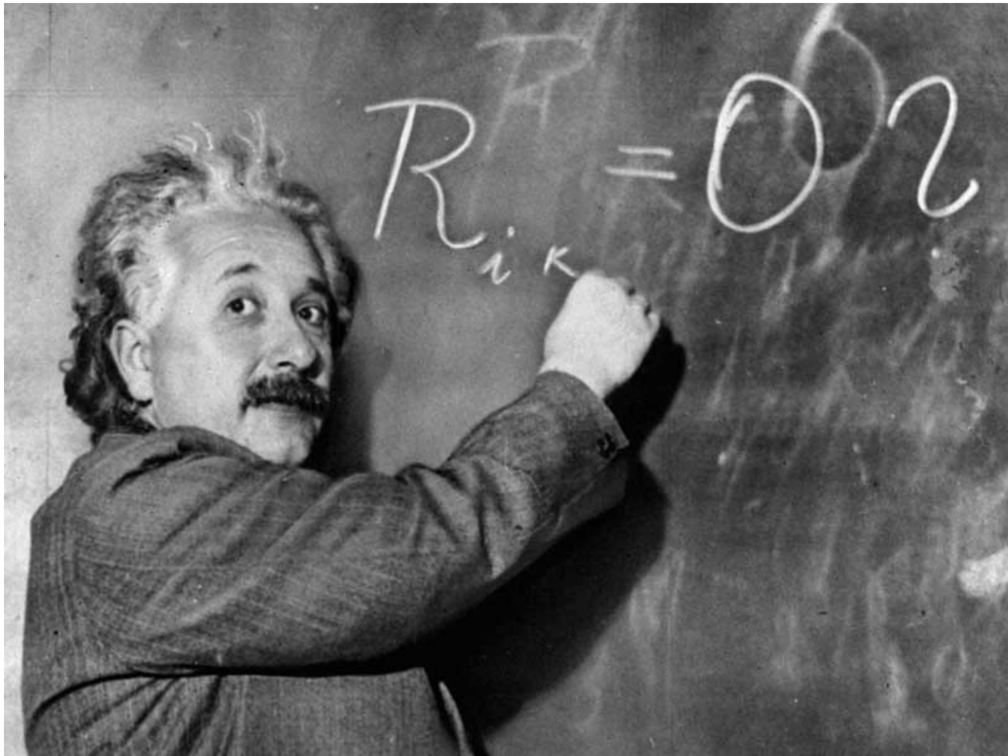
Time notion	Property	Example	Form
Natural language time	memory	brain	?
Time-with-a-present	present	biology	R
Thermodynamical time	direction	thermodynamics	A
Newtonian time	unique	newtonian mechanics	M
Special relativistic time	external	special relativity	M^3
Cosmological time	spatially global	cosmological time	m
Proper time	temporally global	world line proper time	m^∞
Clock time	metric	clocks in GR	c
Parameter time	one dimensional	coordinate time	L^∞
No-time	none	quantum gravity	none



[Rovelli, “quantum gravity”]

The strangest aspect of time

- The present "exists", the past and the future don't



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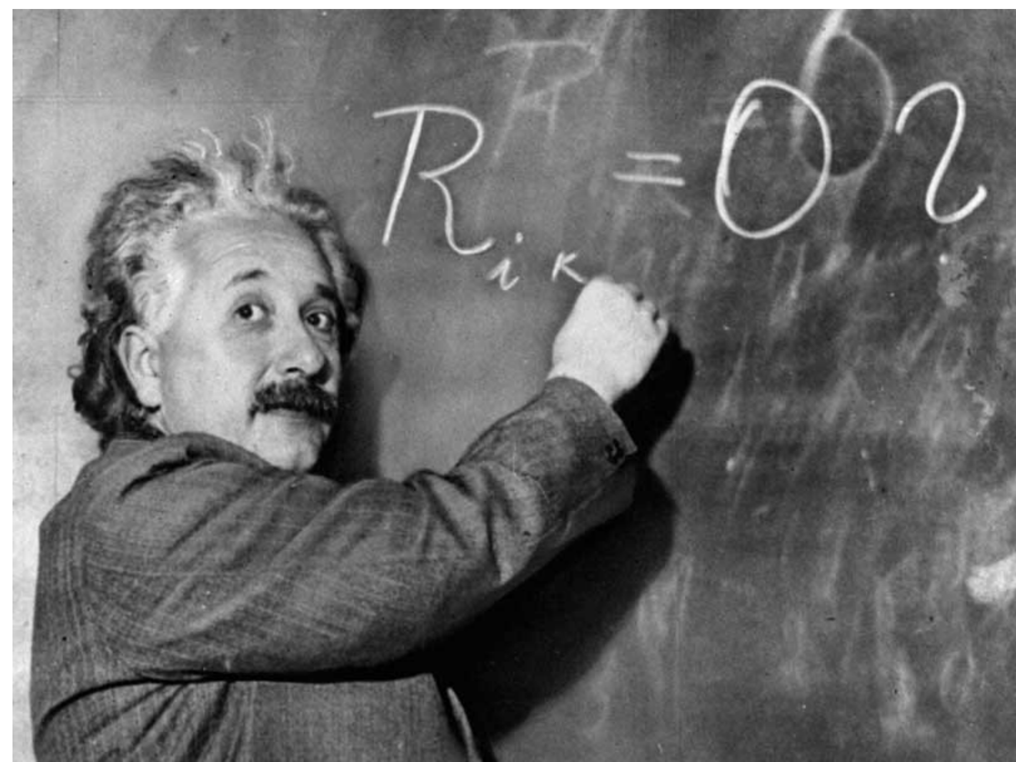

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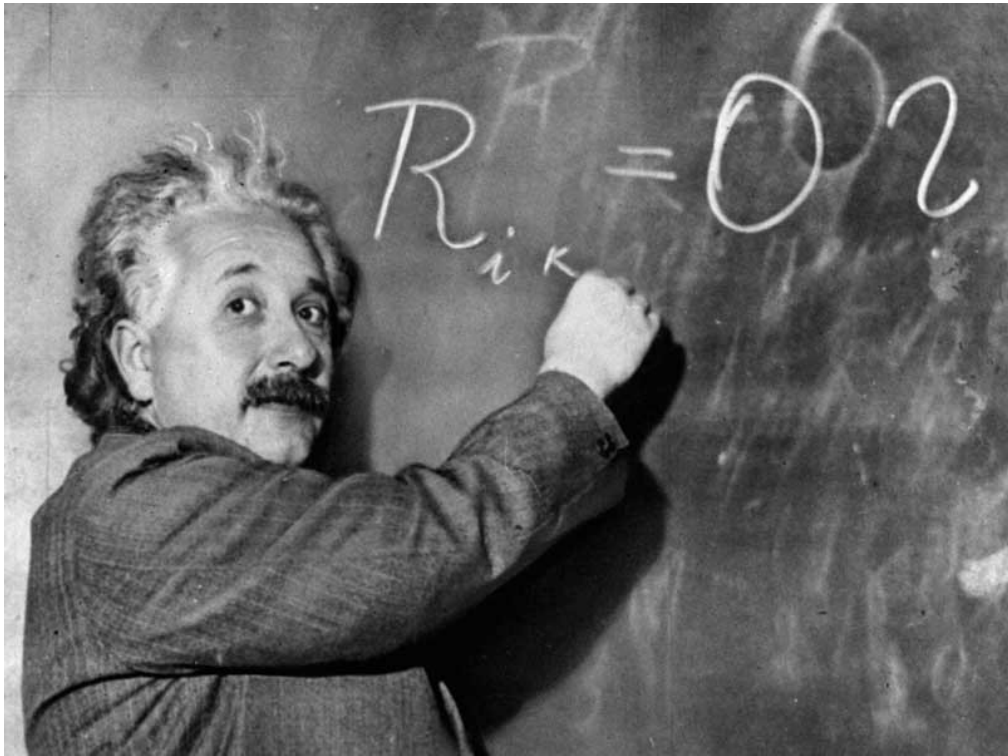

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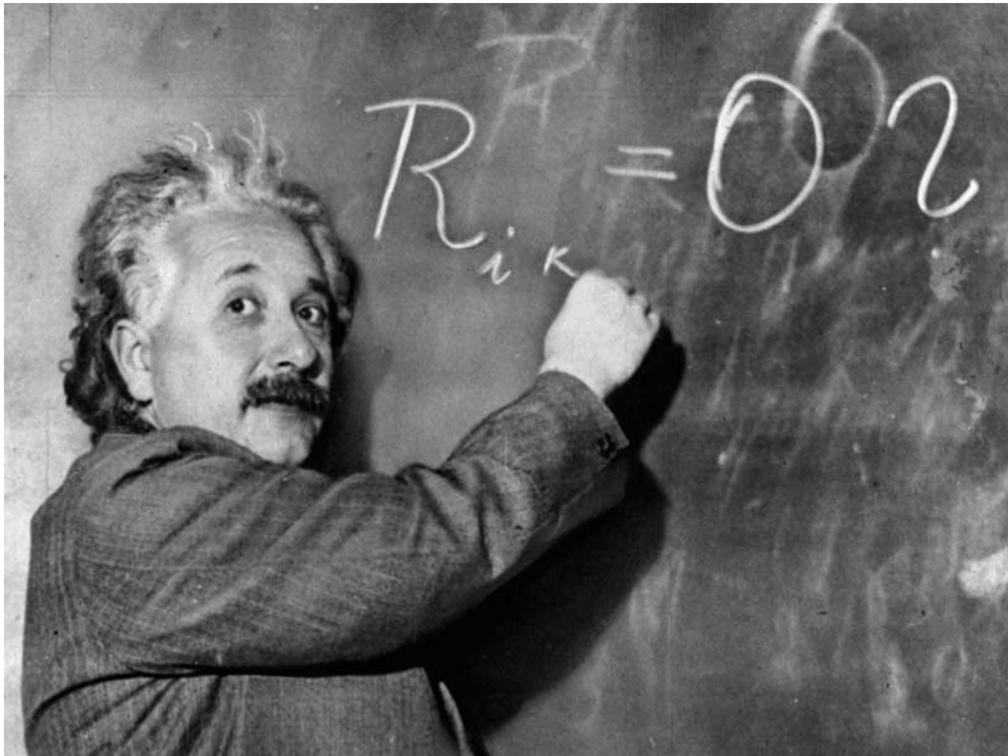
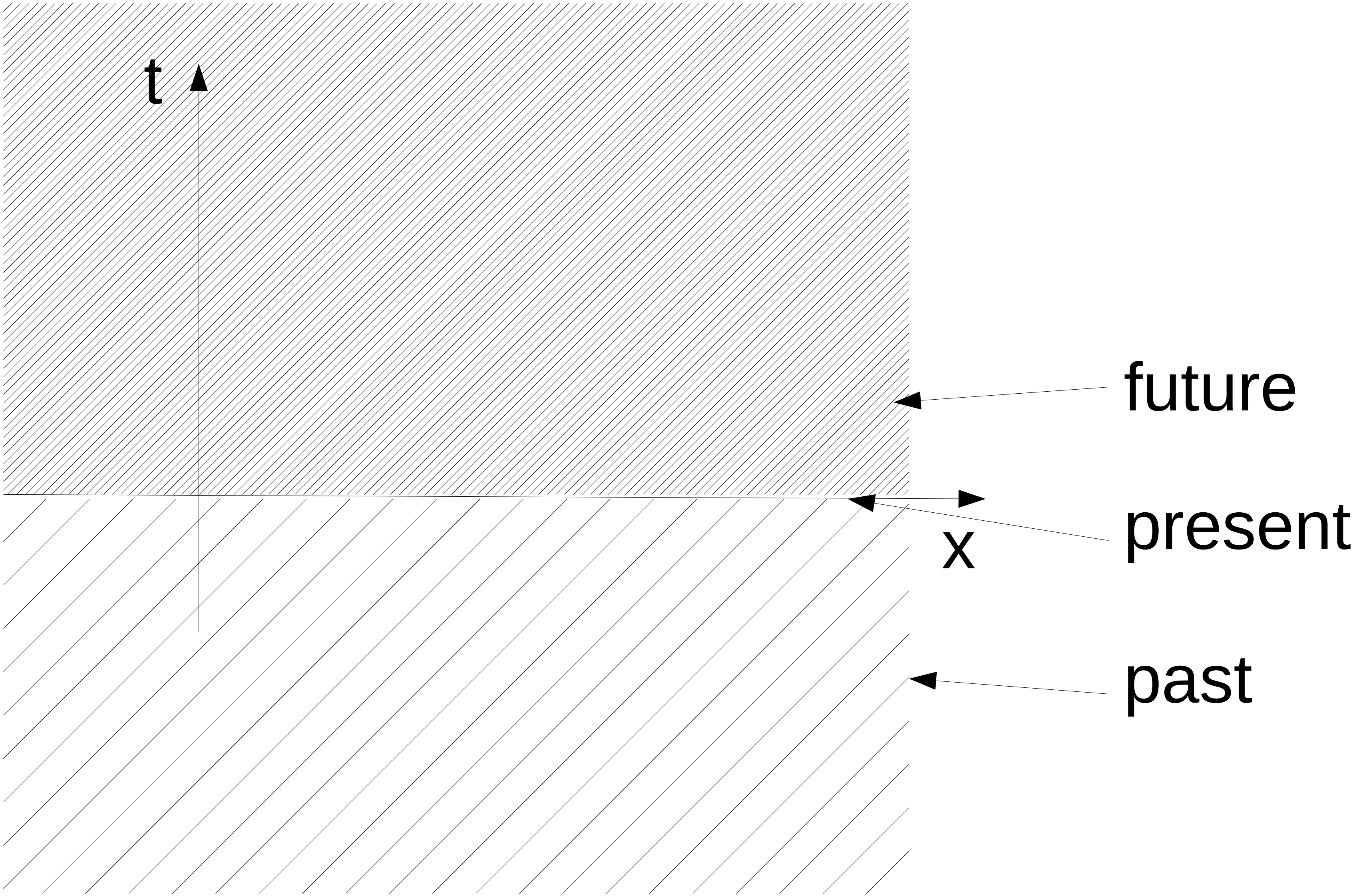
“NO”?!? why?



Relativity of simultaneity

Observers in relative motion divide spacetime in past-present-future in different ways.



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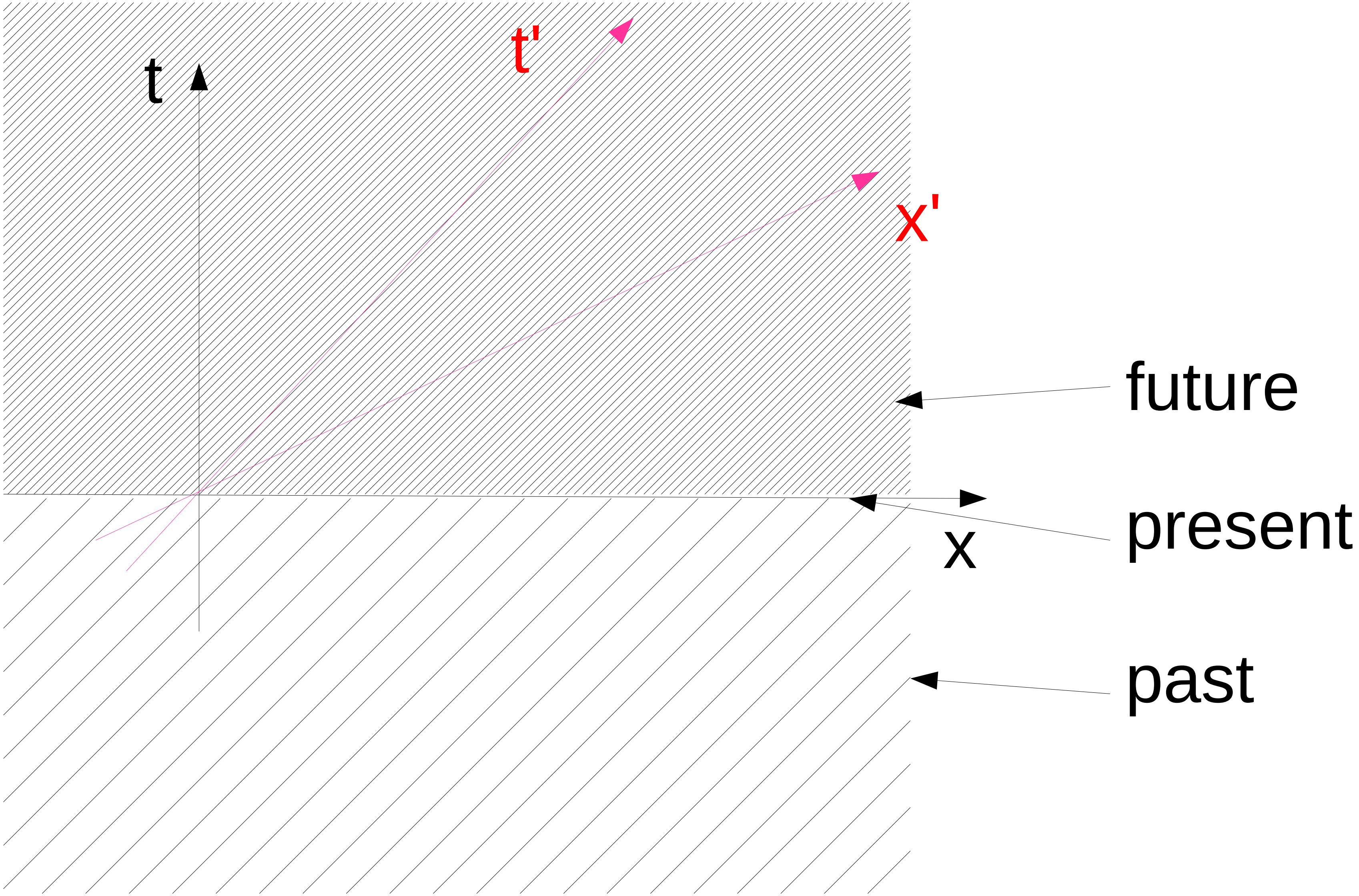

