Secrets Kept, Truth Proved: The Magic of Zero Knowledge Proofs

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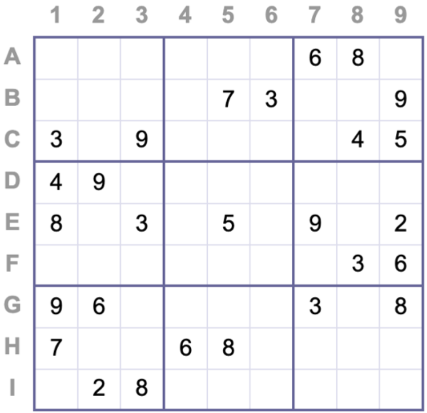
CCSS 2025

June 26, 2025

Solving Sudoku



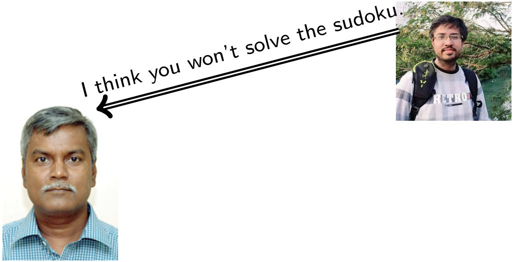
	1	2	3	4	5	6	7	8	9
A							6	8	
B					7	3			9
C	3		9					4	5
D	4	9							
E	8		3		5		9		2
F								3	6
G	9	6					3		8
H	7			6	8				
I		2	8						

Each cell must contain a number (1–9) that is unique to the row, column, and 3x3 grid.

Motivation

World Sudoku Championship

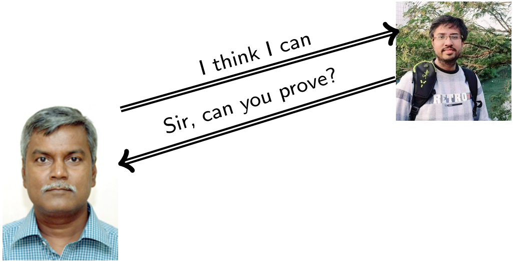
Motivation



I think you won't solve the sudoku.

World Sudoku Championship

Motivation



I think I can

Sir, can you prove?

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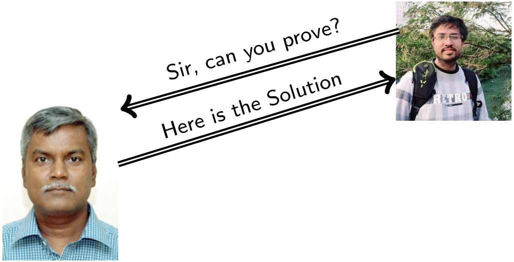
World Sudoku Championship

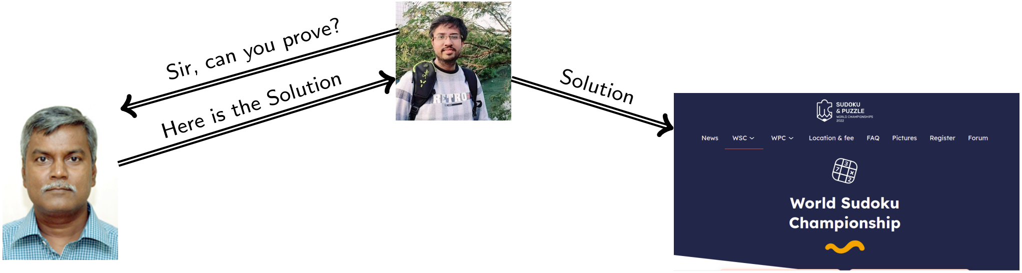
Motivation



Sir, can you prove?

Here is the Solution

Motivation



Sir, can you prove?

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Solution

Motivation

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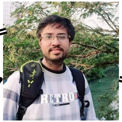

Solution

World Sudoku Championship

Zero Knowledge Proof [GMR]

- Conceived in 1985 by Shafi Goldwasser, Silvio Micali, and Charles Rackoff **[SIAM'85]**.
- Received **Godel Prize** in 1993 for advances in Theoretical Computer Science.

I. Proofs and Proof System

Notion of Proof

What is Proof ?

Notion of Proof

What is Proof ?

“A proof is whatever convinces me” – Shimon Even (1978)

Notion of Proof

A proof involves two parties:

- Prover – One who supplies the proof in favor of the statement
- Verifier – One who verifies the proof

Notion of Proof

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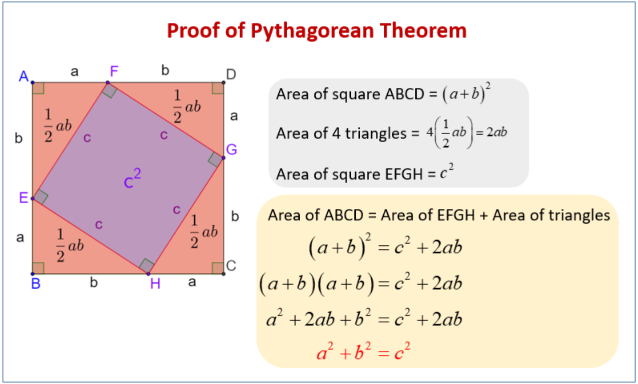
A proof should be easily verifiable.

Types of Proofs: Classical Proof



Proof of Pythagorean Theorem

Area of square ABCD = $(a+b)^2$

Area of 4 triangles = $4\left(\frac{1}{2}ab\right) = 2ab$

Area of square EFGH = c^2

Area of ABCD = Area of EFGH + Area of triangles

$$(a+b)^2 = c^2 + 2ab$$

$$(a+b)(a+b) = c^2 + 2ab$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

$$a^2 + b^2 = c^2$$

Types of Proofs: Classical Proof

- The proof is fixed and written somewhere which is either
    - Self-evident, or
    - Derived from self-evident rules.
- These proofs are static in nature.
- Examples: Mathematical proofs.

Types of Proofs: Interactive Proof

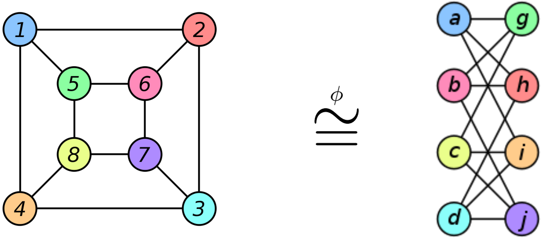
- Proof involves exchanges of information in multiple rounds between prover and verifier.
- Truth is established when the verifier accepts the hypothesis.
- Proof is dynamic in nature.
- Example: Legal Proofs in Court.

