Cryptography - The Journey Begins

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tcg crest
Inventing Harmonious Future

Cryptography

cryptography *noun*

/krɪp'tɒgrəfi/

/krɪp'tɑːgrəfi/

[uncountable]

- the art of writing or solving codes

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the art of writing or solving codes

Cryptology = Cryptography + Cryptanalysis

The Basic Set up

Alice

Bob

The Basic Set up

Alice

Messages

Bob

The Basic Set up

Eve

Messages

Alice

Bob

Notion of An Adversary

- Passive Adversary: Observes the Message
- Active Adversary: Modifies the Message

Why Cryptography?

My Netflix password is ND180722

Why Cryptography?

My Netflix password is ND180722

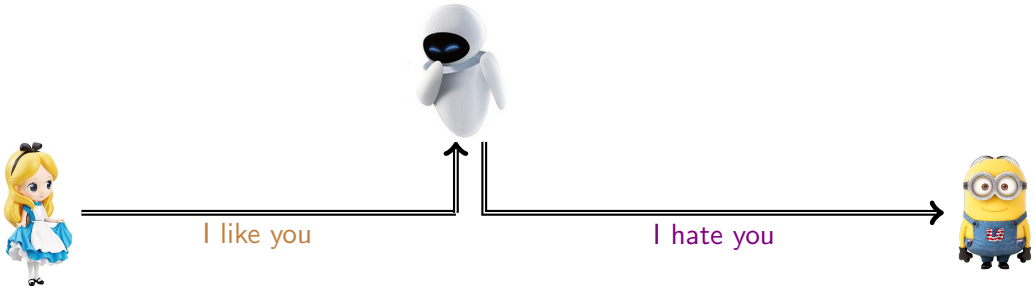
We need Confidentiality

- Bob should read the message.
- Eve should not read the message.

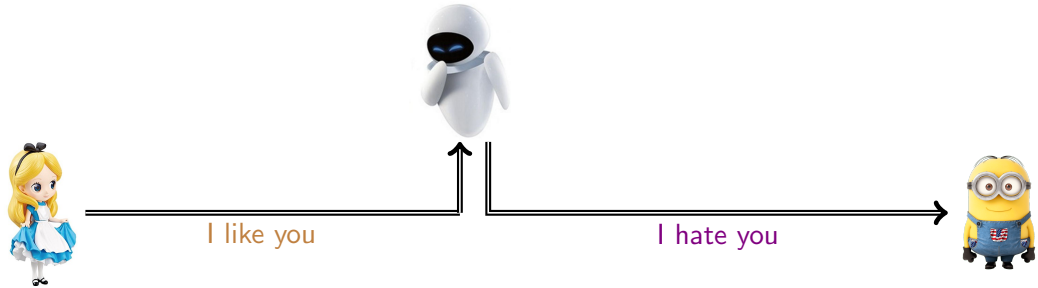
Why Cryptography?

I Like you

Why Cryptography?



Why Cryptography?



We need Authentication/Integrity

Bob should be able to validate the authenticity of the message and the integrity of Alice.

Why Cryptography?

@ * # * *#$$ * # * *@

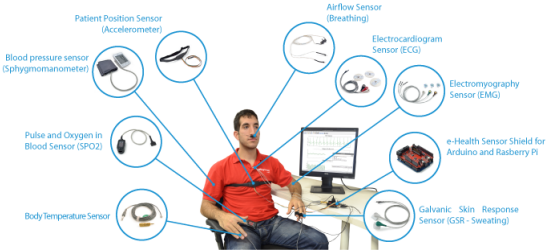
Why Cryptography?

@ ∗ # ∗ ∗ # $$ ∗ # ∗ ∗ @

We need Non-Repudiation

Alice can not deny a message that she sent to Bob.

Real-Life Applications



Cryptography

Symmetric-Key Cryptography

Alice and Bob share a secret key.

Cryptography

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Alice and Bob share a secret key.

Public-Key Cryptography

Each User has a public key and a secret key.