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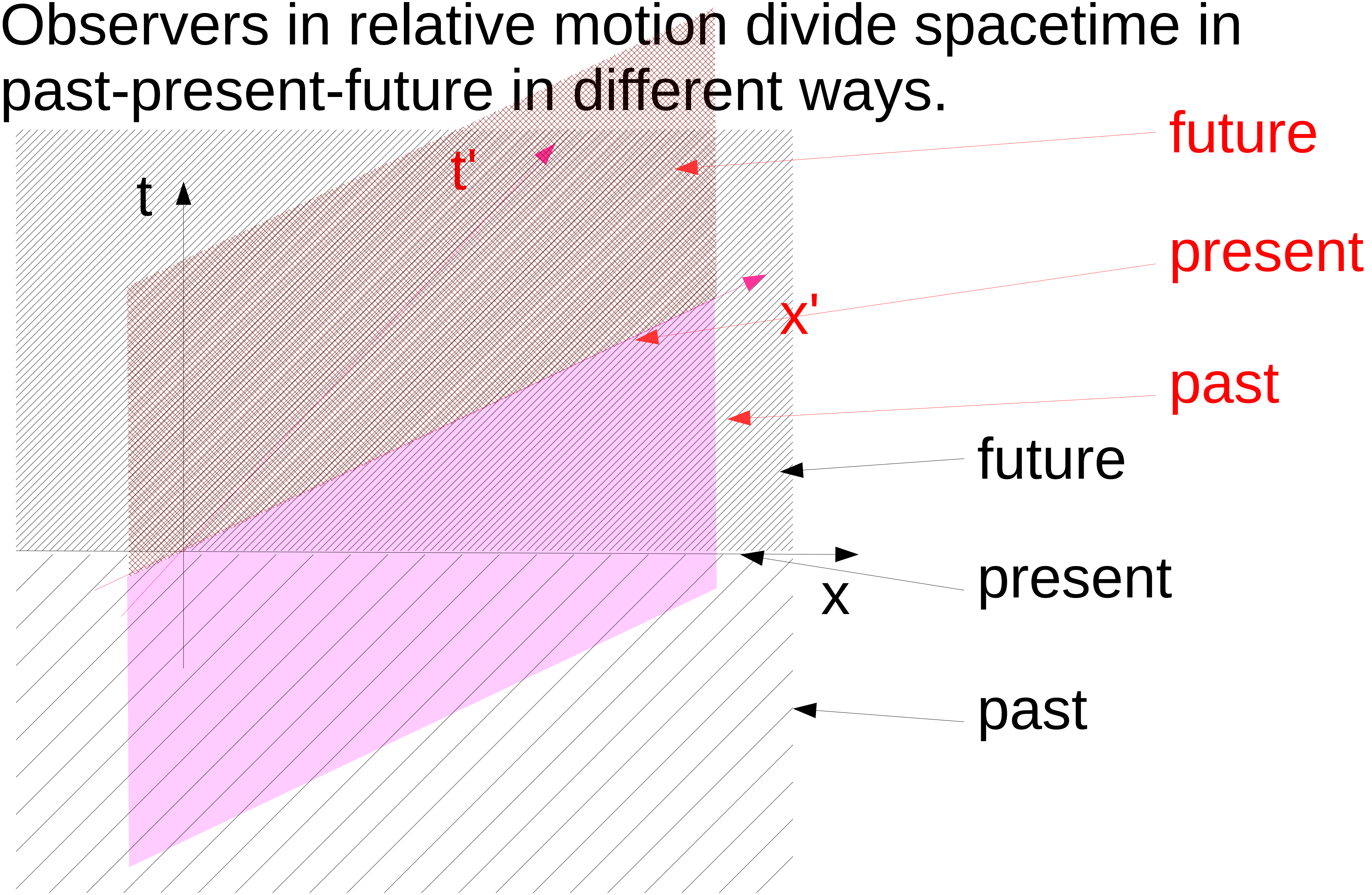

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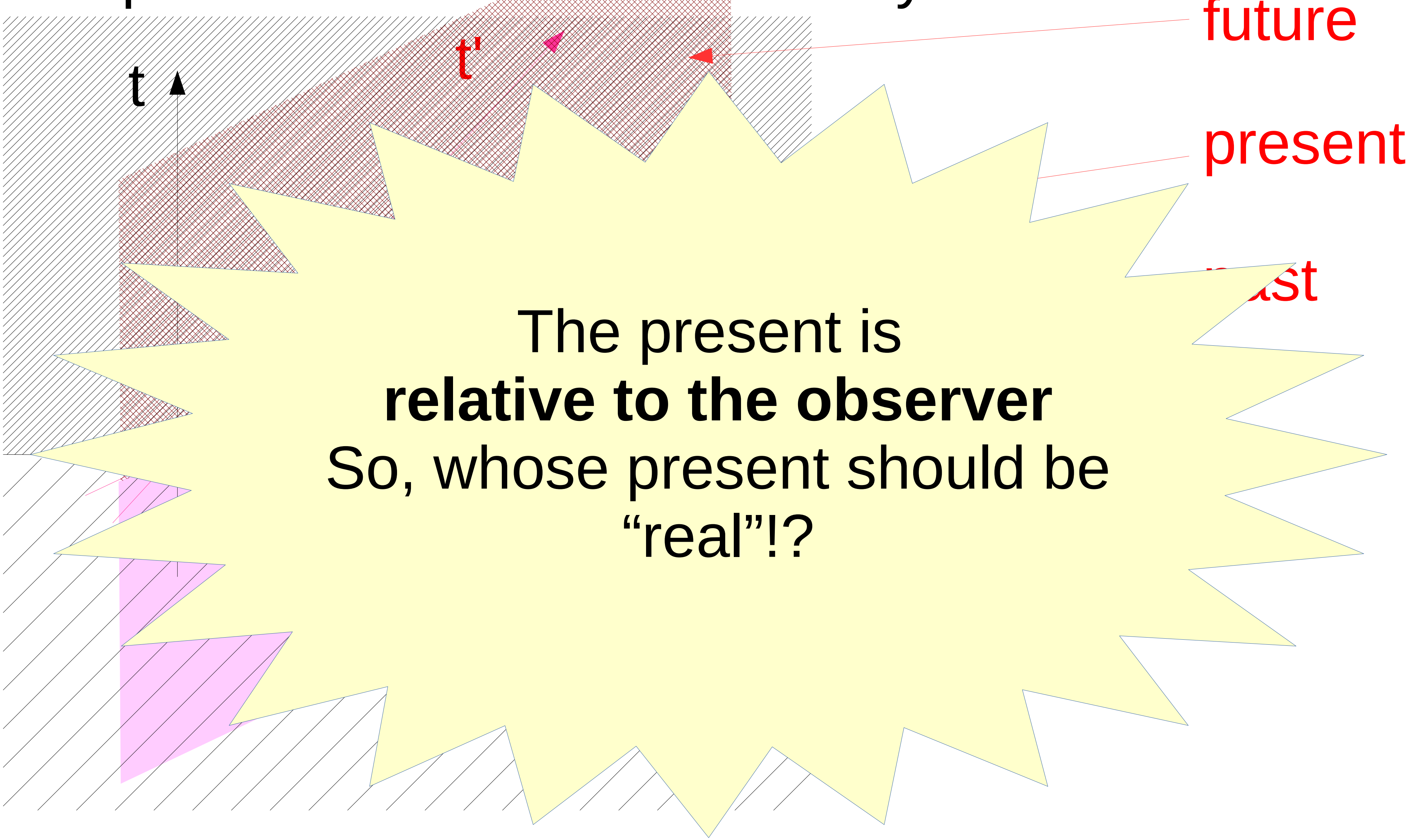
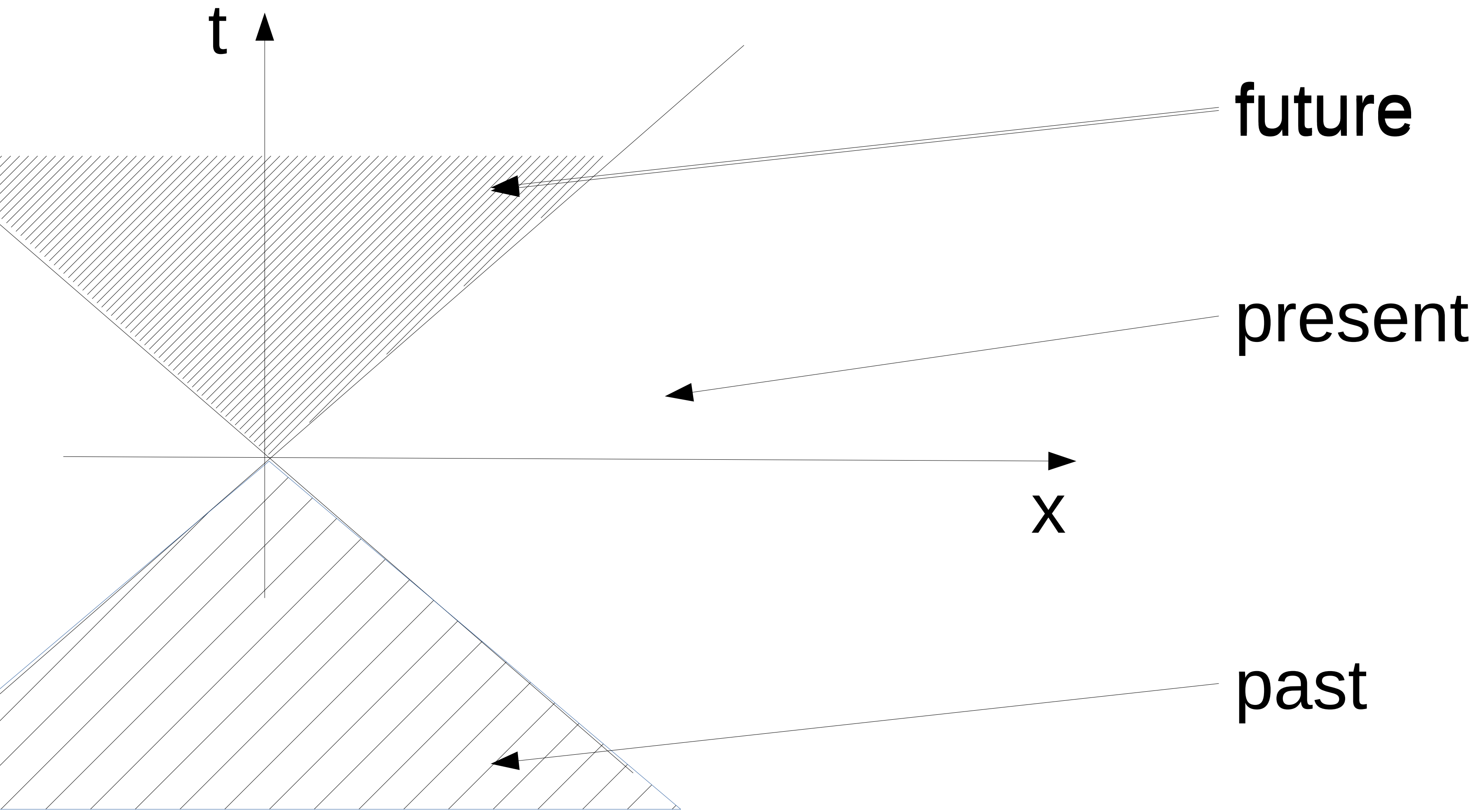


The present is
relative to the observer
So, whose present should be “real”!?

Light cones **are** invariant:
can we use them to define past-future-present?



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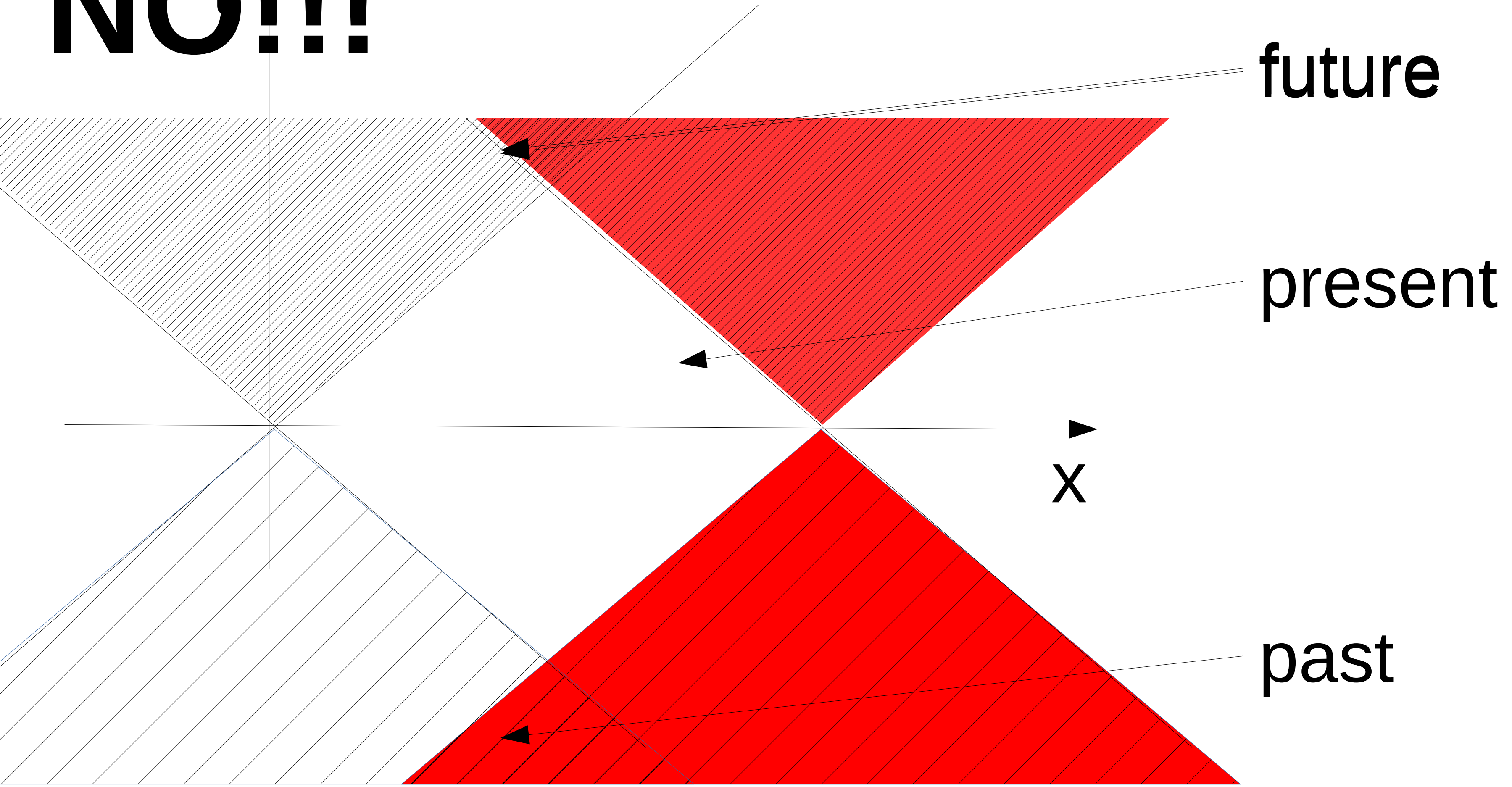
NO!!!



Now even observers in the same reference disagree on what is “real”

Special relativity forces us to give the **same degree of existence to past, present and future**

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(not incompatible with observations, but incompatible with textbook relativity).