The Notion of Proof System

- Efficient - Verification of the proof should be simple
- Completeness - True statement must have a proof
- Soundness - False statement does not have any proof

An Example of a Classical Valid Proof System

Graph Isomorphism



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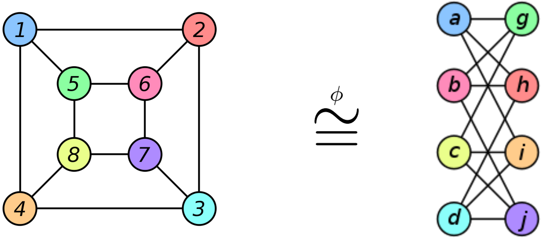
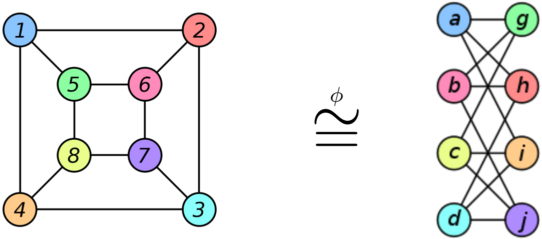


Isomorphism

$$\{\phi(1) = a, \phi(2) = h, \phi(3) = d, \phi(4) = i, \phi(5) = g, \phi(6) = b, \phi(7) = j, \phi(8) = c\}$$

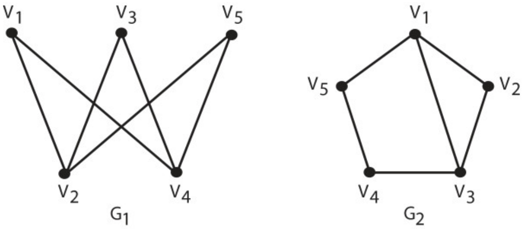
An Example of a Classical Valid Proof System



- Prover (P) sends ϕ as the proof to the verifier (V)
- V verifies ϕ is a valid permutation.
- Proof is complete and sound.
- No interaction between P and V

An Example where a Classical Proof System does not Work

Graph Non-Isomorphism (GNI)



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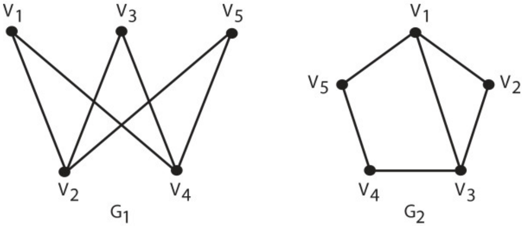

- Prover (P) sends all possible permutations ϕ on G_1 to V.
- Hard to verify..!!

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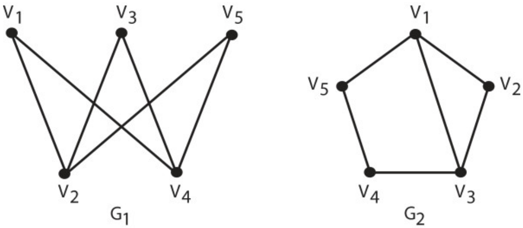

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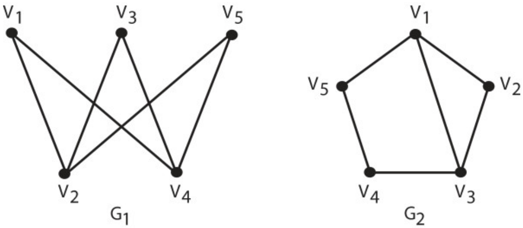
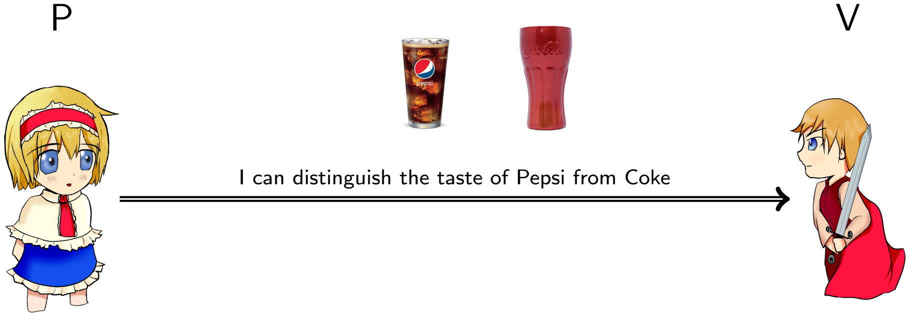


- Need Interactive Proof System.

Distinguishing Problem – An Example of IP



P

V

I can distinguish the taste of Pepsi from Coke

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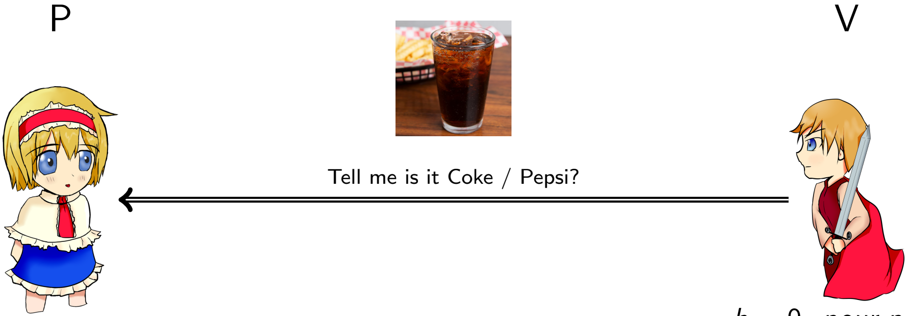
How does P prove her claim to V ?

Distinguishing Problem – An Example of IP



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Tell me is it Coke / Pepsi?

$b = 0$, pour pepsi
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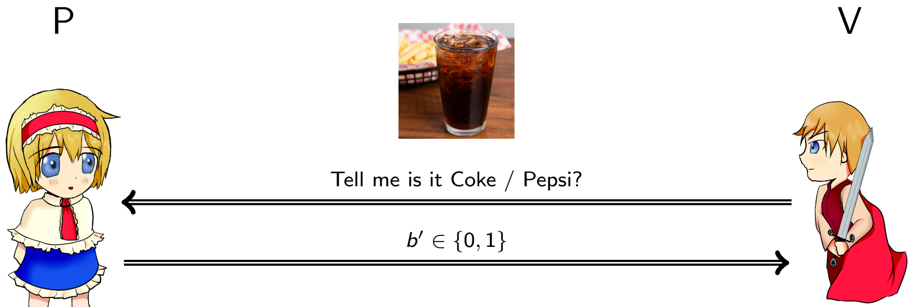

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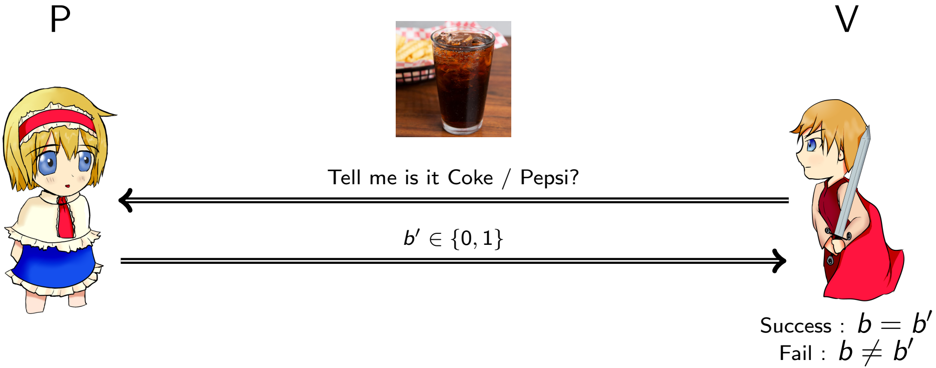