Symmetric-Key Encryption

Message Space: $\mathcal{M}$, Ciphertext space: $\mathcal{C}$, Key-space $\mathcal{K}$.

- Keygen: $K \leftarrow_{\$} \mathcal{K}$.
- Encryption: $C \leftarrow Enc_K(M)$.
- Decryption: $M := Dec_K(C)$.

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Correctness: For any M, K, $Dec_K(Enc_K(M)) = M$.

Kerckhoffs' Principle

- The cryptosystem is known to the adversary.
- But the key is not known to the attacker.
- The secrecy of the cryptosystem lies in the key.

Public-Key Encryption

Receiver's Key: $(R.PK, R.SK)$

- Encryption: $C \leftarrow Enc_{R.PK}(M)$.
- Decryption: $M := Dec_{R.SK}(C)$.

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Correctness: For any $M, (U.PK, U.SK)$, $Dec_{U.SK}(Enc_{U.PK}(M)) = M$.

Historic and Classical Ciphers

The "Art" of Message Hiding

The “Art” of Message Hiding

Ohw wkh Jdph Ehjlq..

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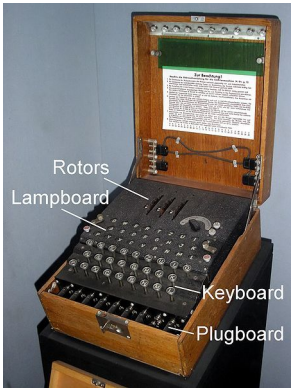
Historic Ciphers: Enigma Machine

https://www.youtube.com/watch?v=ybkkiGtJmkM



Classical Ciphers

Shift Cipher

$\mathcal{M} = \mathcal{C} = \mathcal{K} = \mathbb{Z}_{26}$.

- Keygen: $K \leftarrow_{\$} \mathbb{Z}_{26}$.
- Encryption: $C := (K + M) \mod 26$.
- Decryption: $M := (K + C) \mod 26$.

Shift Cipher - Cryptanalysis

Can you decipher the following text?

JBCRCLQRWCRVNBJENBWRWN,

Shift Cipher - Cryptanalysis

Can you decipher the following text?

JBCRCLQRWCRVNBJENBWRWN,

After applying brute force search:

jbcrclqrwcrvnbjenbwrwn
iabqbkpqvbqumaidmavqvm
hzapajopuaptlzhclzupul
gyzozinotzoskygbkytotk
fxynyhmnsynrjxfajxsnsj
ewxmxglmrxmqiweziwrmri
dvwlwfklqwlphvdyhvqlqh
cuvkvejkpvkogucxgupkpg
btujudijoujnftbwftojof
astitchintimesavesnine

Affine Cipher

$\mathcal{M} = \mathcal{C} = \mathbb{Z}_{26}$, $\mathcal{K} = \mathbb{Z}_{26}^{\star} \times \mathbb{Z}_{26}$.

- Keygen: $(K_1, K_2) \leftarrow_{\$} \mathbb{Z}_{26}^{\star} \times \mathbb{Z}_{26}$.
- Encryption: $C := (MK_1 + K_2) \mod 26$.
- Decryption: $M := K_1^{-1}(C - K_2) \mod 26$.

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How to cryptanalyze affine cipher?

Substitution Cipher

$\mathcal{M} = \mathcal{C} = \mathbb{Z}_{26}$, $\mathcal{K} = \mathbb{P}(26)$.

- Keygen: $\pi \leftarrow_{\$} \mathbb{P}(26)$.
- Encryption: $C := \pi(M)$.
- Decryption: $M := \pi^{-1}(C) \mod 26$.