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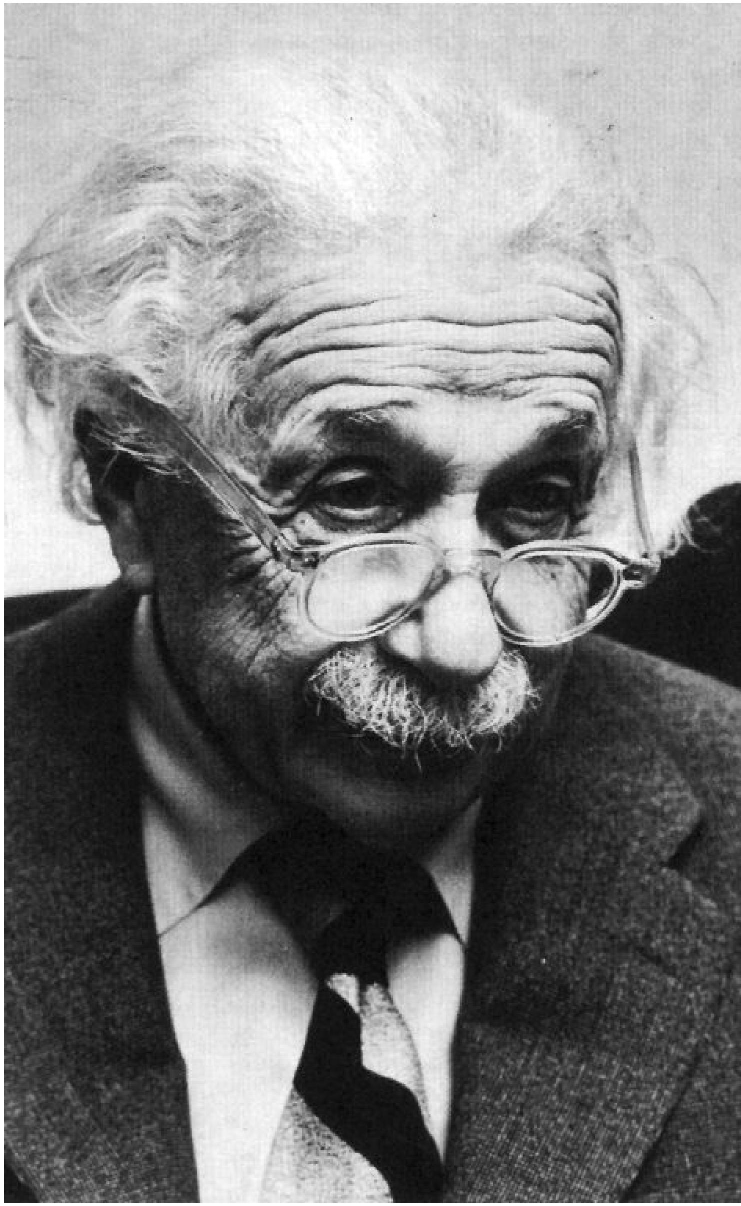
Technically:
Presentism
vs
Eternalism

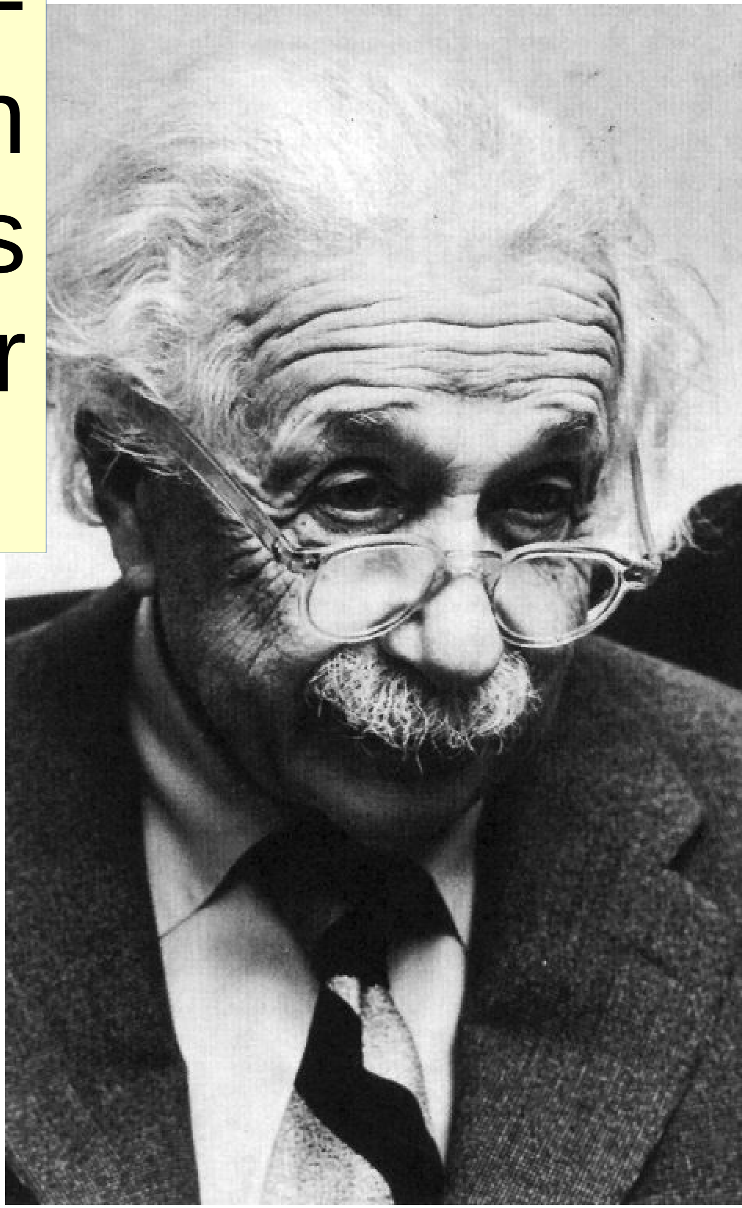
What did Einstein say about this?



What did Einstein say about this?

"For us convinced physicists the distinction between past, present, and future is only an illusion, however persistent."

Albert Einstein, 21 May 1955



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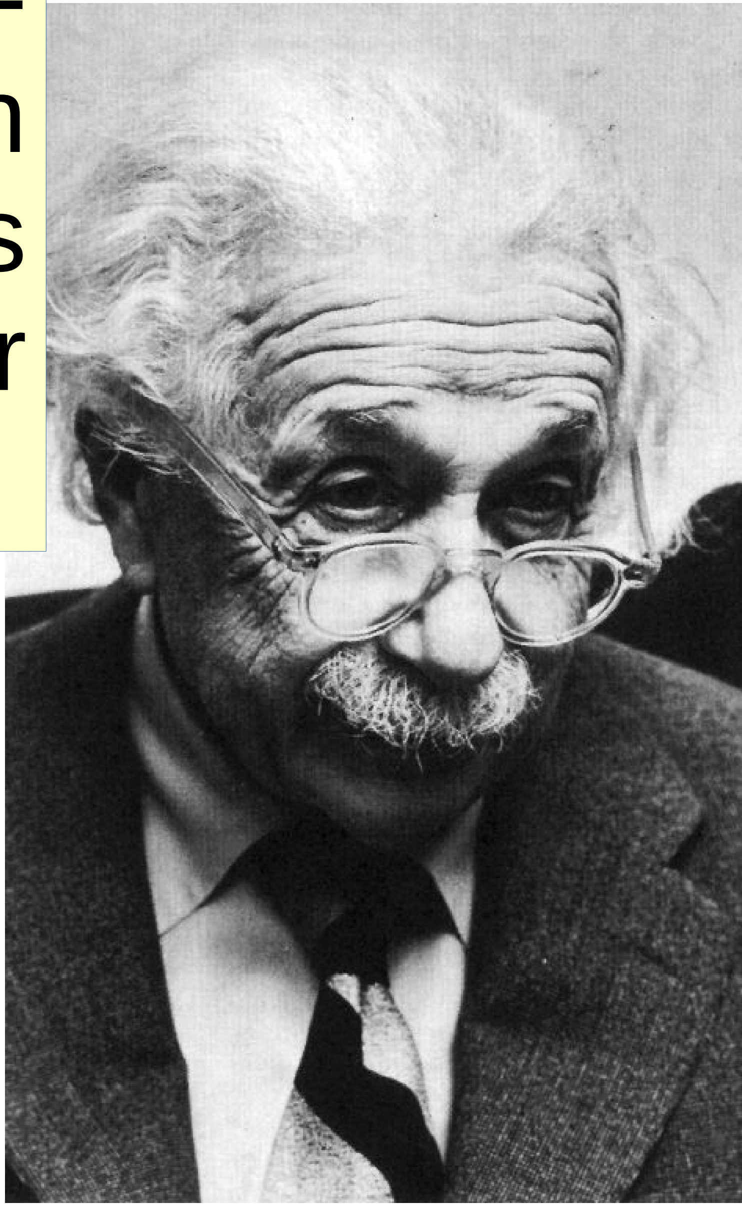

Albert Einstein, 21 May 1955

In a letter to the widow of his dear friend Michele Besso: trying to console her (or himself?) with special relativity.

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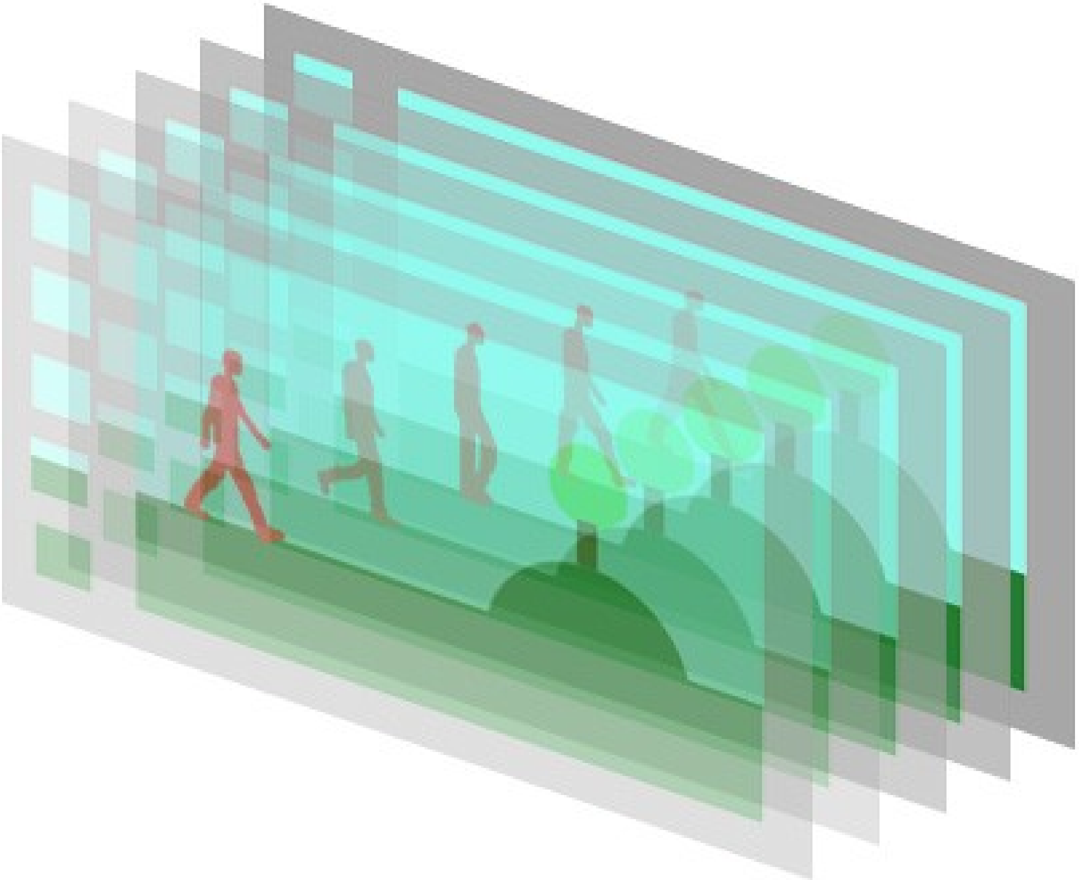
Consequence: Block universe



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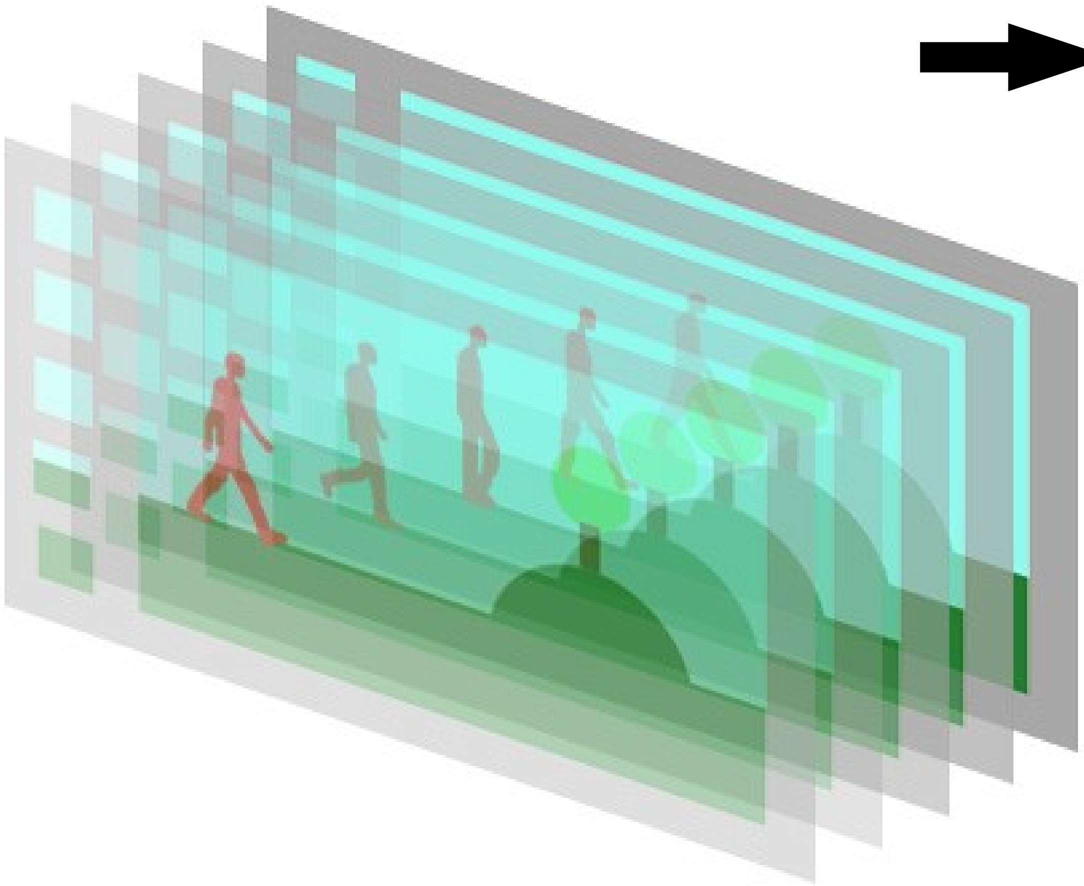

we'd like a quantum description of time that contains the BU

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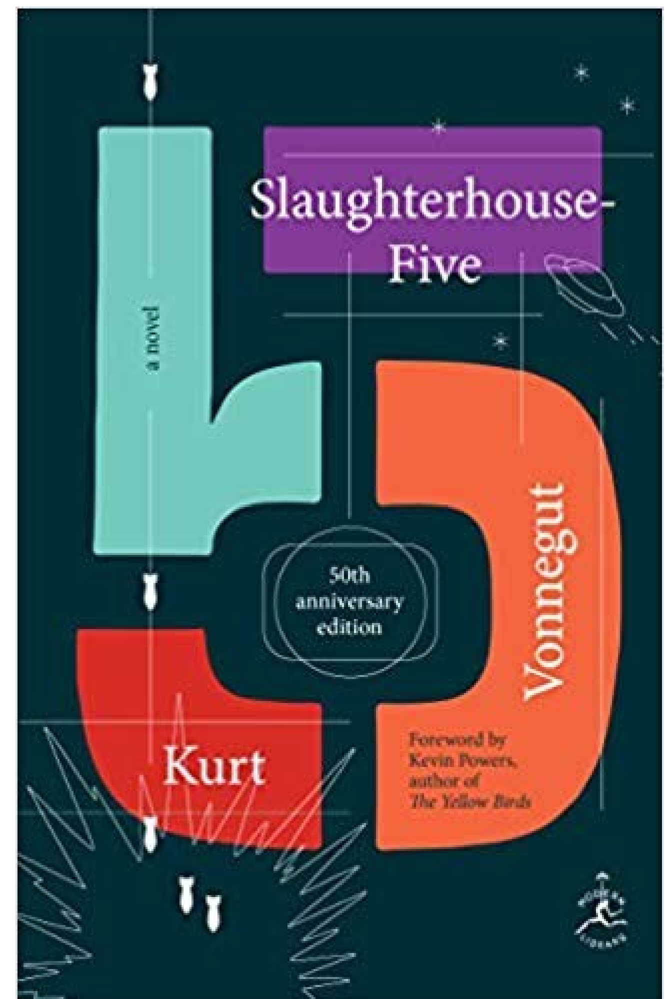
Our intuition fails badly...

"Story of Your Life" by Ted Chiang.



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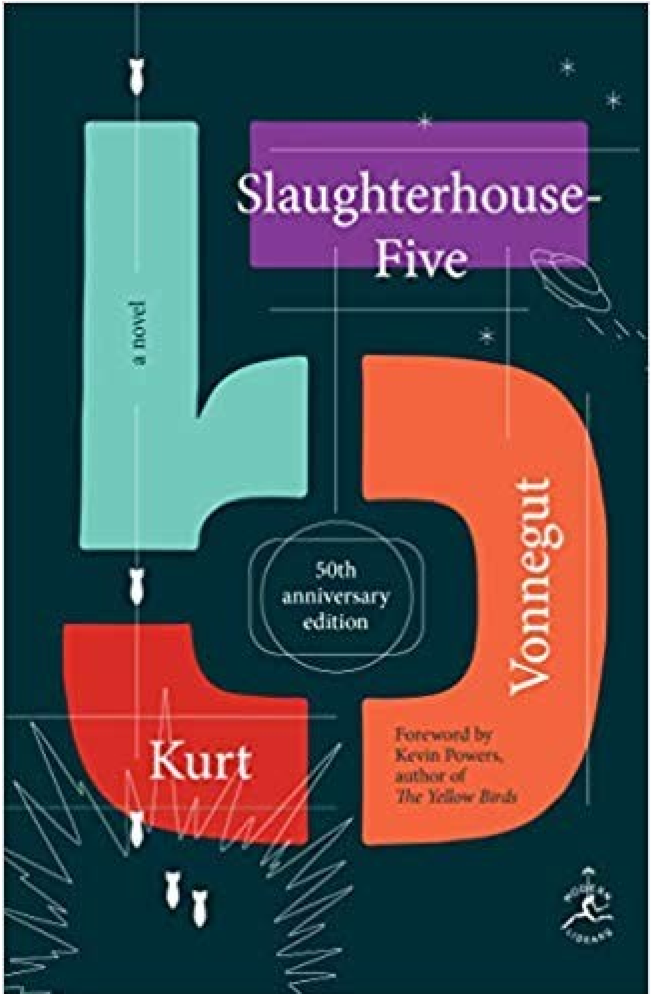
our perception of time is incompatible with relativity:

- we perceive time locally (only the present)
- we perceive space globally (we don't perceive only our own location)



...join GR and QM?!?