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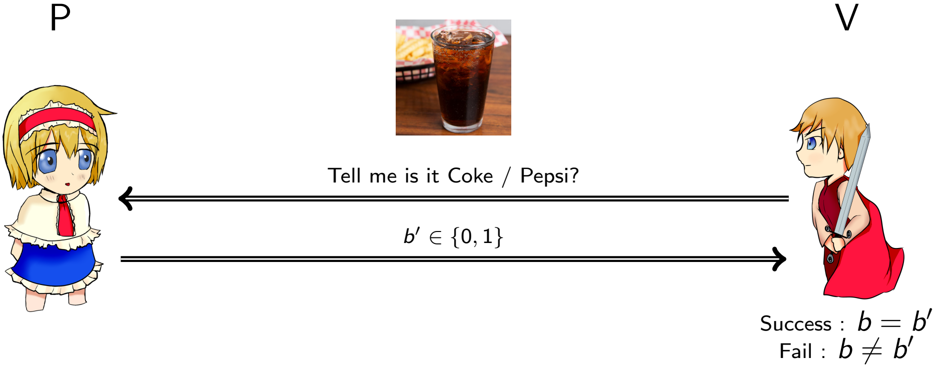

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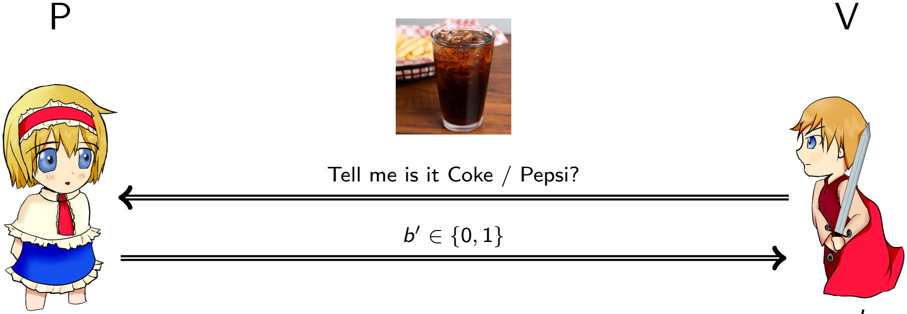

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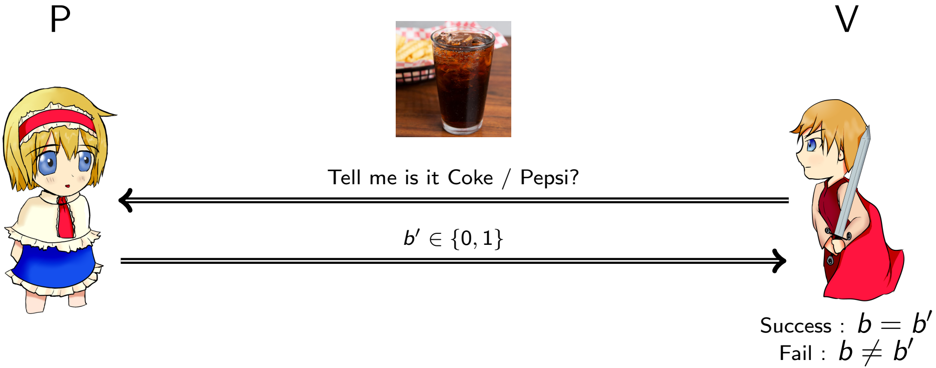

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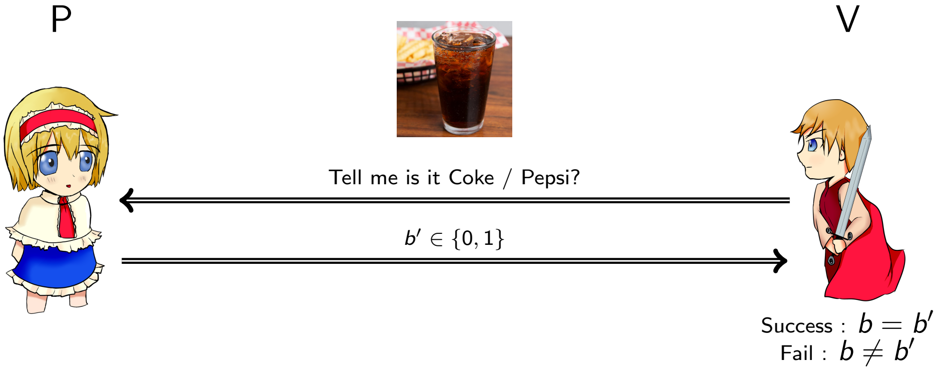

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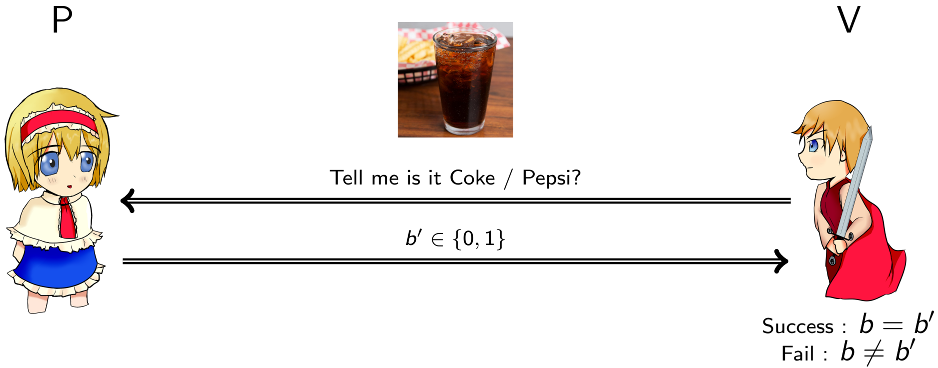

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- Repeat the expermient afresh and continues for 10 times.
- If P does not know, then it fails with probability (?)

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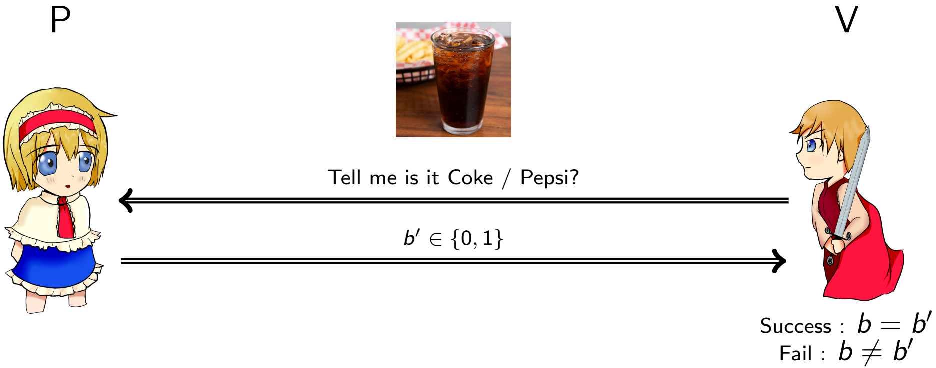

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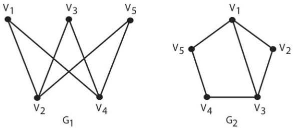
- Repeat the expermient afresh and continues for 10 times.
- If P does not know, then it fails with probability (?) 1023/1024
- **Soundness error** 2^{-10}

GNI has an Interactive Proof

P



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I can prove these two graphs are non-isomorphic

GNI has an Interactive Proof

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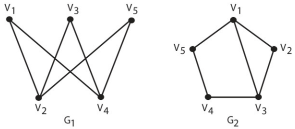

V

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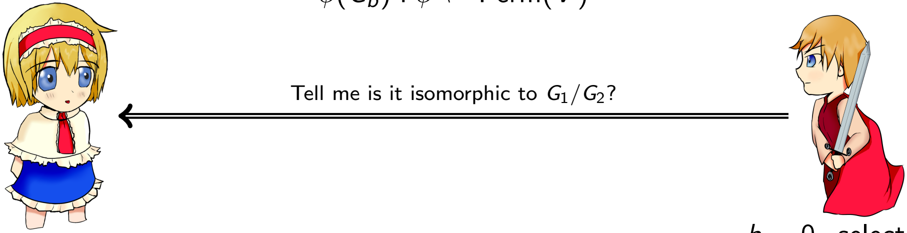
← I don't believe you

GNI has an Interactive Proof



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$\phi(G_b) : \phi \leftarrow \mathsf{Perm}(V)$

Tell me is it isomorphic to G_1/G_2?

$b = 0$, select G_1
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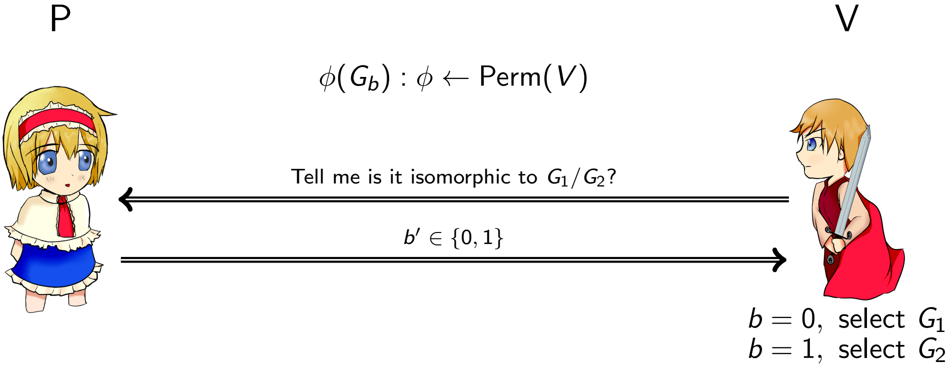

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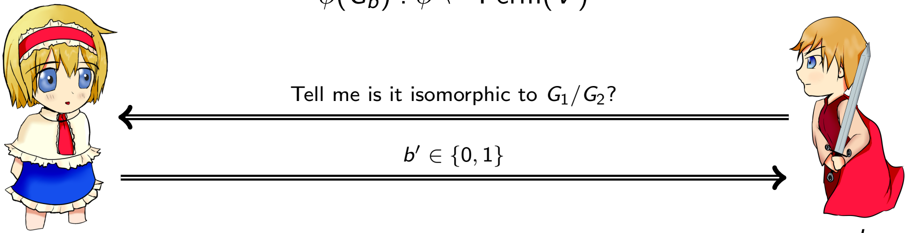

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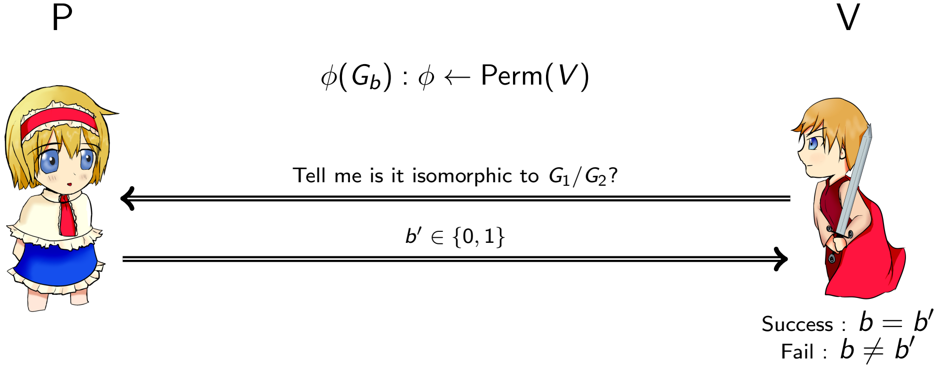

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- Completeness holds.
- Repeat the experiment for t times, Soundness error 2^{-t}.

II. (Zero) Knowledge

The Notion of Knowledge

What is Knowledge ?

The Notion of Knowledge

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Knowledge is the ability to complete a new task – Rafael Pass and Abhi Shelat

The Notion of Knowledge

What is Knowledge ?

Knowledge is the ability to complete a new task – Rafael Pass and Abhi Shelat

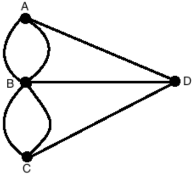
A conversation between two parties conveys knowledge when it allows the recipient to complete a “new” task that she could not complete before

The Notion of Zero Knowledge

The Notion of Zero Knowledge

We can't but we can define Zero-Knowledge.

The Notion of Knowledge– An Example



“I know this figure can not be drawn in a paper without lifting the pen”

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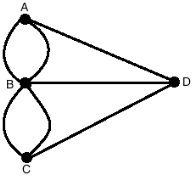

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Does the message convey any knowledge ?

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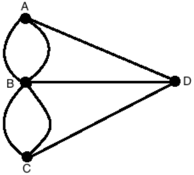

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Does the message convey any knowledge ?

No! I can very well compute whether the graph is “eulerian” or not.

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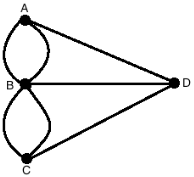

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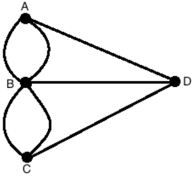
Does the message convey any knowledge ?

No! I can very well compute whether the graph is “eulerian” or not. – (**Zero-Knowledge**)

The Notion of Knowledge– An Example



“I know this figure can not be drawn in a paper without lifting the pen”

Information which conveys no knowledge is called **zero knowledge**

III. (I) + (II) ⇒ Zero Knowledge Proof.

Notion of Zero Knowledge Proof (ZKP)

Can a classical proof system be a zero knowledge proof system ?

Notion of Zero Knowledge Proof (ZKP)

Can a classical proof system be a zero knowledge proof system ?

It must be an interactive proof system

- Efficient - Verification of the proof should be simple
- Completeness - A prover should be able to prove a valid statement.
- Soundness - A dishonest prover should not be able to prove an invalid statement.

Notion of Zero Knowledge Proof (ZKP)

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- Completeness - A prover should be able to prove a valid statement.
- Soundness - A dishonest prover should not be able to prove an invalid statement.

How do we model the proof system does not convey any knowledge ?

Modeling Zero Knowledge

Notion of Simulator

The verifier can produce a transcript that “looks similar” to the transcript that results from the interaction between the honest prover and the verifier.

Modeling Zero Knowledge

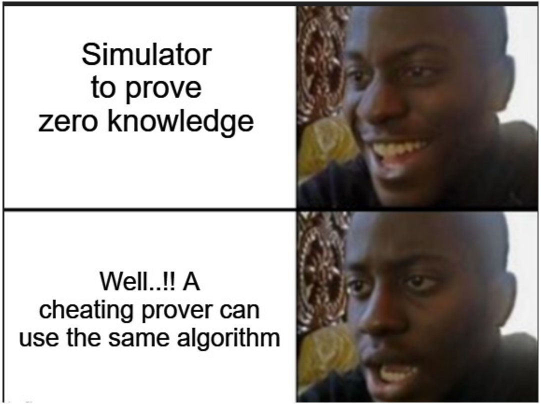
Notion of Simulator

The verifier can produce a transcript that “looks similar” to the transcript that results from the interaction between the honest prover and the verifier.