Canonical quantization of GR →

Wheeler-De Witt equation:

$$\hat{H}|\Psi\rangle = 0$$

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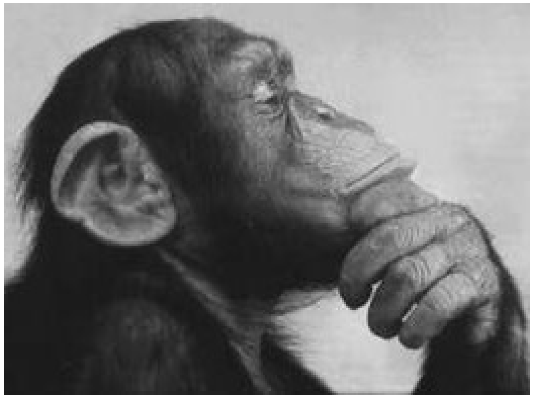

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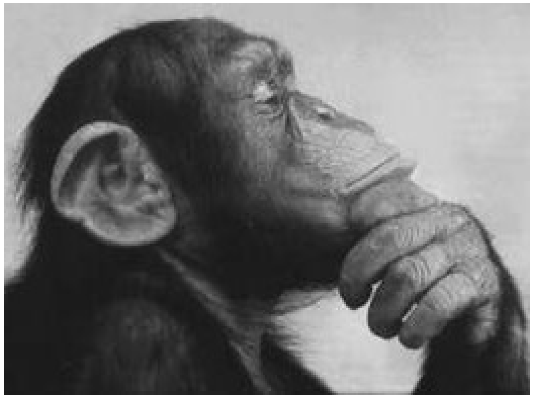

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"Problem of time":
how to interpret this?!

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Wheeler-De Witt equation:

Is W-dW wrong?

Is it right and it
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Take a step back…
Time in non-relativistic QM

Time in quantum mechanics:

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a classical parameter in the Schroedinger eq.

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Time in quantum mechanics?

Inconsistency in the formulation of QM
(inconsistency in one of its postulates)

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Quantum Time

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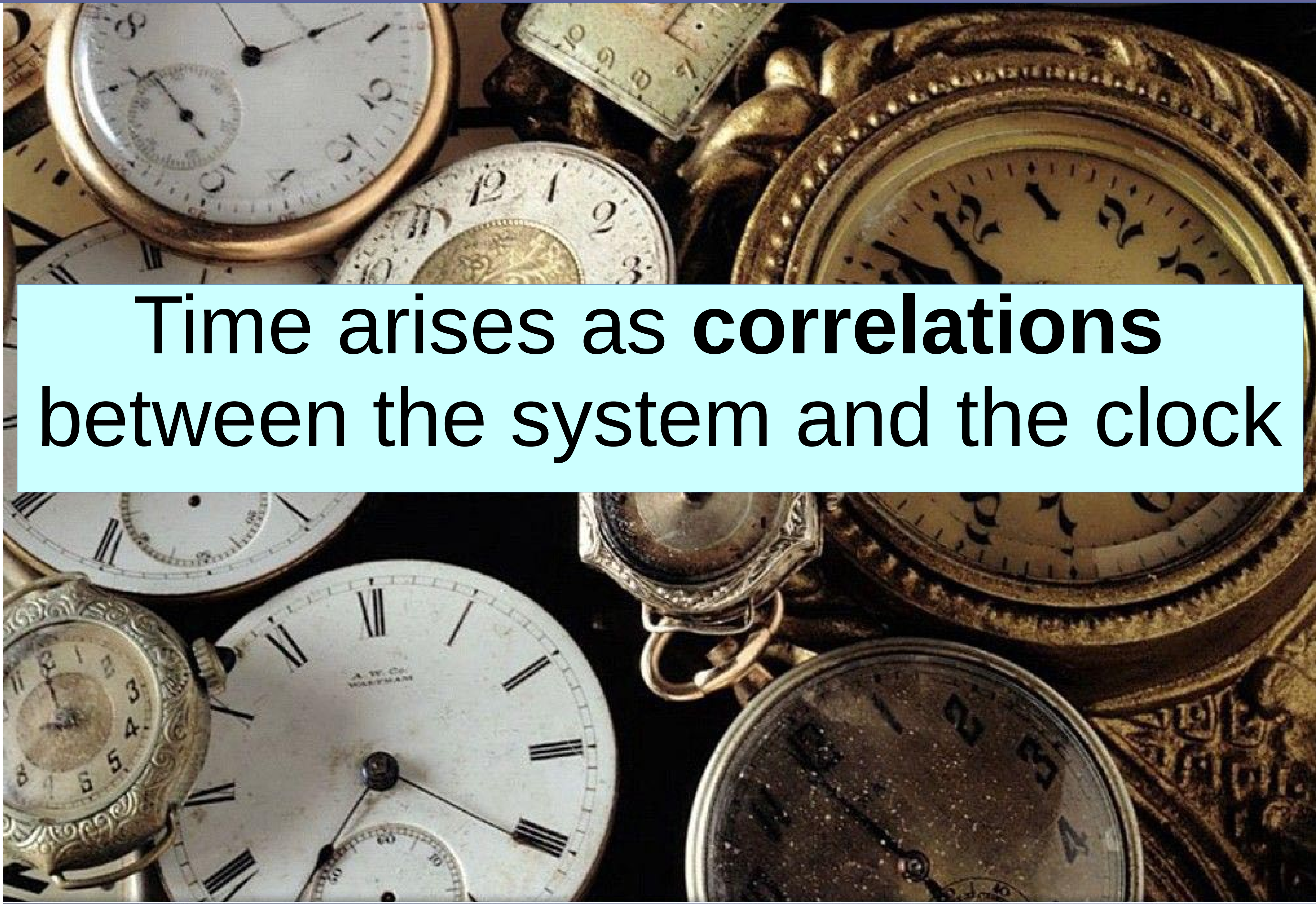
$$\mathcal{H} \equiv \mathcal{L}^2(\mathbb{R})$$

eigenbasis $\{|x\rangle\}$

$|t\rangle$

Time and entanglement



Time arises as **correlations** between the system and the clock

The PWAK mechanism

Page and Wootters [PRD **27**,2885 (1983)]
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clock “momentum”

system Hamiltonian

$$\hat{\mathbb{J}} := \hbar\hat{\Omega} \otimes \mathbb{1}_S + \mathbb{1}_T \otimes \hat{H}_S \; ,$$

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This means that for physical states the *system* Hamiltonian is the generator of *clock* time translations

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position representation:

$$\langle x|\hat{P} = -i\frac{\partial}{\partial x}\langle x|$$

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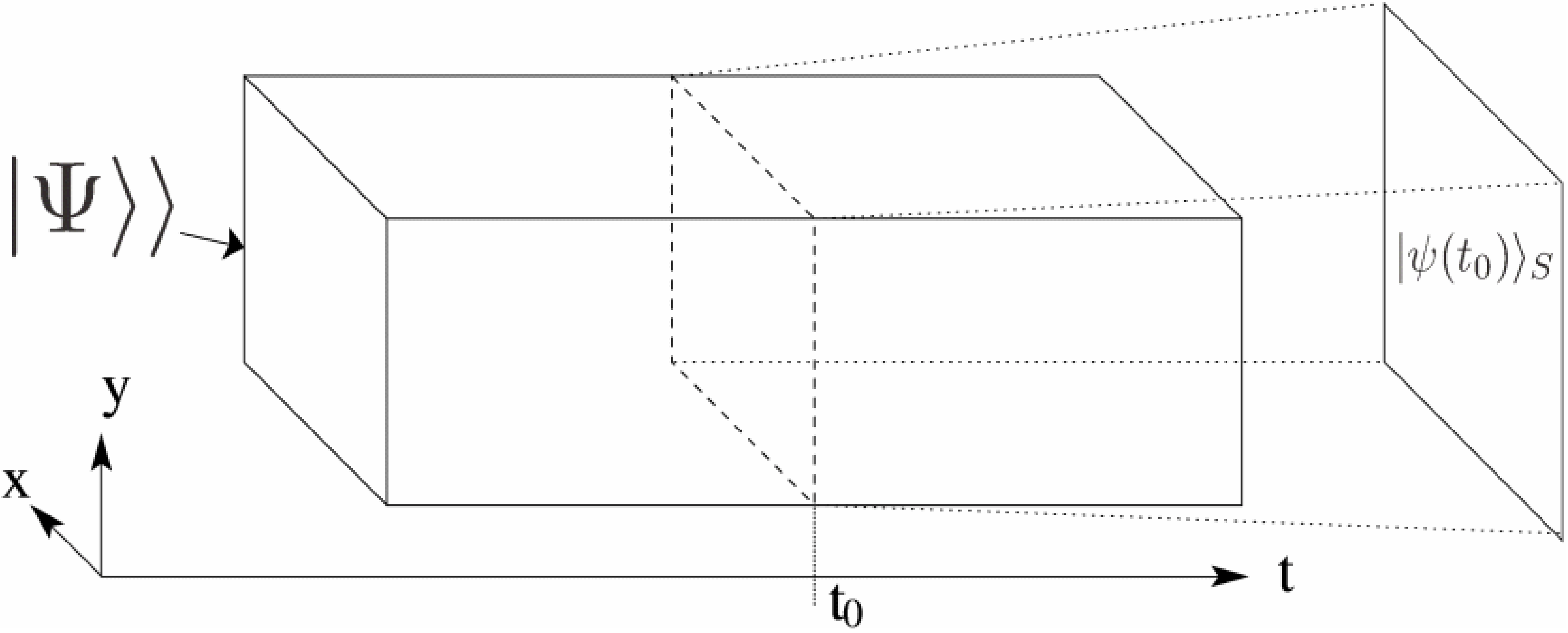
"momentum" representation=time indep. Schr eq.

what I've been saying is that

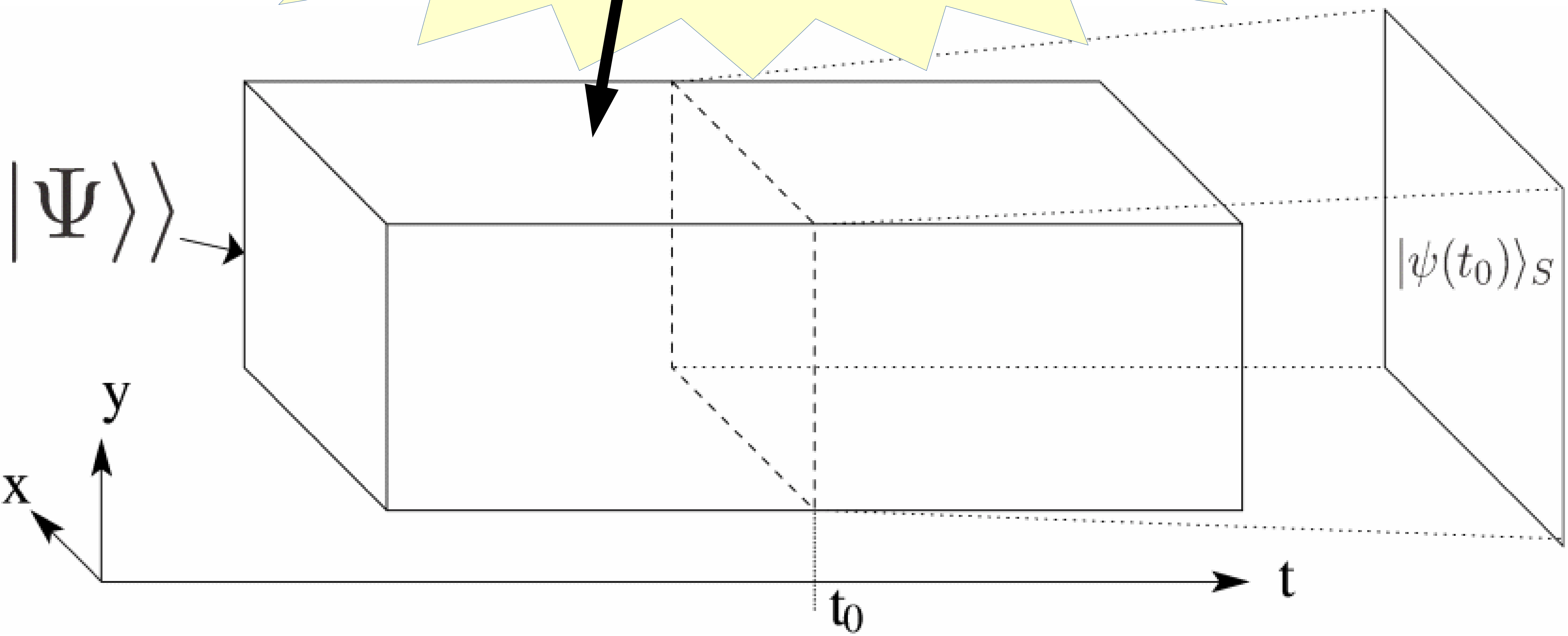
conventional qm arises in this framework through conditioning.



what I've been saying ... tha

co ... this
fra ... ing.

Non relativistic
Block universe picture!!



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All pure solutions to the WdW eq. $\hat{\mathbb{J}}|\Psi\rangle\rangle = 0$

are of the form:

$$|\Psi\rangle\rangle = \int dt\ |t\rangle_T \otimes |\psi(t)\rangle_S$$

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are of th

which mean ... e

system at time t ... $|\phi$ ...

Remember this!
Relation QM-Relativity

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time is here a **quantum degree of freedom** (with its Hilbert space) and can be entangled

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this does **not** necessarily imply that time is **discrete!!**

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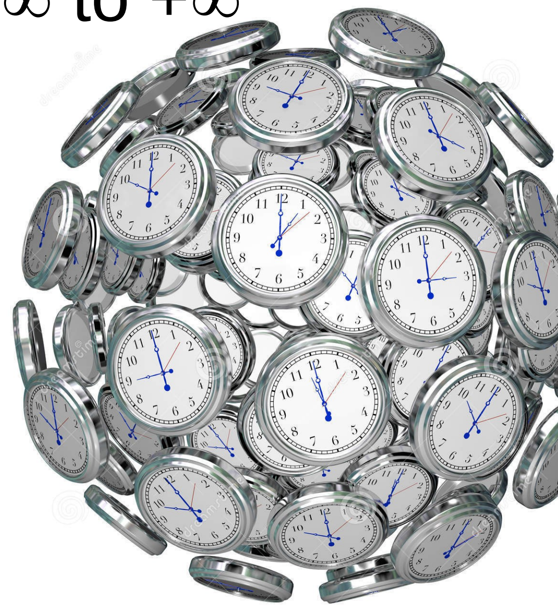

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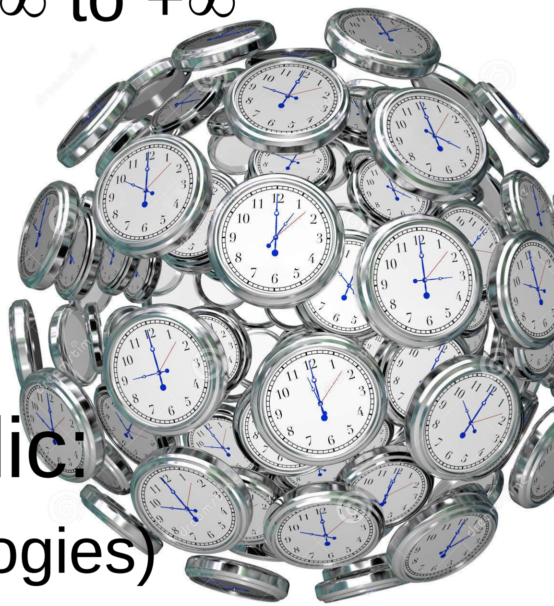

if the clock has finite energy, time is cyclic: e.g. a spin (appropriate for certain closed cosmologies)

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BUT, a physical interpretation of the time Hilbert space is **un-necessary**

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alternative:

It can be seen as an **abstract purification space**

Is entanglement important?
Could we do with classical correlations?

$$|\Psi\rangle\rangle = \int dt\ |t\rangle_T \otimes |\psi(t)\rangle_S$$

$$= \int d\mu(\omega)\ |\omega\rangle_T \otimes |\psi(\omega)\rangle_S\ ,$$

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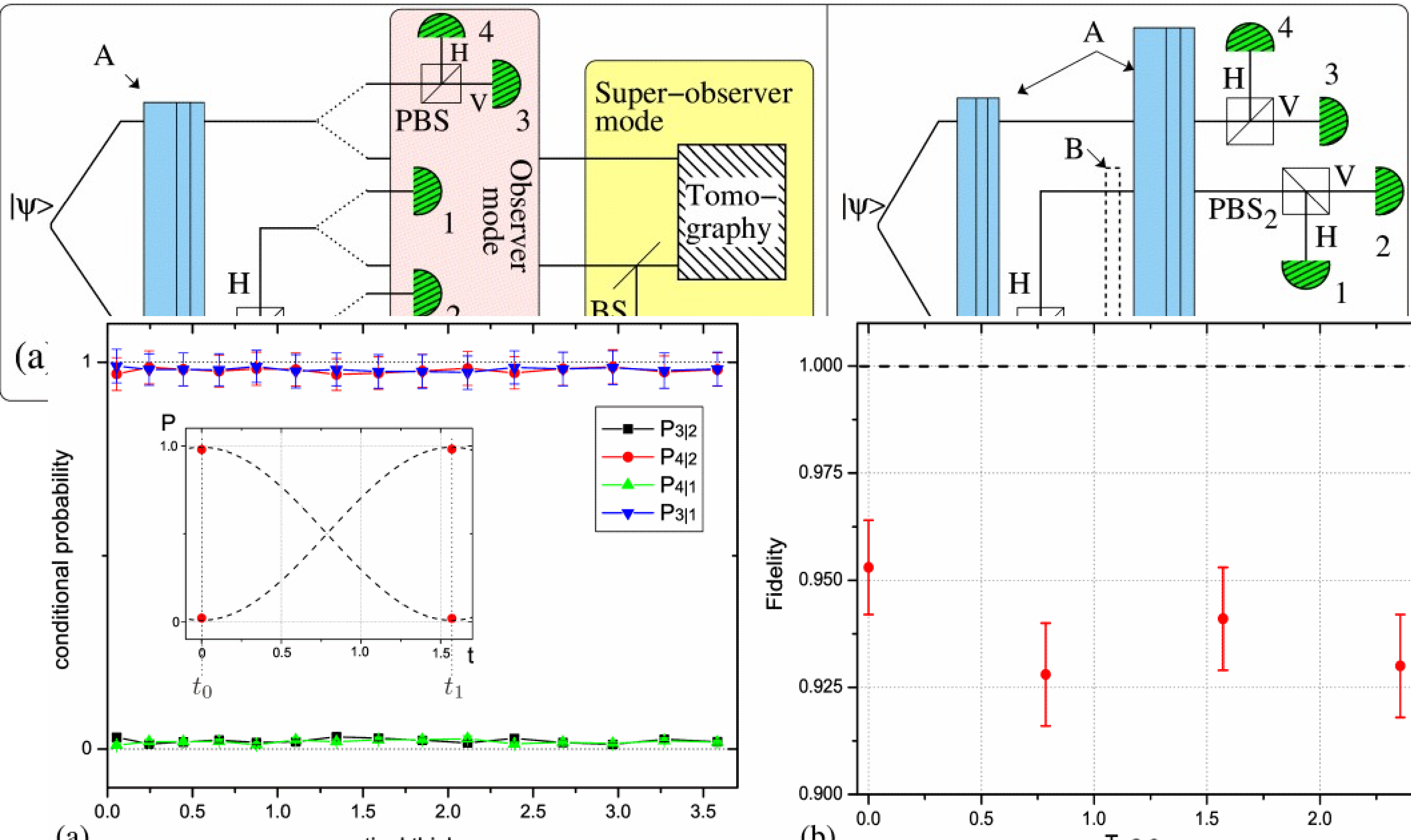
$$\int dt|t\rangle|t\rangle|\psi(t)\rangle,\ \int dt|t\rangle|t\rangle|t\rangle|\psi(t)\rangle, \cdots$$

Experimental illustration
(collaboration with the INRIM group)



Our contribution

These ideas were basically abandoned in the 80s: because of objections (Kuchar, Unruh, etc.)