Rationale of Simulator

- It postulates that whatever a party can do “efficiently” by itself cannot be considered a gain from interaction with the outside.
- What matters is that any “real gain” can NOT occur whenever we are able to present a simulation.

Modeling Zero Knowledge


Simulator to prove zero knowledge
Well..!! A cheating prover can use the same algorithm

Modeling Zero Knowledge

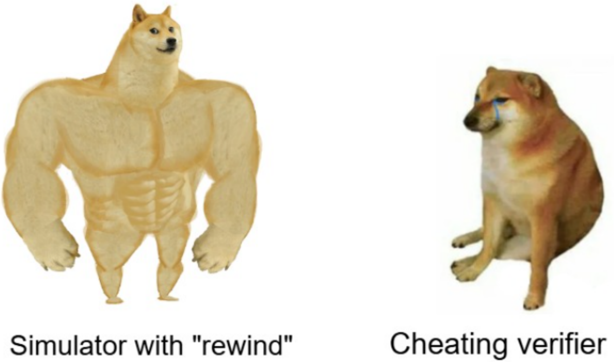
There must be something that the simulator can do but a cheating prover can not..!!

Modeling Zero Knowledge

There must be something that the simulator can do but a cheating prover can not..!!

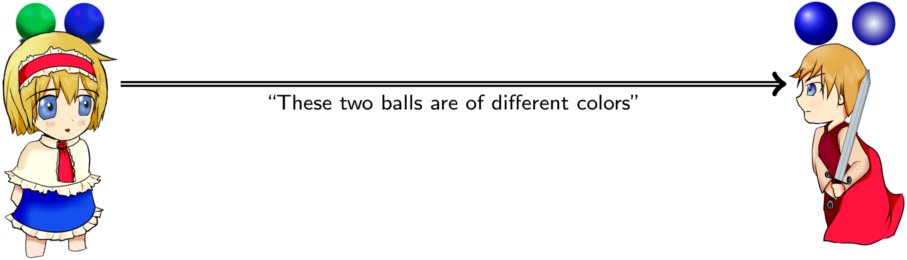
- The simulator can rewind the interaction.
- In fact, one should be able to construct simulators corresponding to a cheating verifier as well.

Modeling Zero Knowledge



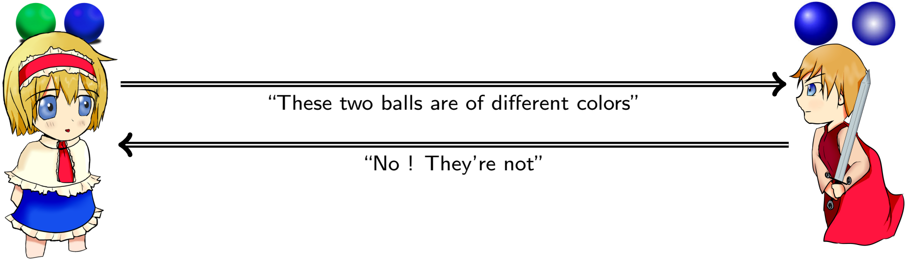
A Simple Example

Examples of ZKP : Two Balls and the Color-Blind Friend



“These two balls are of different colors”

Examples of ZKP : Two Balls and the Color-Blind Friend



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“No ! They’re not”

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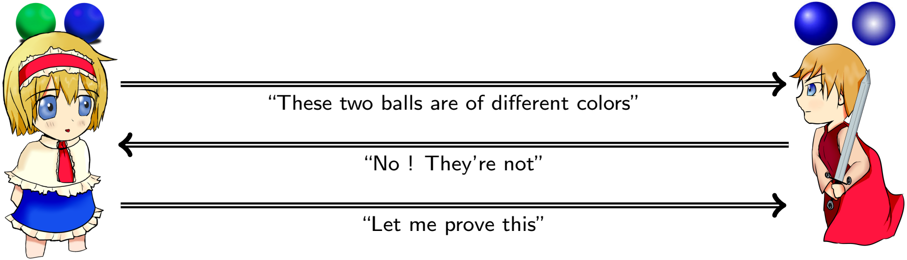

“These two balls are of different colors”

“No ! They’re not”

“Let me prove this”

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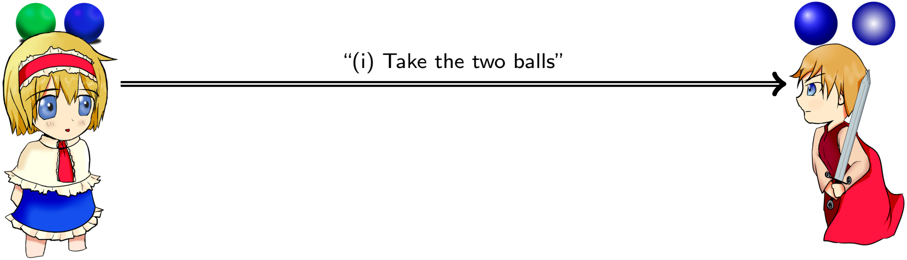

“(i) Take the two balls”

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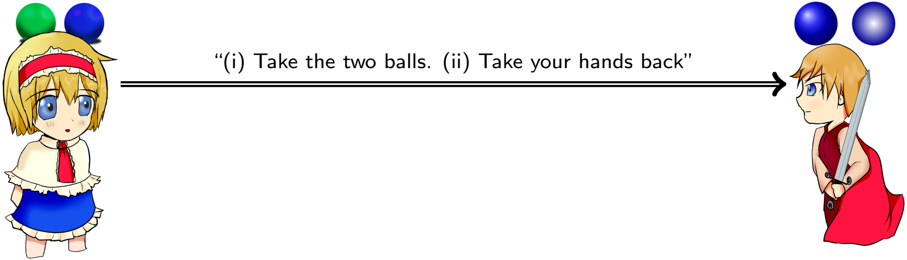
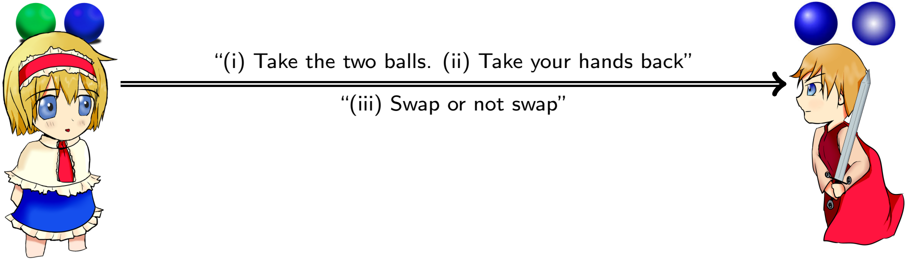


“(i) Take the two balls. (ii) Take your hands back”

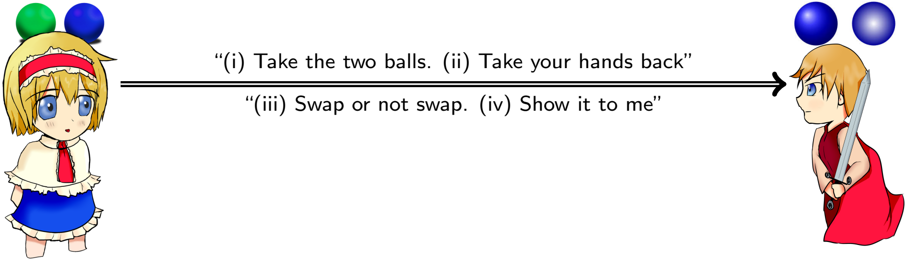
Examples of ZKP : Two Balls and the Color-Blind Friend



“(i) Take the two balls. (ii) Take your hands back”

“(iii) Swap or not swap”

Examples of ZKP : Two Balls and the Color-Blind Friend



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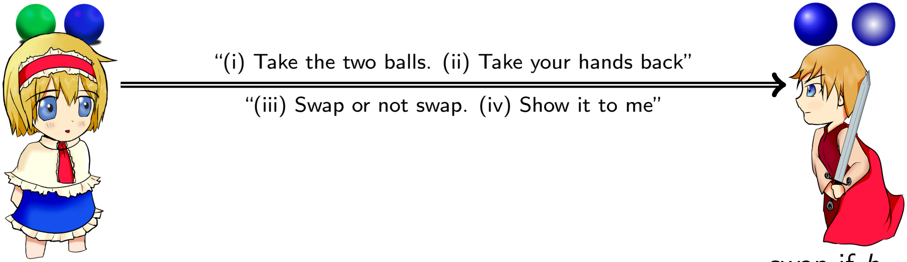

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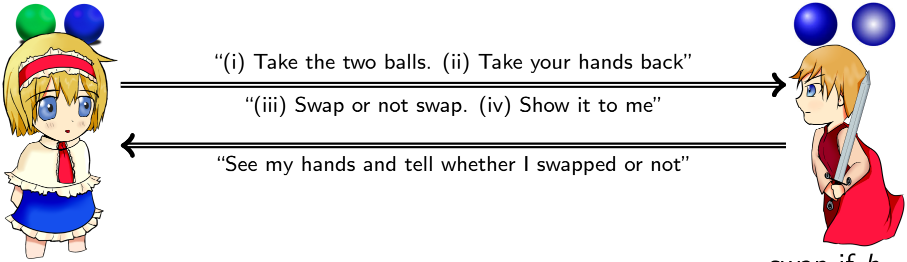

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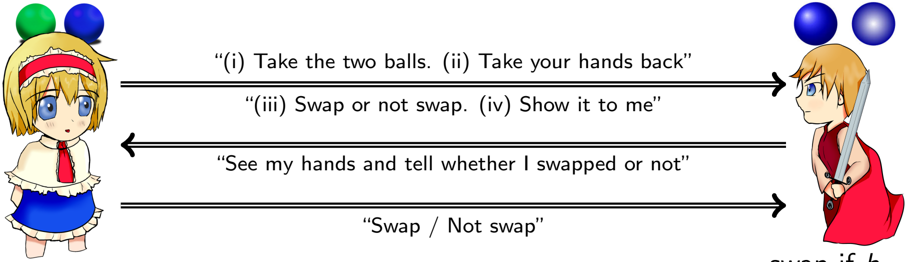

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- If Alice really knows the balls are distinguishable, then she always wins – (**Complete**)

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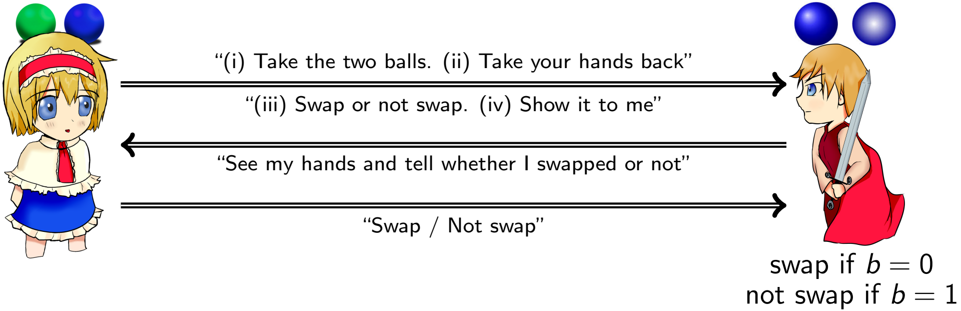

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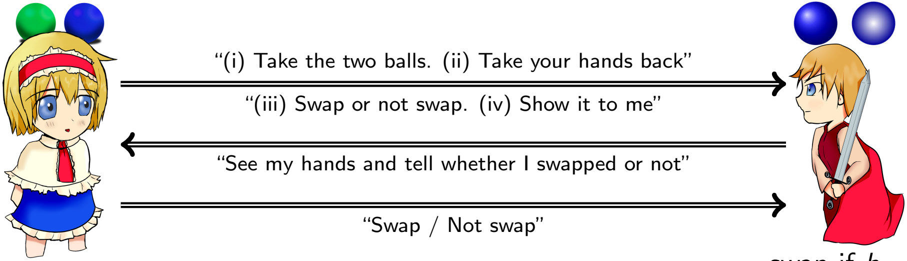
- If Alice really knows the balls are distinguishable, then she always wins – (**Complete**)
- If Alice does not know then she fails with probability 2^{-t} after $'t'$ many repetitions – (**Soundness error**)

Examples of ZKP : Two Balls and the Color-Blind Friend



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After the experiment, Bob does not know which ball is of which color – (**Zero Knowledge**)

A Cryptographic Example

Example of ZKP: Cryptographic Protocol

Discrete Logarithm Problem

- Take any large prime p, and consider $\mathbb{Z}_p^\star = \{1, 2, \ldots, p-1\}$
- Let g be a generator of $\mathbb{Z}_p^\star$, primitive element modulo p:

$$\langle g \rangle = \mathbb{Z}_p^\star = \{1, g, g^2, \ldots, g^{p-2}\} \bmod p.$$

- Given g and x, it is easy to calculate $g^x \bmod p$.
- However, given g and y, it is hard to find x in the range of 0 to $p-2$ that satisfies

$$g^x \equiv y \bmod p.$$

Use of Discrete Logarithm Problem in Locker Access

$$\langle g \rangle = \mathbb{Z}_p^*$$

$$x \in \mathbb{Z}_p^*, y \leftarrow g^x \mod p$$

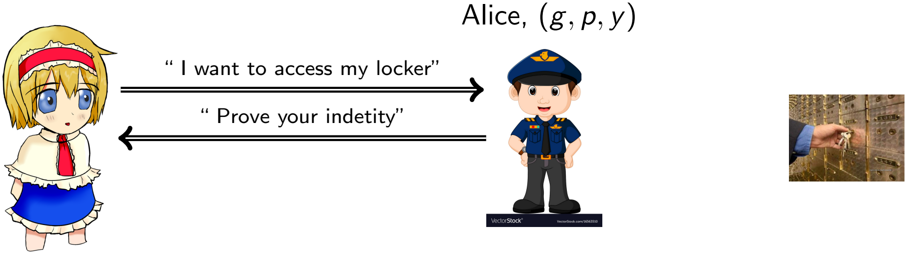
$$\xRightarrow{(g, p, y)}$$

Use of Discrete Logarithm Problem in Locker Access

“ I want to access my locker”

Alice, (g, p, y)

Use of Discrete Logarithm Problem in Locker Access



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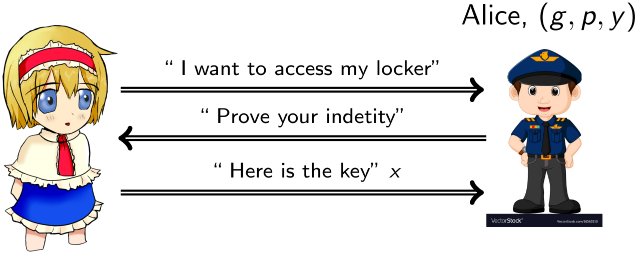
" I want to access my locker"