We removed these objections

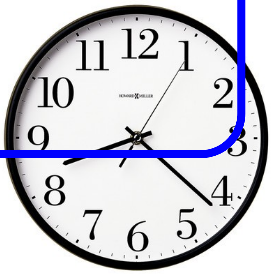
... and also perfected the model

(e.g. role of entanglement, momentum representation)

Criticisms to time quantizations

The Pauli argument

Pauli: “time **cannot be quantized**, because a time operator that is a generator of energy translations implies that the energy is un-bounded (also from below)”



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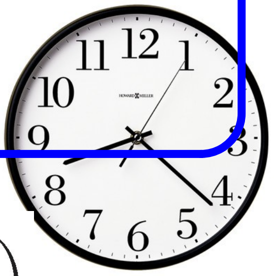

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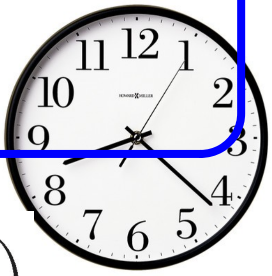

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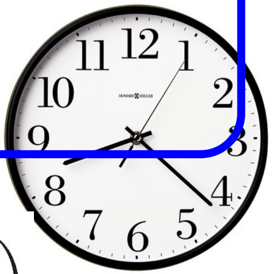

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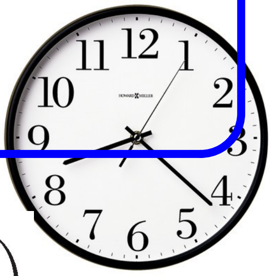

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In other words, the **Pauli argument fails** in our case because the energy-time connection is not enforced dynamically as

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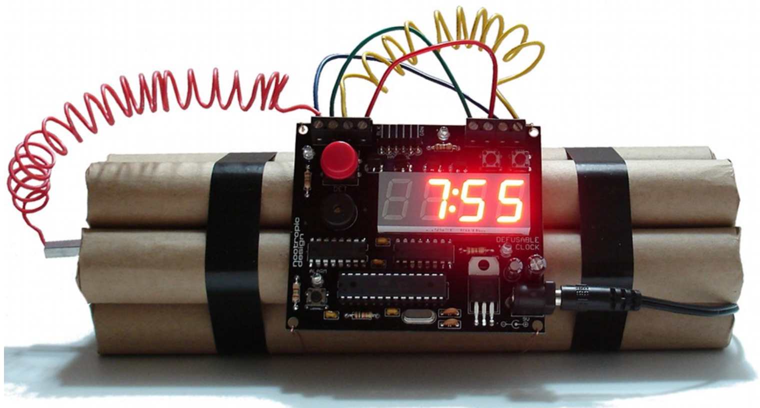

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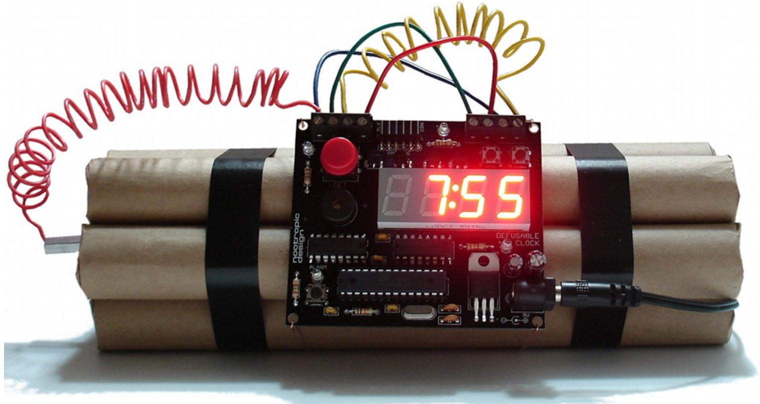

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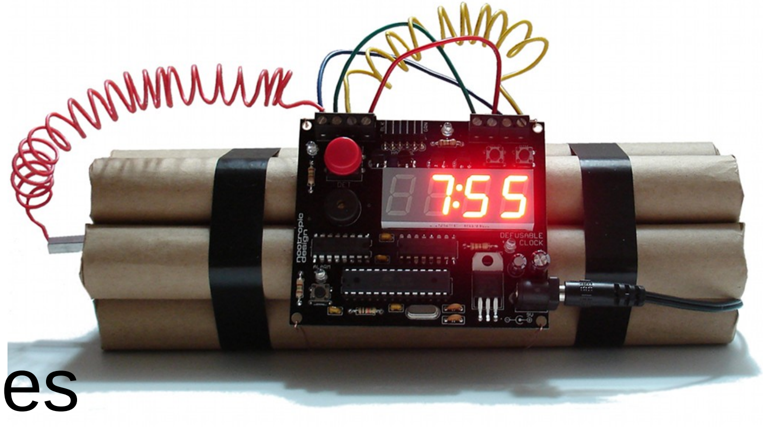

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- in conventional qm, time is not a dynamical variable $\Rightarrow$ no problem.

- in our case, time *is* a dynamical variable, but its translations are NOT generated by $\hat{H}_S$ (but by $\hat{\Omega}$)

The Kuchar argument against PaW

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$$|\Psi\rangle\rangle = \int dt\ |t\rangle_T \otimes |\psi(t)\rangle_S$$

↓ time t

$$|\psi(t)\rangle$$

after a measurement of time, the state collapses to $|\psi(t)\rangle$: successive measurements give wrong statistics: **no more evolution**

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The second measurement is a joint measurement on the system and on the d.o.f. that stored the outcome of the first.

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Measurement of observable with eigenstates $|a\rangle$ at t_0:

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$$\Rightarrow \boxed{p(a|t_0) = |\langle a|\psi(t_0)\rangle|^2 = \| {}_m\langle a|_T \langle t_0|\Psi\rangle\rangle \|^2}$$

$$= |\psi_a(t_0)|^2$$

(Born's rule)

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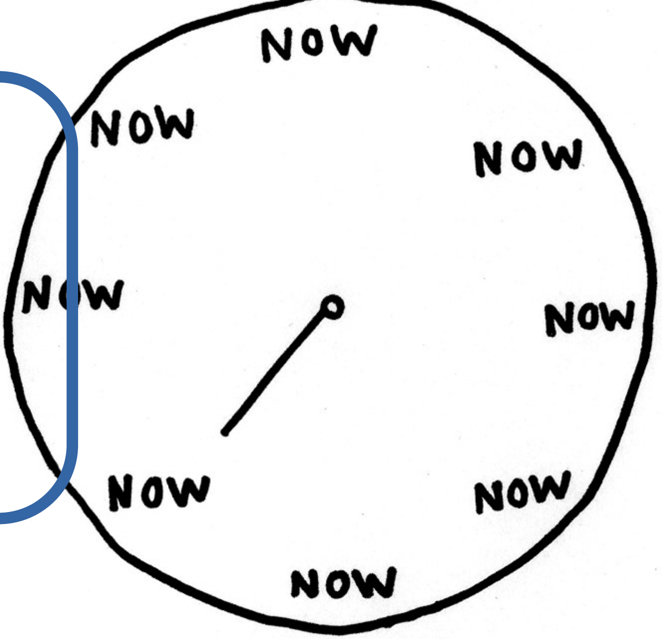
The expected outcome!!

(Born's rule)

⇒ Kuchar's objection is defeated!



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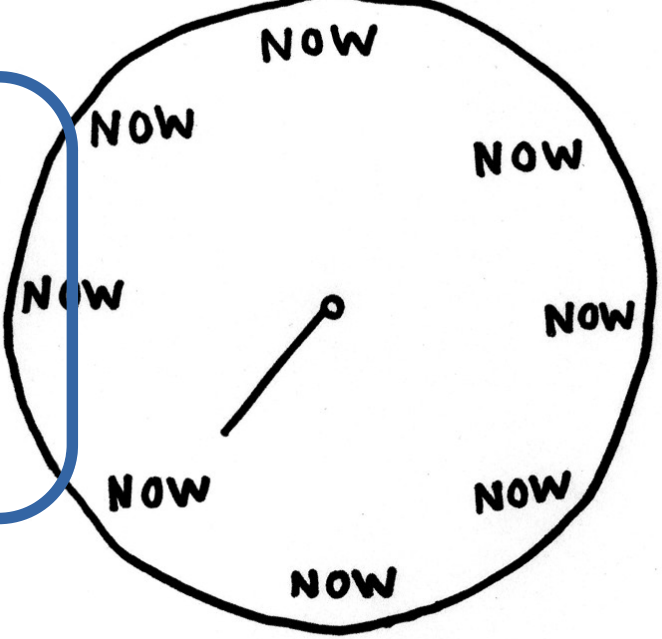

this argument can be extended to POVMs, propagators, etc...

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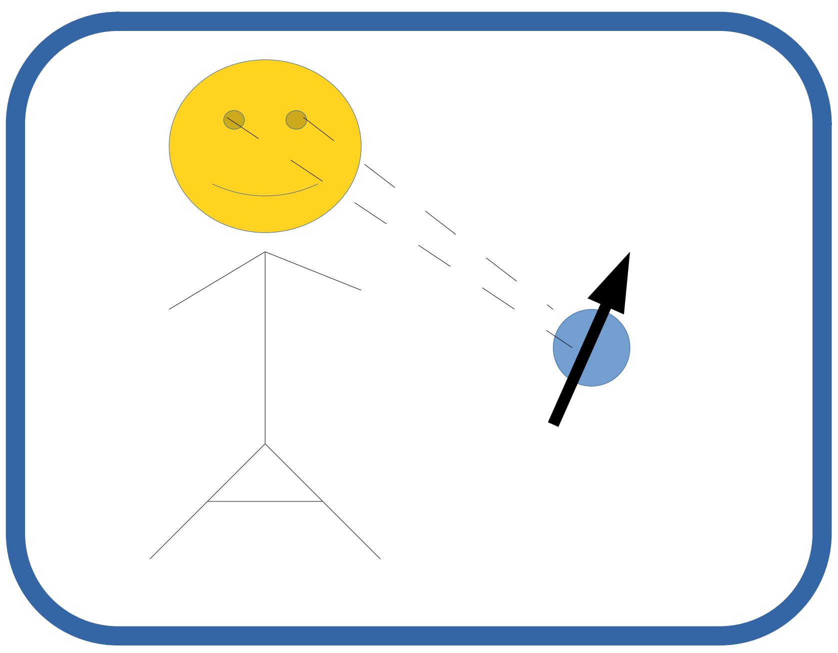

Alice's lab

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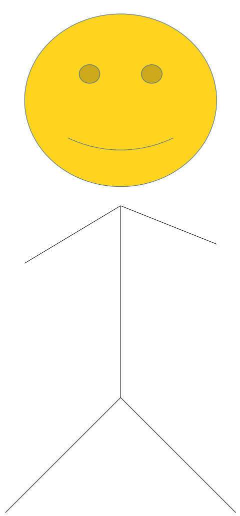
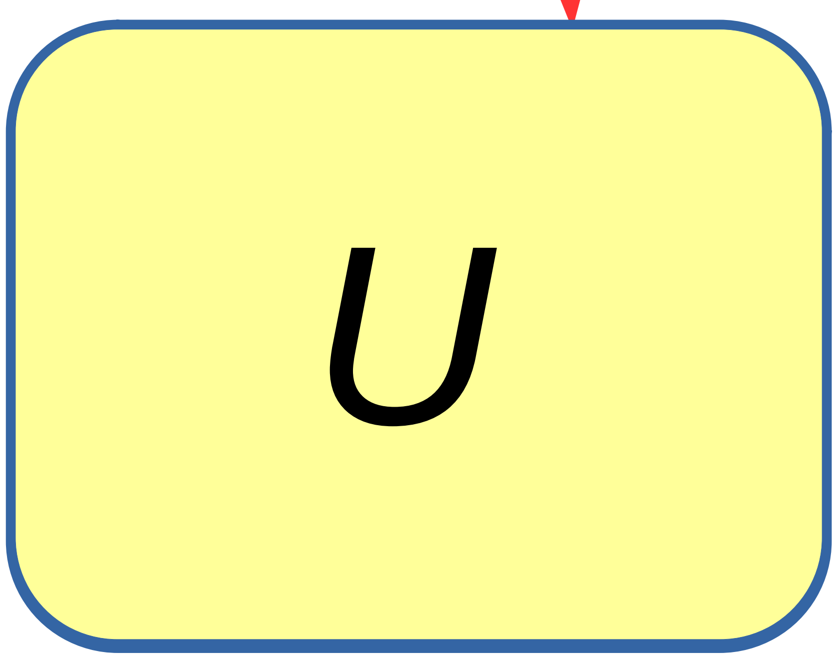


Bob's point of view



Alice's lab

Different "times":

- Quantum time
- Time of arrival (quantum mechanics)
- Proper time (or clock time)
- Coordinate time
- Entropic time (arrow of time)
- WdW (no time)
- Conscience time
- "Time" (??)
- ...

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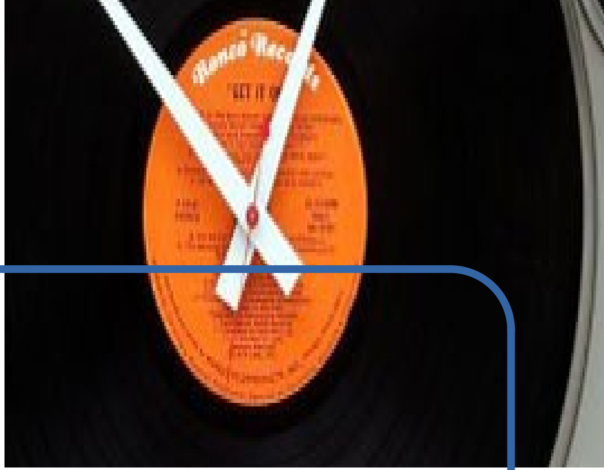

QM

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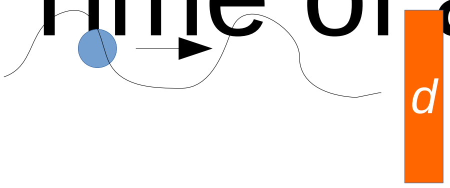
YES!

(joint work with Krzysztof Sacha)

Time of arrival



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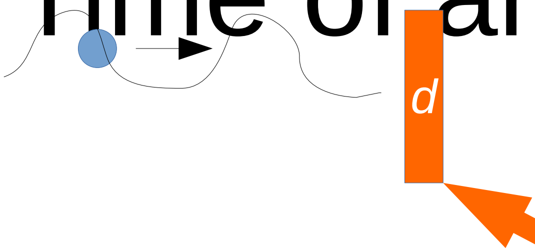


particle's spatial degrees of freedom

clock

$$|\Psi\rangle = \frac{1}{\sqrt{T}} \int_T dt\, |t\rangle |\psi(t)\rangle$$

Projective POVM:

$$\forall t : \Pi_t \equiv |t\rangle\langle t| \otimes P_d \; ; \quad \Pi_{na} = \mathbb{1} - \int dt\, \Pi_t$$

Outcome *t*=”particle is at position *d* at time *t*”, outcome *na*=”not arrived”

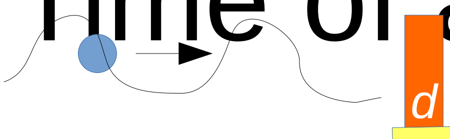
$\Downarrow$

Time of arrival quantum observable:

$$A = \int dt\, t\, |t\rangle\langle t| \otimes P_d + \mathbb{1}_c \otimes \lambda \int_{x \notin D} dx\, |x\rangle\langle x|$$

A joint observable for clock $\otimes$ particle

Time of arrival



particle's spatial degrees of freedom

clock

$$\int \ldots |\psi(t)\rangle$$

Projec…

$\forall t$: … $\mathbb{1}_t$

Outcome t– … not arrived”

A property ***of the CLOCK***!!
(not *of the particle*, as in
~~most~~ competing proposals)
all?

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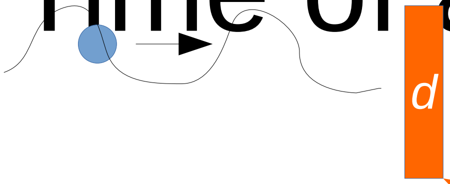

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$$\forall t : \Pi_t \equiv |t\rangle\langle t| \otimes P_d \,; \qquad \Pi_{na} = \mathbb{1} - \int dt\, \Pi_t$$

Outcome *t*=”particle is at position *d* at time *t*”, outcome *na*=”not arrived”

$\Downarrow$

$$p(t, x=d) = \mathrm{Tr}\big[|\Psi\rangle\langle\Psi|\, \Pi_t\big] = \frac{1}{T} |\psi(d|t)|^2 \,,$$

Born's rule

with $\psi(x|t) \equiv \langle x|\psi(t)\rangle$

$\Downarrow$

$$p(t|x=d) = \frac{p(t, x=d)}{p(x)} = \frac{|\psi(d|t)|^2}{\int_T dt\, |\psi(d|t)|^2} \,,$$

Bayes rule

Time of arrival prob. distribution

Time of arrival

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- build a joint observable from this
- from the joint probability of clock+particle, get the time of arrival prob through the Bayes rule.

Only “time of arrival”?

Only “time of arrival”? → NO!

Extensions to other time measurements:

a **generic** time measurement is

“At what time did the event *E* happen?”

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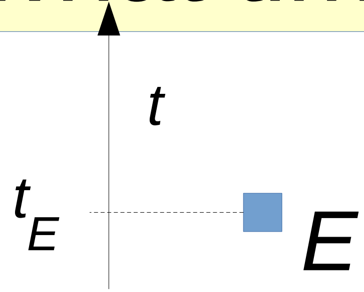

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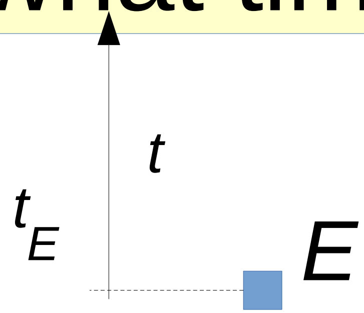

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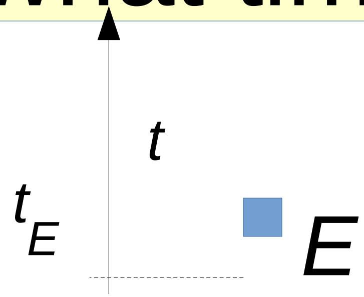

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Use the same trick: a joint projector on the clock and on the system (the projector on the system referring to the event *E*)

e.g. “at what time is the spin up?” The projector is $|\uparrow\rangle\langle\uparrow|$

All usual manipulations →

for observables can be done:

- Expectation values
- Probability distributions
- Eigenstates, eigenvalues, etc.

Advantages with respect to previous proposals:

- Describe situations that prev prop could not (multiple pass, stationary particle, etc.)
- Extension to arbitrary events
- Possibility of describing multiple clocks
- Testable differences: experiment!

Up to now: nonrelativistic QM

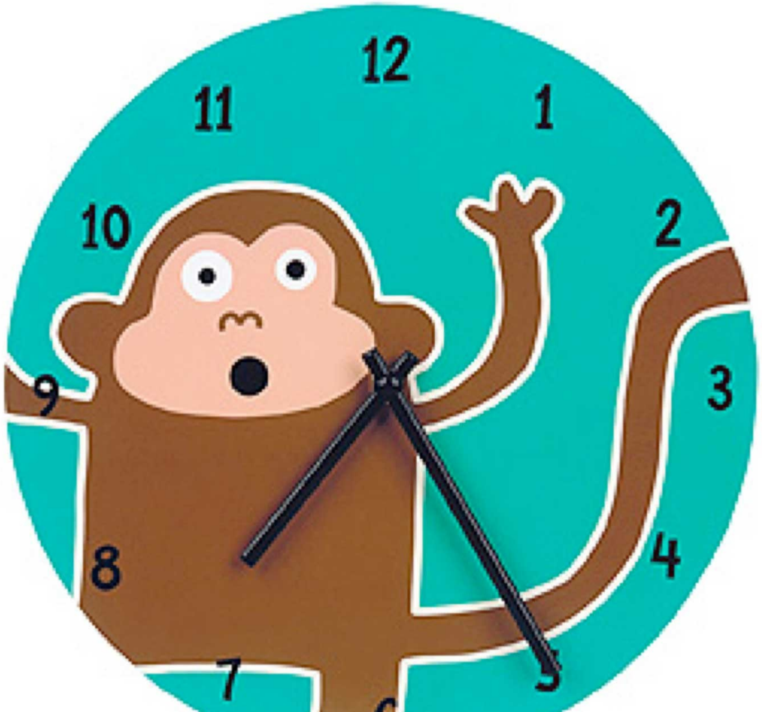
Can we use similar ideas for relativistic QM?

What is the problem?

What is the problem?

- QM ⟶ quantum systems

- GR ⟶ events



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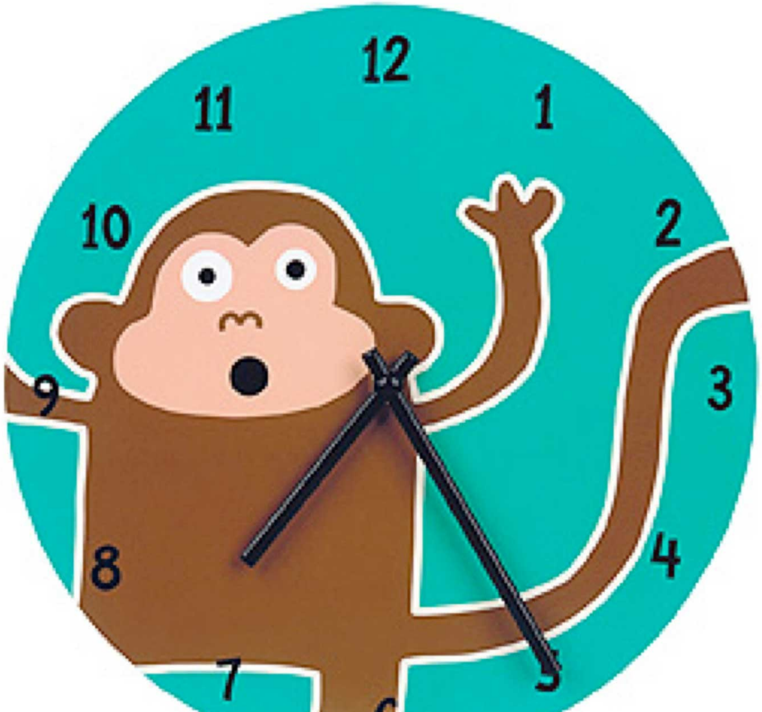

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Which is the most fundamental concept?

What do YOU think?!

Two approaches:

- General relativistic theory of QM

Systems are more fundamental: GR is made of quantum fields

- Quantum theory of GR

Events are more fundamental:

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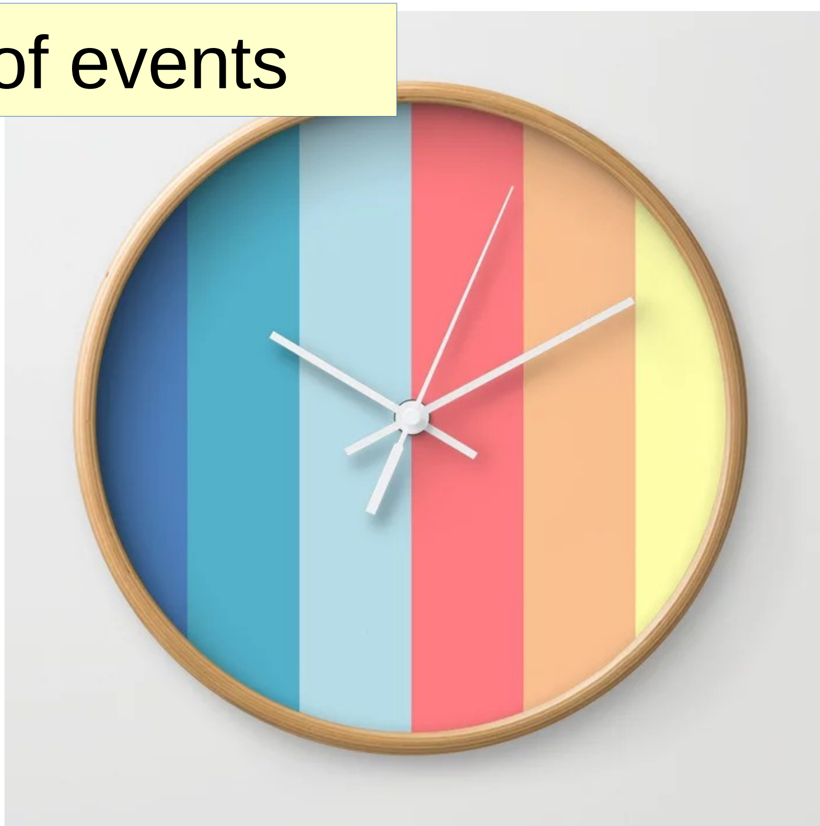

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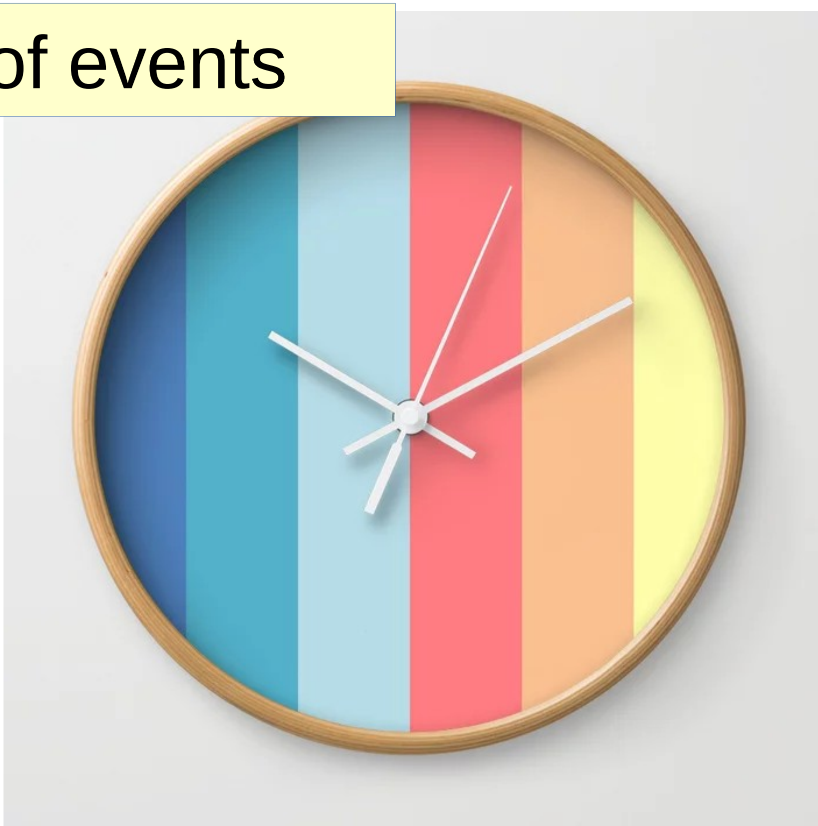

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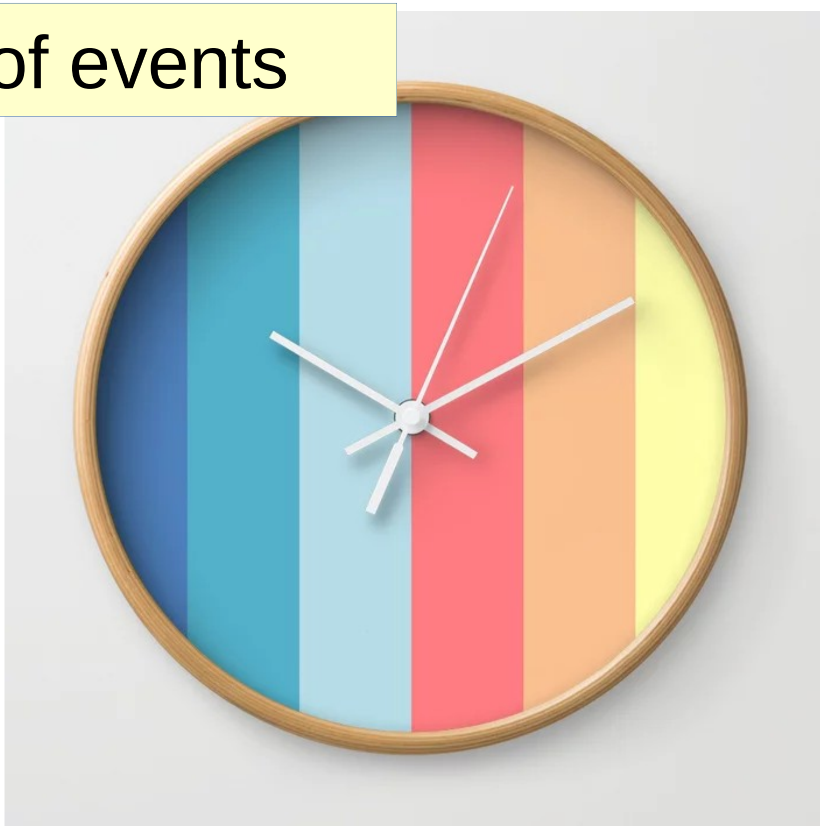

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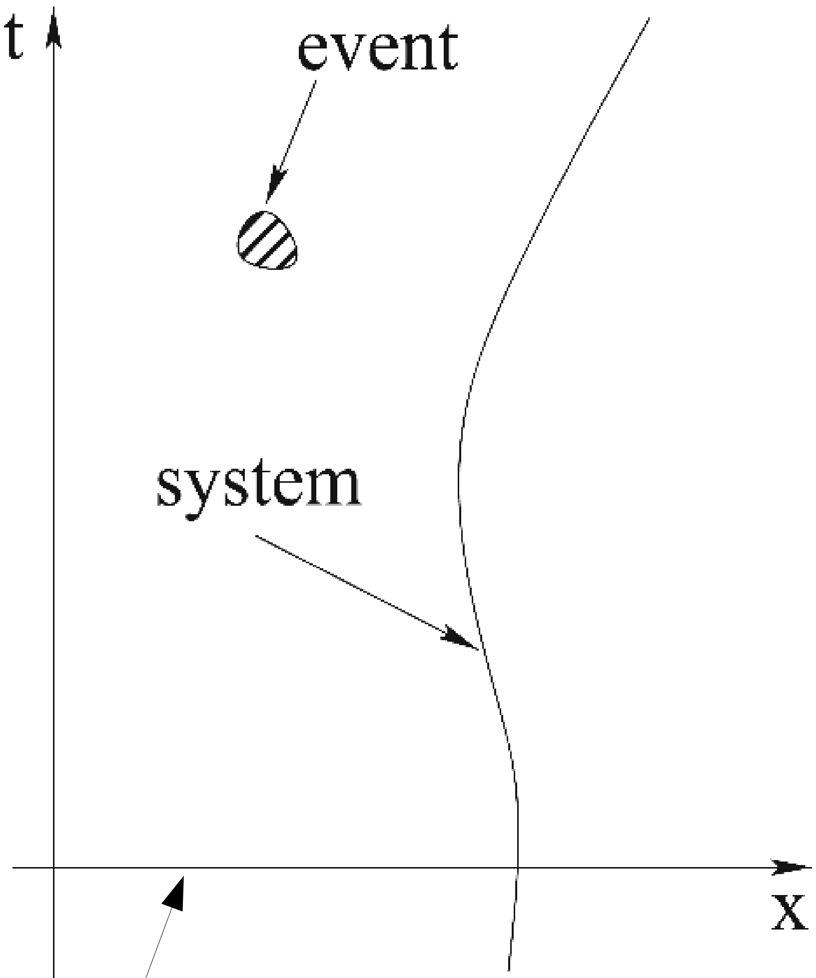
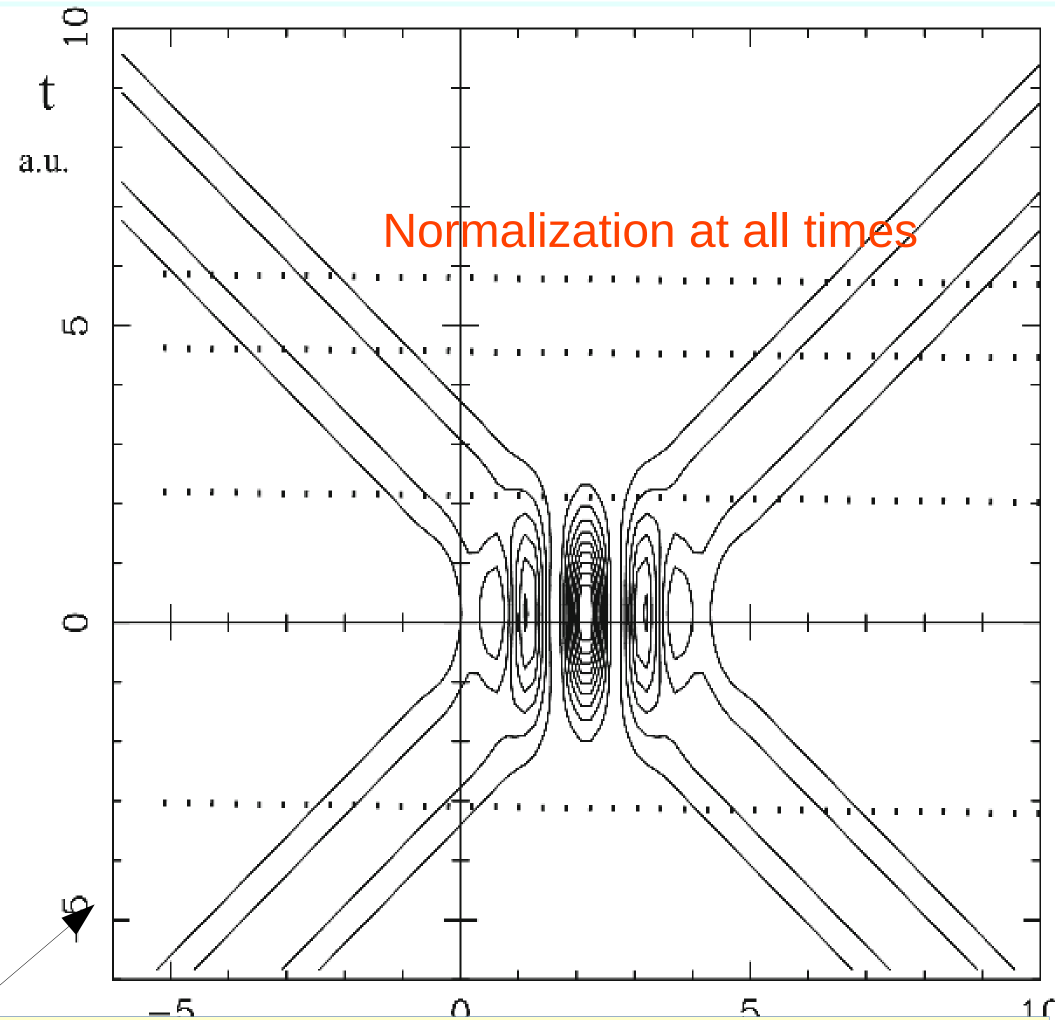
Explore the alternative!!