" Prove your indetity"

Use of Discrete Logarithm Problem in Locker Access



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" Here is the key" x

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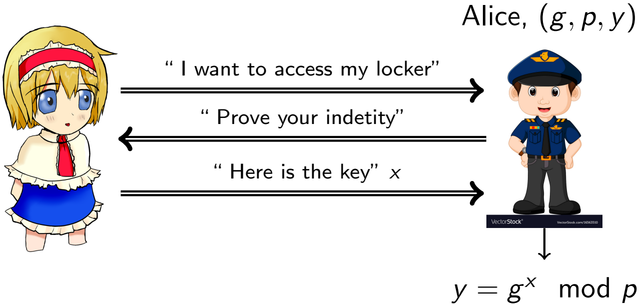

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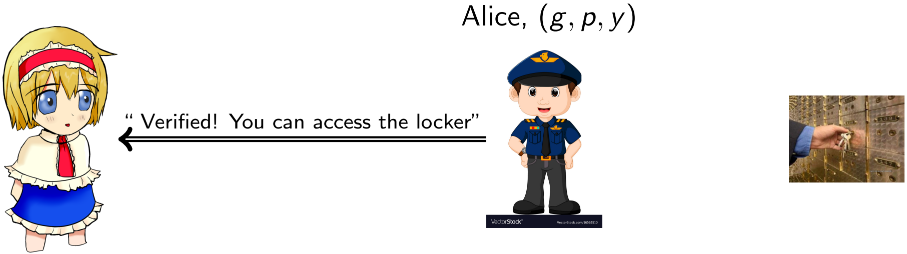
$$y = g^x \mod p$$

Use of Discrete Logarithm Problem in Locker Access



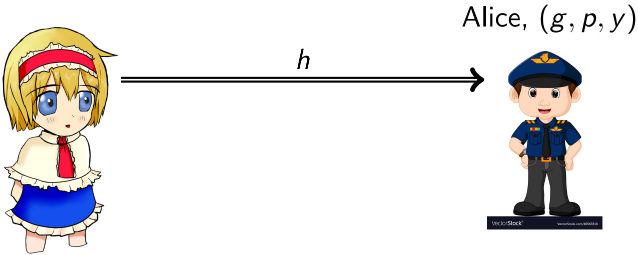
Alice, (g, p, y)

“ Verified! You can access the locker”

Use of Discrete Logarithm Problem in Locker Access

Use of ZKP of Discrete Logarithm Problem in Locker Access

$r \leftarrow \mathbb{Z}_p^*, h \leftarrow g^r \mod p$



Alice, (g, p, y)

h

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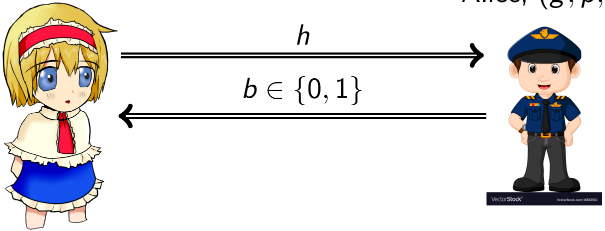

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$b \in \{0, 1\}$

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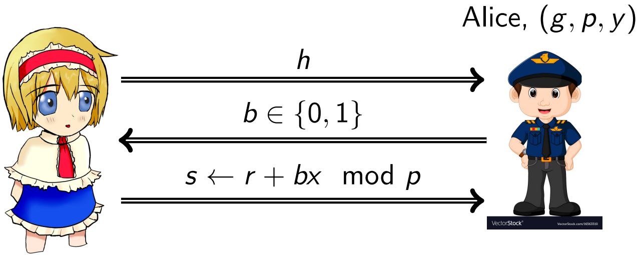

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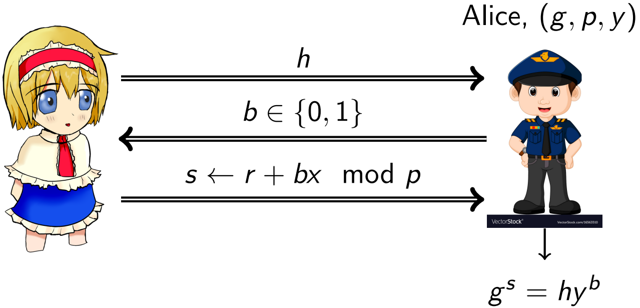


Alice, (g, p, y)

h

$b \in \{0, 1\}$

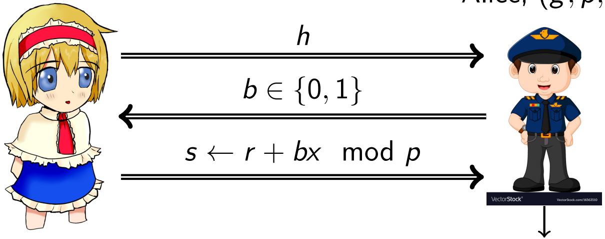
$s \leftarrow r + bx \mod p$

$g^s = hy^b$

Use of ZKP of Discrete Logarithm Problem in Locker Access

$r \leftarrow \mathbb{Z}_p^*, h \leftarrow g^r \mod p$

Alice, (g, p, y)



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$s \leftarrow r + bx \mod p$

$g^s = hy^b$

Repeat the game for t times

Use of ZKP of Discrete Logarithm Problem in Locker Access

Completeness

If Alice knows the secret x, then she will always win the game by computing the s following the protocol.

Soundness

- If Alice cheats, then the probability of winning the game in a trial is $1/2$. (Can you show that?)
- Repeating the experiment t times: Soundness error: 2^{-t}

Use of ZKP of Discrete Logarithm Problem in Locker Access

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Note: Repeating experiment means the randomness is generated freshly.

ZKP of Discrete Logarithm Problem

View of the interaction: $\text{view}_{\text{guard}^*}^{\text{Alice}}(g, p, y) = (h, b, s)$

ZKP of Discrete Logarithm Problem

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Simulator for DL Problem

1. Pick $b' \xleftarrow{\$} \{0,1\}$, $s \xleftarrow{\$} \mathbb{Z}_p^*$
2. Compute $h = \frac{g^s}{y^b} \bmod p$ and send it to the verifier
3. Verifier replies with b. If $b \neq b'$, rewind and execute step 1 again.
4. Transcript of the simulator: $M^*(g, p, y) = (h, b, s)$.

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Observe that $\text{view}_{\text{guard}^*}^{\text{Alice}}(g, p, y) \cong M^*(g, p, y)$

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Can a cheating prover generate the view?

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Can a cheating prover generate the view? No, it receives b only after sharing h.

ZKP for all Problems in $\mathcal{NP}$

Zero Knowledge Proofs for $\mathcal{NP}$

The Class $\mathcal{NP}$

A language L is in $\mathcal{NP}$ if given a witness it can be verified in polynomial time.

$\mathcal{NP}$-Completeness

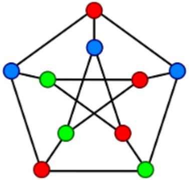
A language L is $\mathcal{NP}$-Complete if

- $L \in \mathcal{NP}$, and
- Each $L' \in \mathcal{NP}$ is polynomially reducible to L.

Examples: 3-COL, 3-SAT, CLIQUE, Vertex Cover.

3-COL Problem

A graph G is 3-colorable if the vertices of a given graph can be colored with only three colors, such that no two vertices of the same color are connected by an edge. Given a graph can you make it 3-colorable?