Current QM not able to deal with q events





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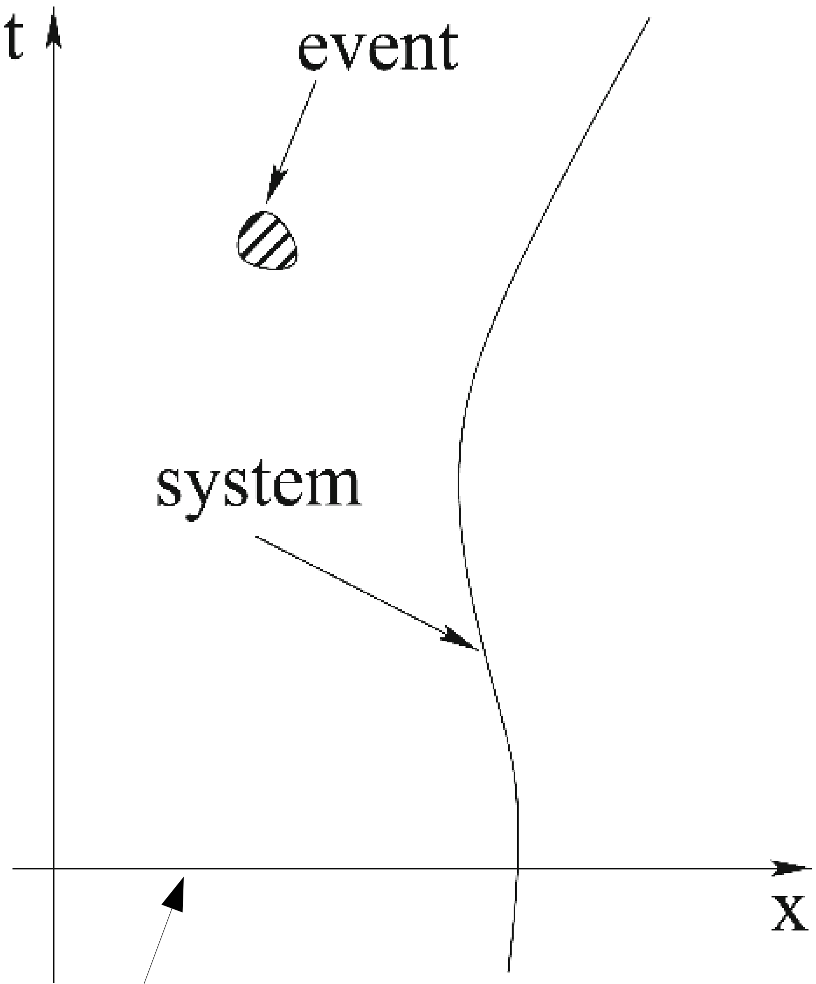

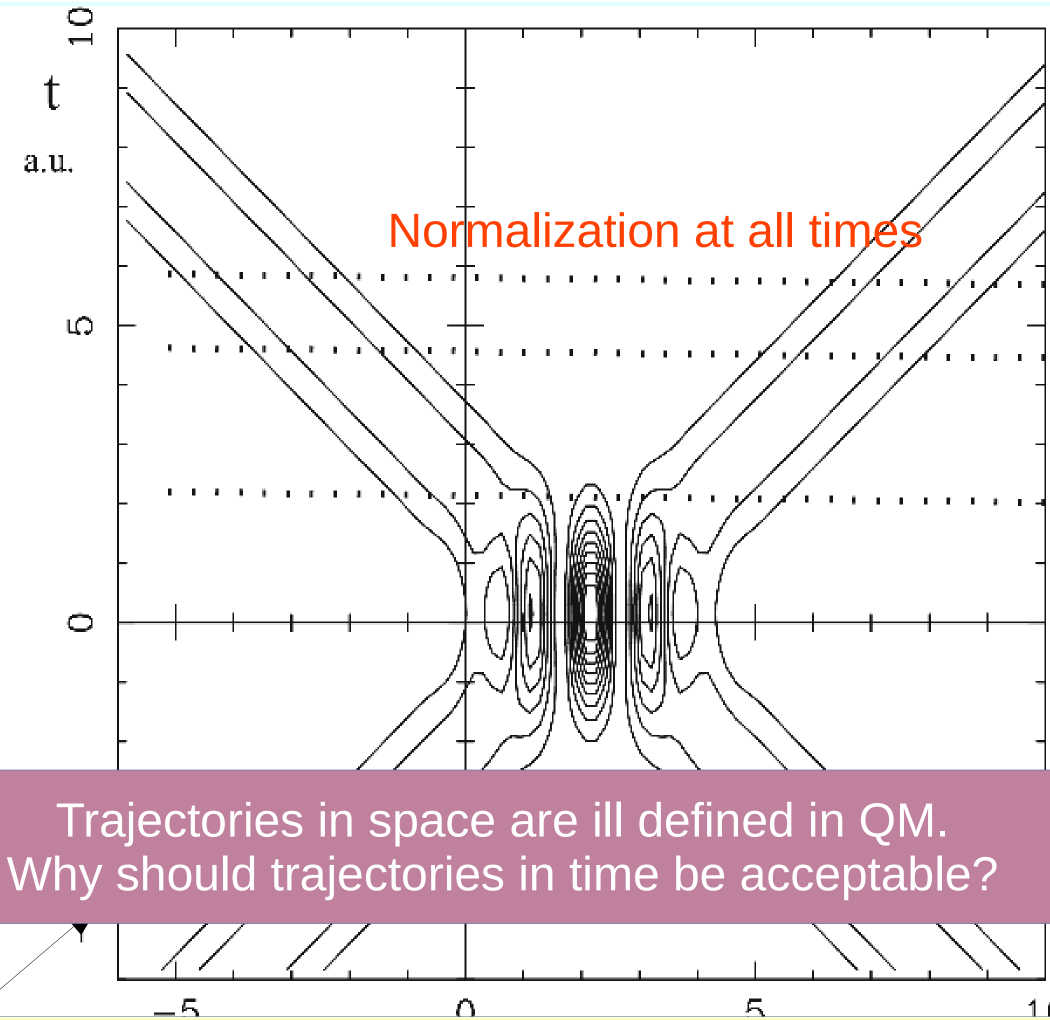

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Why should trajectories in time be acceptable?

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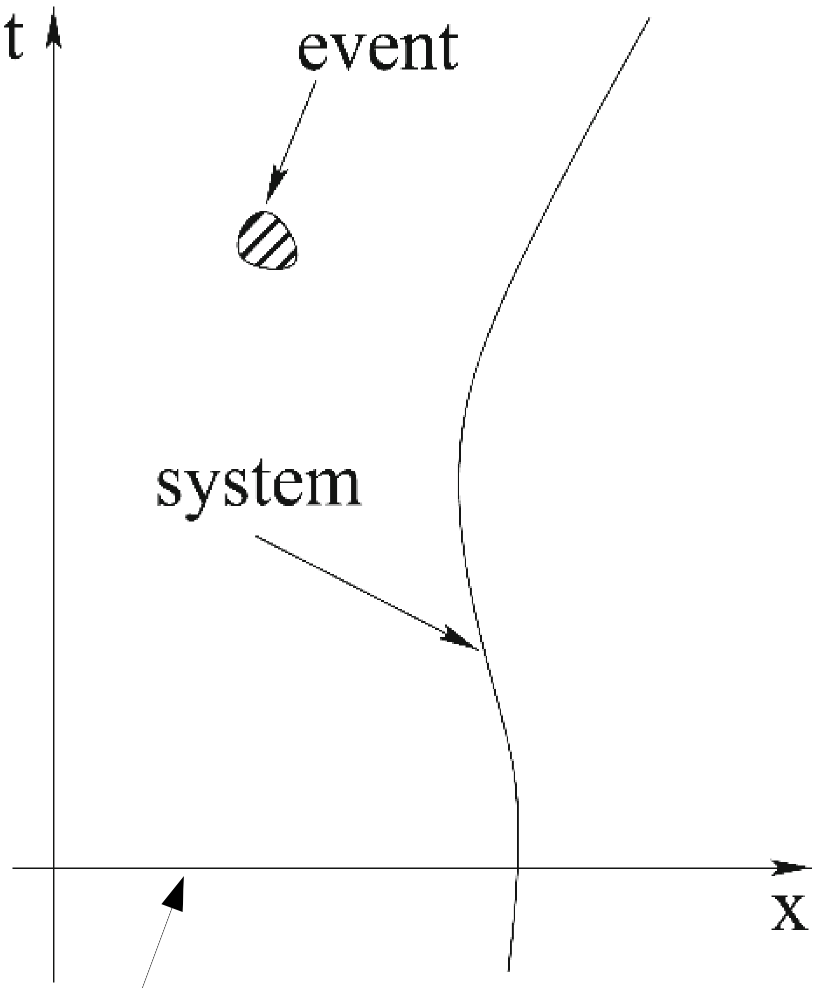

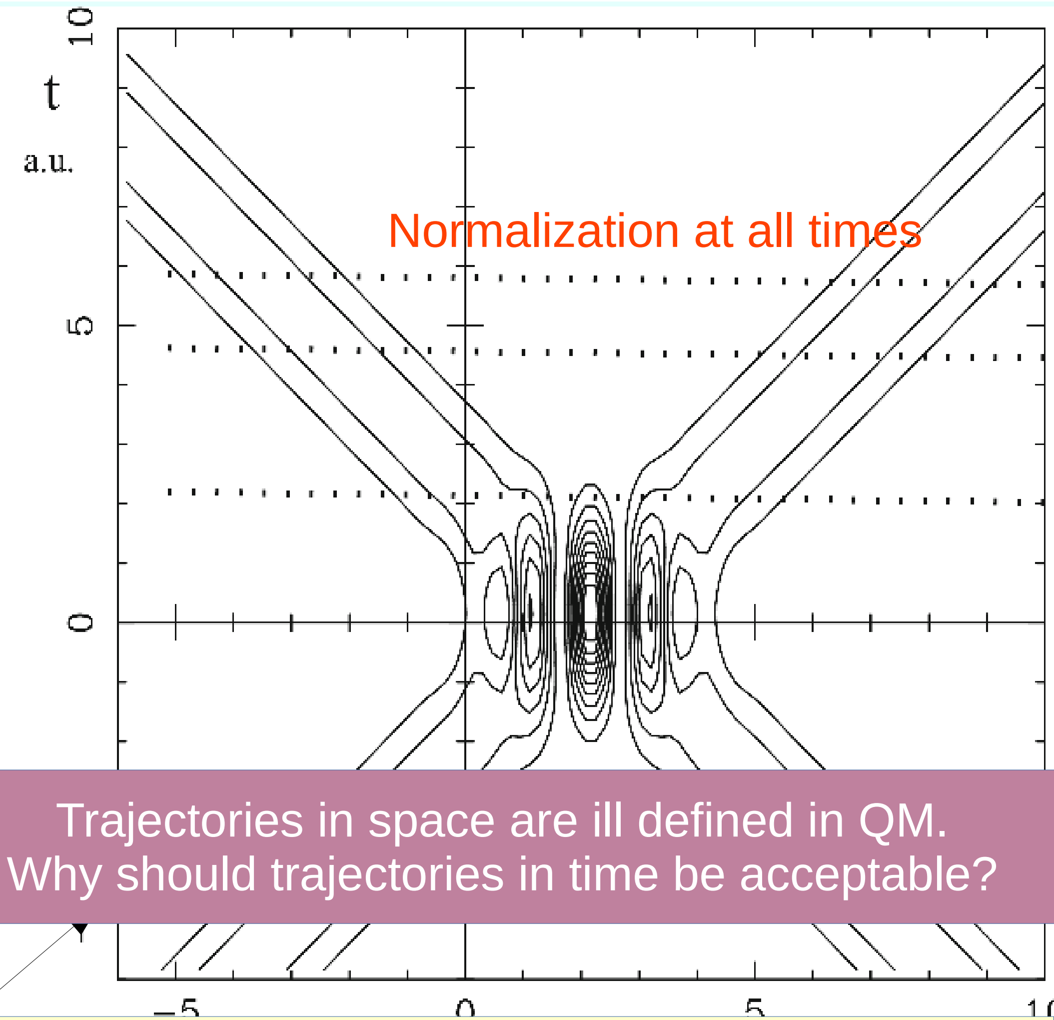

Normalization at all times

Trajectories in space are ill defined in QM.
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Need: Hilbert space for events (and its composition rule!)

Start with SPECIAL relativity

Let's deal with GR in the future (much in the future!)

- QM uses **time conditioned quantities**

Textbook QM and QFT

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$$|\psi(t)\rangle$$ States (Schroedinger picture)

$$X(t)$$ Observables (Heis. picture)

Textbook QM and QFT

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$$\Downarrow$$

CANNOT be relativistically covariant

(covariance=“formulas look the same in all reference frames”)

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2) Use a state that is invariant for Lorentz transforms, e.g the vacuum $|0\rangle$

Our approach: Geometric Event-Based QM

GEB

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GEB

quantum events → fundamental

Our approach: Geometric Event-Based QM

GEB

quantum events ⟶ **fundamental**

quantum systems ⟶ derived: a **quantum state for a succession of events** in q spacetime

A quantum event has a **position** in spacetime, but also an **energy-momentum**

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(without energy nothing can happen, without momentum, nothing can be localized)

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Poincare' algebra:

$$\begin{aligned} [M^{\mu\nu}, P^\rho] &= -i(\eta^{\mu\rho} P^\nu - \eta^{\nu\rho} P^\mu), \\ [M^{\mu\nu}, M^{\rho\sigma}] &= i(\eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\rho} M^{\nu\sigma} \\ &\quad - \eta^{\mu\sigma} M^{\rho\nu} + \eta^{\nu\sigma} M^{\rho\mu}) \end{aligned}$$

Now we can do GEB of easy systems (scalar KG and Dirac). Can we extend to more complex fields?

Can we extend to GR?

A universe with a single event

$$|\Phi\rangle = \int d^4x \; \Phi(\overline{x}) \; |\overline{x}\rangle = \int d^4p \; \tilde{\Phi}(\overline{p}) \; |\overline{p}\rangle$$

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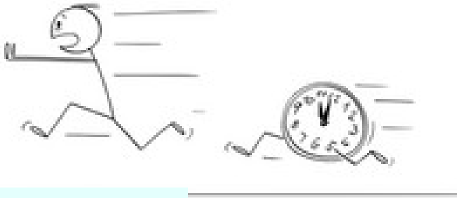

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Physical states are the ones that satisfy some constraints (e.g. PW WdW).

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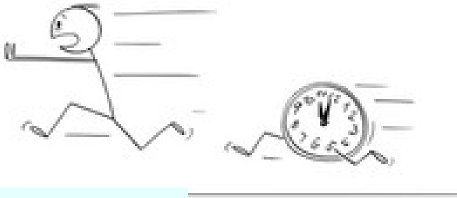

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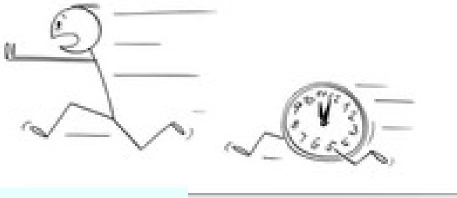

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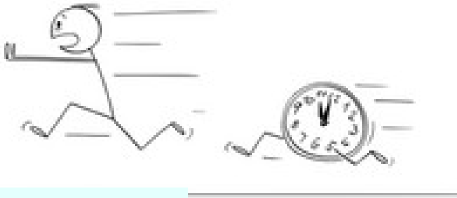

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GEB

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Probability that the particle is at position $\vec{x}$ *given* that the time is t!!!

QM

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QM probabilities are NOT covariant
GEB probabilities ARE covariant

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$$\Delta X^\mu \Delta P^\mu \geqslant 1/2, \quad \mu = 0, 1, 2, 3$$

UR!!!