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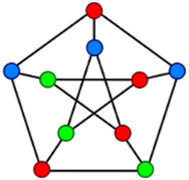

3-COL problem is an $\mathcal{NP}$-complete Problem

Zero Knowledge Proofs for $\mathcal{NP}$

The Basic Idea

- 3-COL is $\mathcal{NP}$-complete
- Any problem in $\mathcal{NP}$ can be reduced to the 3-COL problem
- We will show a Zero Knowledge Proof for 3-COL problem

ZK Proof of Graph 3-Colorability

- Common Input: A 3-colorable graph $G(V, E)$, $|V| = n$.
- Auxiliary Input (Prover): A 3-coloring $\phi : V \rightarrow \{1, 2, 3\}$

Interactive Protocol

- P1: Execute the following:
  - $\pi \leftarrow_{\$} \{1, 2, 3\}$, sets $\psi(i) = \pi(\phi(i))$, $\forall i = 1(1)n$.
  - Choose $s_1, \ldots, s_n \leftarrow_{\$} \{0, 1\}^n$.
  - Computes $c_i = C_{s_i}(\psi(i))$ and sends $c_1, \ldots, c_n$.

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- P2: Upon receiving $e = (u, v) \in E$, reveals $(s_u, \psi(u))$ and $(s_v, \psi(v))$.

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- V1: $(u, v) \leftarrow_\$ E$ and sends to P.
- P2: Upon receiving $e = (u, v) \in E$, reveals $(s_u, \psi(u))$ and $(s_v, \psi(v))$.
- V2: Upon receiving (s, σ) and (s', σ'), verifies $c_u = C_s(\sigma)$, $c_v = C_{s'}(\sigma')$ and $\sigma \neq \sigma'$. If all the conditions hold, accept; otherwise, reject.

ZK Proof of Graph 3-Colorability

Main Result

If the commitment scheme satistisfies the hiding and the binding requirements, then the construction constitutes an auxiliary-input zero-knowledge interactive proof for G3C.

Properties

- Completeness bound: 1.
- Soundness bound: $1/|E|$.

Some Real Life Applications

Use of ZKP in e-Auction Protocol

- Fairness: All bids should remain confidential, no bidder should be able to modify the committed bid, lowest bid must win.
- Confidentiality: Except the winning bid all the other bids must remain confidential
- Anonymity: Information about the identity of the bidders (except the winner) must be confidential.

Use of ZKP in e-Auction Protocol

- The auction repository stores all committed bids, not their openings.
- The bidders commit their bids by submitting the cryptographic commitments of their bid value.
- After the commitment phase is finished, the auctioneer opens all commitments.
- It is generated one proof for each losing bid. This proof demonstrate that the difference between the losing value and winning value is positive.
- Each proof can be publicly verified by any interested party

Use of ZKP in e-Voting

- A voter can cast his/her vote to 0 or 1 in an encrypted way.
- The authority gets all the encrypted votes, add all ballots using the scheme's add algorithm, decrypts the sum.
- A voter can encrypt an invalid vote and the authority would decrypt the sum incorrectly.
- Any one can verify that no votes have been modified, added, or deleted during the process.
- No one should be able to find your casted vote.

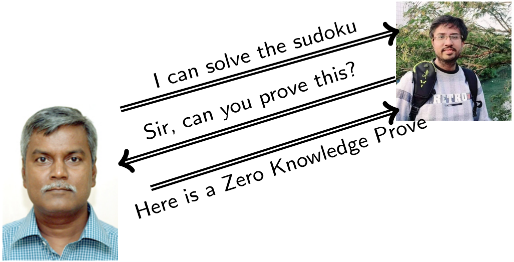
Applications of ZKP in Crypto currencies

- Sending private blockchain transactions should not reveal
  - source of the actual money,
  - how much money was sent, or
  - the identity of the final recipient.
- Traitional methods may reveal some relevant information.

Now-a-days, ZKP is widely used in several cryptocurrencies: ZCASH, Monero, PIVX, Zerocoin.

Revisiting the Sudoku Problem

Solving the Sudoku



I can solve the sudoku

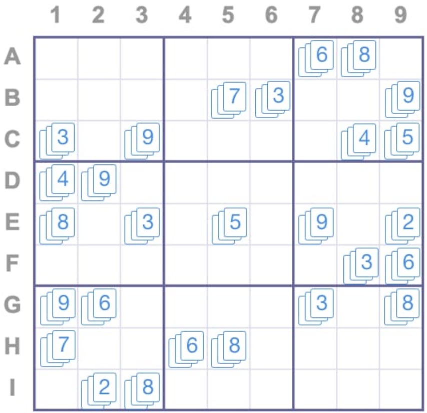
Sir, can you prove this?

Here is a Zero Knowledge Prove

Zero Knowledge Proof for Sudoku

	1	2	3	4	5	6	7	8	9
A							6	8	
B					7	3			9
C	3		9					4	5
D	4	9							
E	8		3		5		9		2
F								3	6
G	9	6					3		8
H	7			6	8				
I		2	8						

Zero Knowledge Proof for Sudoku



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E	8		3		5		9		2
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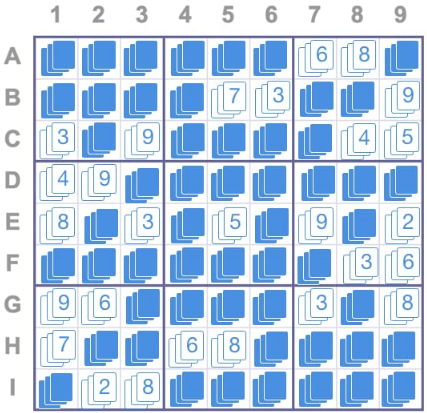
Zero Knowledge Proof for Sudoku



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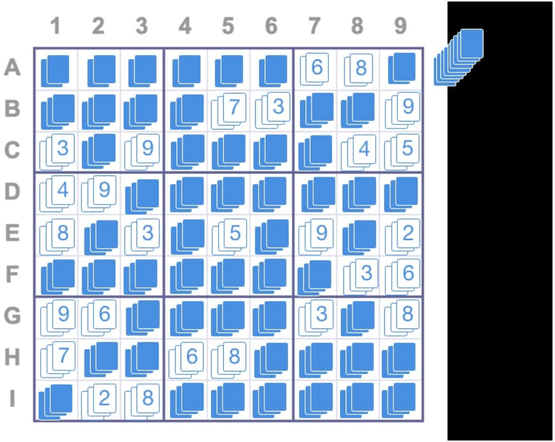
Zero Knowledge Proof for Sudoku



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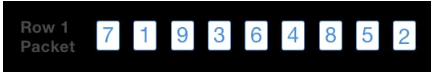
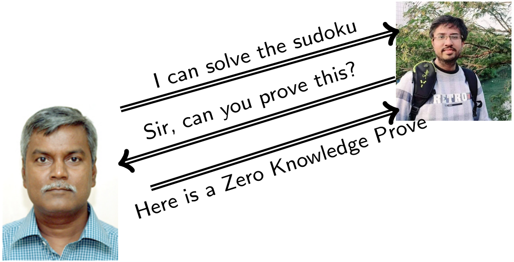
Zero Knowledge Proof for Sudoku



All the cards are shuffled: Zero Knowledge.

Solving the Sudoku



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World Sudoku Championship

Effect of ZKP

Thank You

Hope you have gained some knowledge about "Zero Knowledge"..!!