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Cannot localize an event in time unless it has an energy spread

LORENTZ TRANSFORMS IN GEB

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Just a **unitary transformation on the GEB state** (Wigner's prescription on how to describe symmetries of a theory)

$$|\Phi'\rangle = U_{\Lambda}|\Phi\rangle$$

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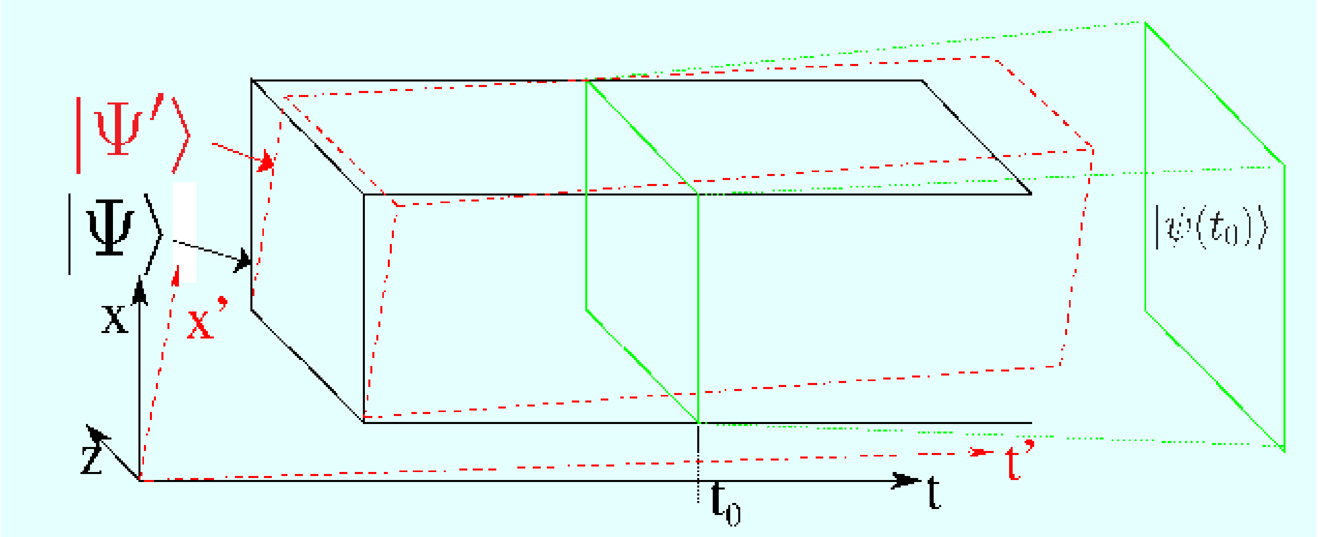

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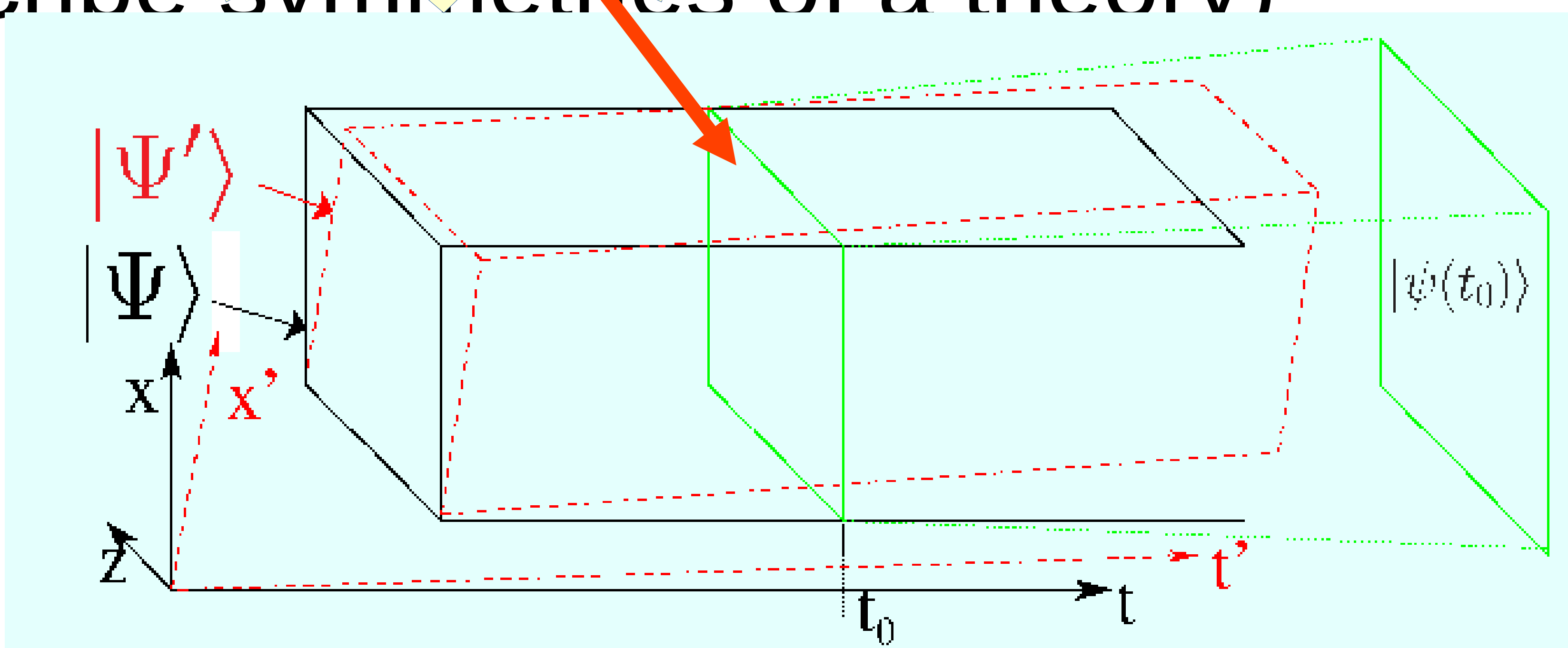
Relativistic
block universe picture!!

... **on the GEB** ... ption on how to describe ... of a theory)



LORENTZ TRANSFORMS IN QFT

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… or you can take a shortcut through GEB

LORENTZ TRANSFORMS IN QFT

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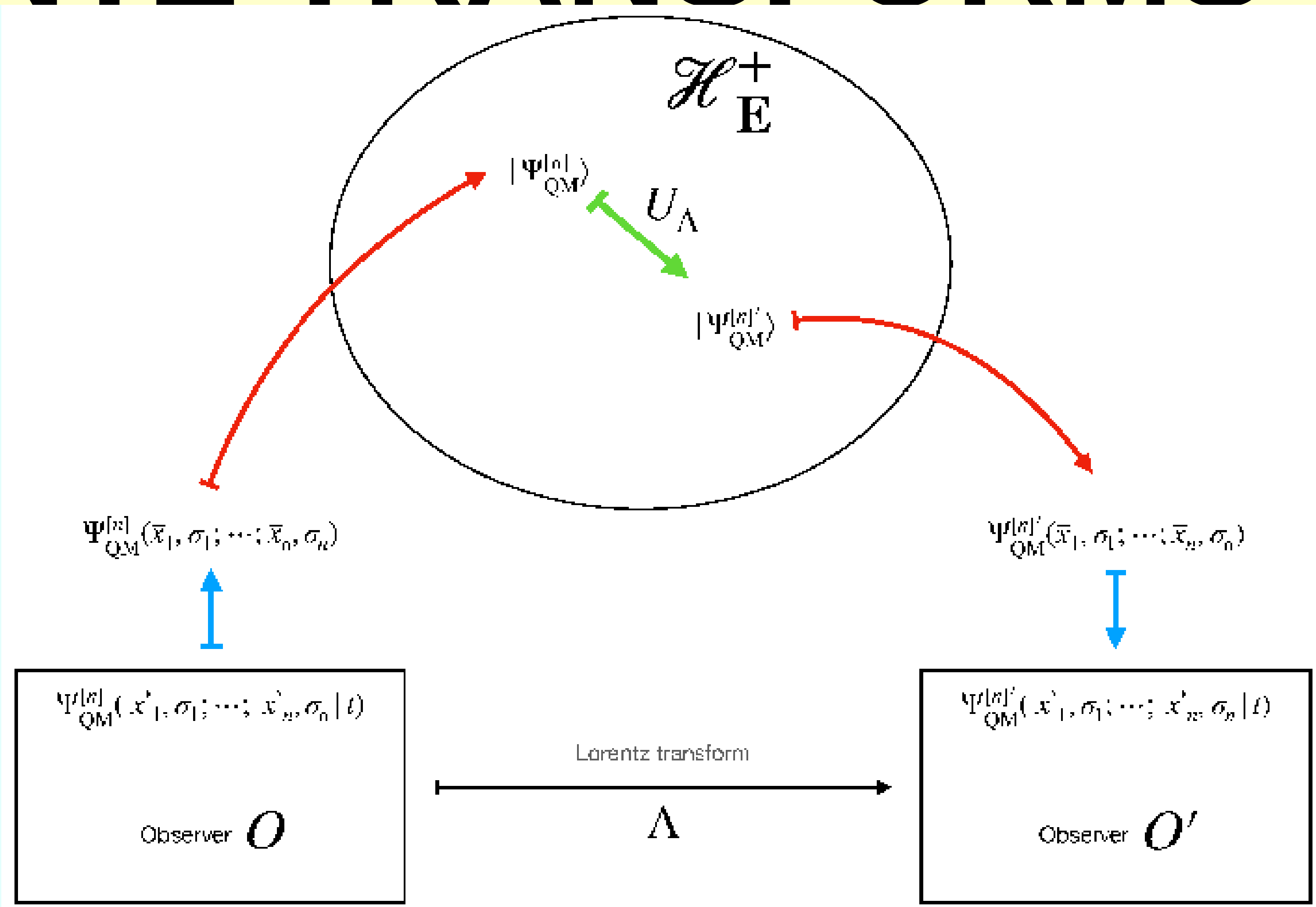

… GEB

Easier than requantizing everything:
a good first motivation for GEB

Multiple events: tensor products!

(if fixed number of events n)

$$|\Phi^{[n]}\rangle = \sum_{\sigma_1,\cdots,\sigma_n} \int d^4x_1 \cdots d^4x_n \, \Phi^{[n]}(\overline{x}_1,\sigma_1;\cdots;\overline{x}_n,\sigma_n) \Big|\overline{x}_1,\sigma_1;\cdots;\overline{x}_n,\sigma_n\Big\rangle$$

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Fock space

(otherwise)

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Creation operators: create an event at position x_1

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EACH EVENT WITH ITS OWN TIME!!!!

(cfr Dirac's multiparticle-multitime)

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$$= \frac{1}{\sqrt{n!}} \sum_{\sigma_1, \cdots, \sigma_n} \int d^4x_1 \cdots d^4x_n \, \Phi^{[n]}(\overline{x}_1, \sigma_1; \cdots ; \overline{x}_n, \sigma_n) \; a^\dagger_{\overline{x}_1, \sigma_1} \cdots a^\dagger_{\overline{x}_n, \sigma_n} |0\rangle_4$$

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Commutators:

$$\text{Bose: } [a_{\overline{x},\sigma}, a^\dagger_{\overline{x}',\sigma'}] = \delta_{\sigma,\sigma'}\delta^{(4)}(\overline{x}-\overline{x}'),\ [a_{\overline{x},\sigma}, a_{\overline{x}',\sigma'}] = 0,$$

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Bosonic events ⟶ Bosons

Fermionic events ⟶ Fermions

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4D vacuum

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Different state from the 3D vacuum of QFT ($|0\rangle_3$=no particles at time t=0 in the Heis pic)

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$$|0\rangle_3 = \text{foliate}(a^\dagger_{\overline{p}=0}|0\rangle_4)$$

Event state of **zero 4-momentum**: ground state of the field

Relativistic QM FROM GEB

Use **constraints** (as in the quantum time P&W, WdW, etc.)!

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(prob amplitude of finding particle in x given that the time is t)

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(GEB state describing the whole dynamics of the **particle as a state of a sequence of events**)

3) Write it as an eigenstate of a constraint op. →

$$K|\Psi_{\rm QM}\rangle = 0$$

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Similarly for the Dirac eq. constraint.

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WHY?

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WHY?

To get a better ontology?
To go further than QFT can go?

Conclusions

What did I say?!?

- Philosophical considerations about time

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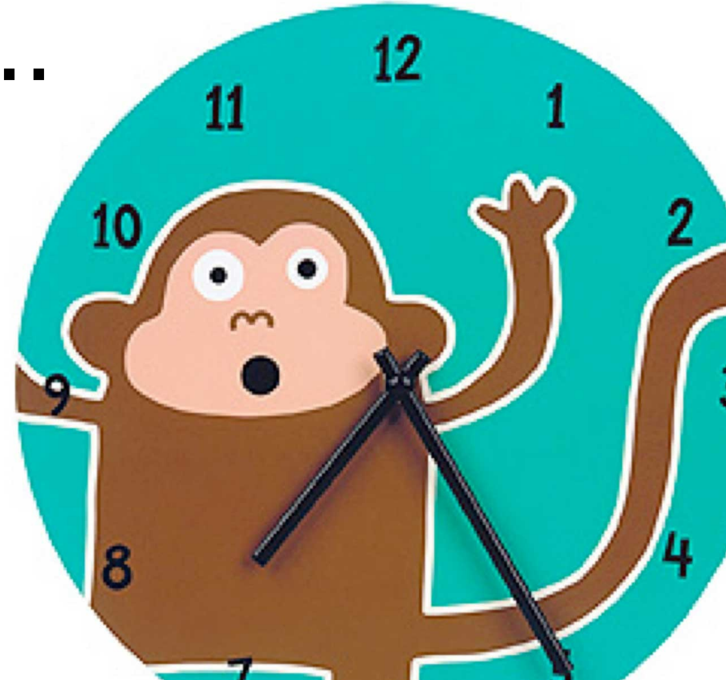

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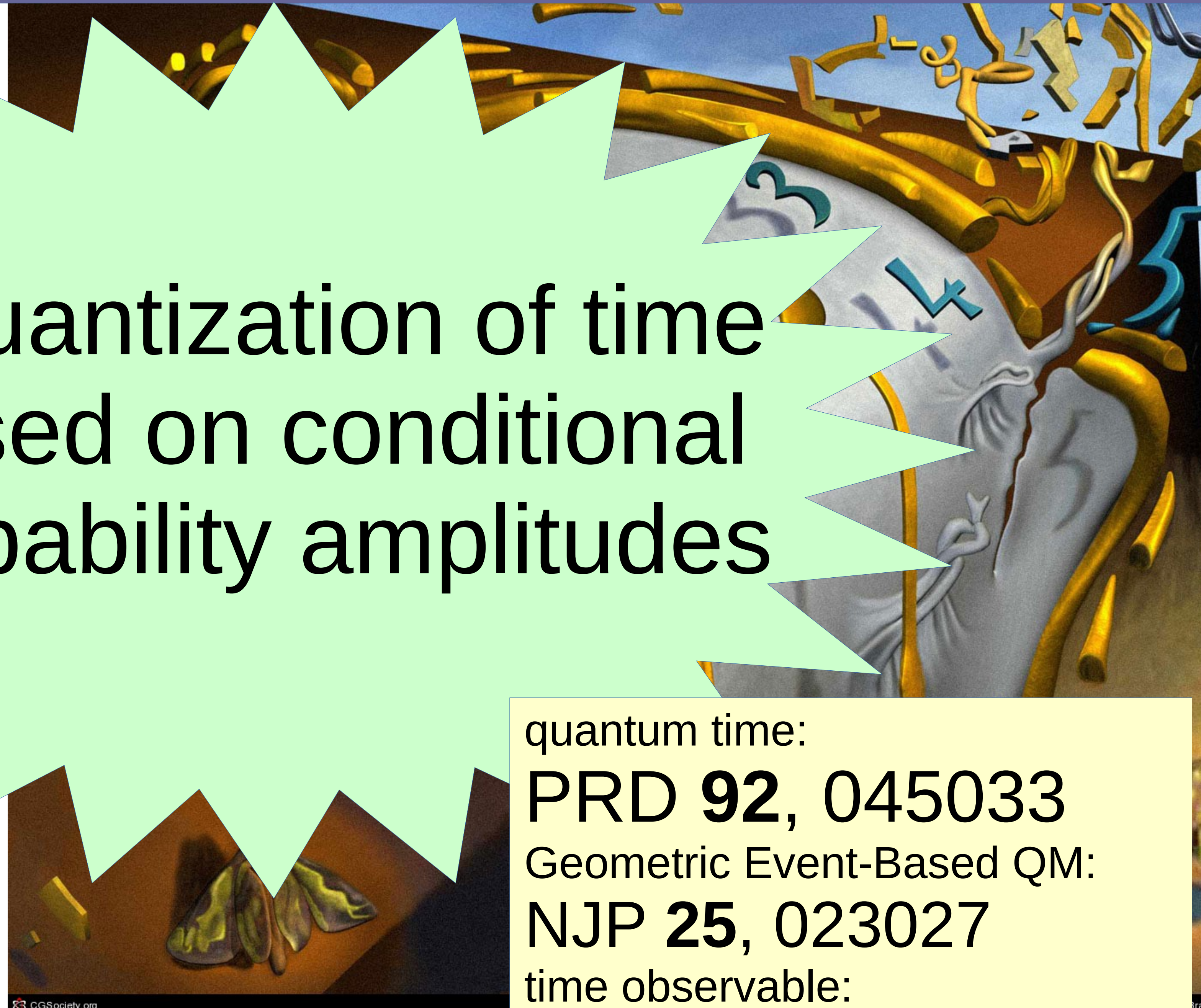
Take home message



A quantization of time based on conditional probability amplitudes

quantum time:
PRD **92**, 045033

Geometric Event-Based QM:
NJP **25**, 023027

time observable:
PRL **124**, 110402

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