Cryptanalysis of Substitution Cipher

Can you decipher the following text?

```
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJ
NDIFEFMDZCDMQZKCEYFCJMYRNCWJCSZREXCHZUNMXZ
NZUCDRJXYYSMRTMEYIFZWDYVZVYFZUMRZCRWNZDZJJ
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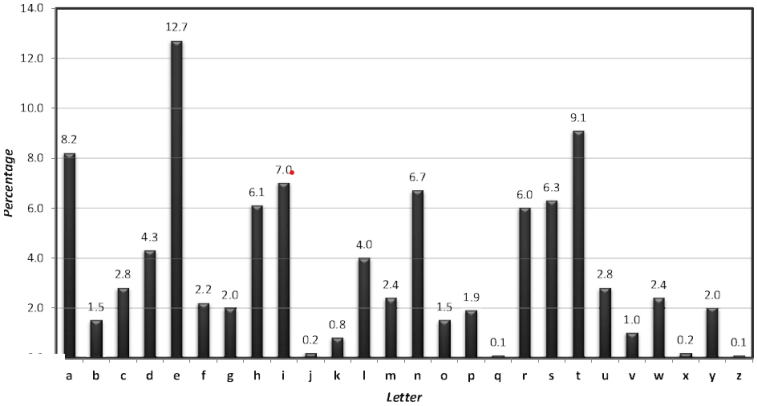
```

How about applying a brute-force search?

Cryptanalysis of Substitution Cipher

Cryptanalysis of Substitution Cipher

Use Frequency Table:



Cryptanalysis of Substitution Cipher: Frequency Table

```
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJ
NDIFEFMDZCDMQZKCEYFCJMYRNCWJCSZREXCHZUNMXZ
NZUCDRJXYYSMRTMEYIFZWDYVZVYFZUMRZCRWNZDZJJ
XZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJNZDIR
```

letter	frequency	letter	frequency
A	0	N	9
B	1	O	0
C	15	P	1
D	13	Q	4
E	7	R	10
F	11	S	3
G	1	T	2
H	4	U	5
I	5	V	5
J	11	W	8
K	1	X	6
L	0	Y	10
M	16	Z	20

Cryptanalysis of Substitution Cipher

- Z occurs 20 times: $d_K(Z) = e$

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- C, D, F, J, M, R, Y might be encryptions of t, a, o, i, n, s, h, r, but difficult to predict.

Cryptanalysis of Substitution Cipher: Frequency Table

Look at di-grams:

$$TH, HE, IN, ER, AN, RE, ED, ON, ES, ST,$$
$$EN, AT, TO, NT, HA, ND, OU, EA, NG, AS,$$
$$OR, TI, IS, ET, IT, AR, TE, SE, HI, OF.$$

Cryptanalysis of Substitution Cipher: Frequency Table

Look at di-grams:

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Also look at tri-grams:

$$THE, ING, AND, HER, ERE, ENT,$$
$$THA, NTH, WAS, ETH, FOR, DTH.$$

Cryptanalysis of Substitution Cipher: Frequency of Occurances

```
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJ
NDIFEFMDZCDMQZKCEYFCJMYRNCWJCSZREXCHZUNMXZ
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- Look at digrams of the form $\_Z$ and $Z\_$.

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- $ZW : 4$, $WZ : 0$, W less frequent: $d_K(W) = d$
- $DZ : 4$, $ZD : 2$: $d_K(D) \in \{r, s, t\}$

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- Z occurs 20 times: $d_K(Z) = e$
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- Look at digrams of the form $\_Z$ and $Z\_$.
- $ZW : 4$, $WZ : 0$, W less frequent: $d_K(W) = d$
- $DZ : 4$, $ZD : 2$: $d_K(D) \in \{r, s, t\}$
- $ZRW : 1$, $RW : 2$, R frequent: $d_K(R) = n$

Cryptanalysis of Substitution Cipher

```
------end---------e----ned---e-----------
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJ

--------e----e---------n--d---en----e----e
NDIFEFMDZCDMQZKCEYFCJMYRNCWJCSZREXCHZUNMXZ

-e---n------n------ed---e---e--ne-nd-e-e--
NZUCDRJXYYSMRTMEYIFZWDYVZVYFZUMRZCRWNZDZJJ

-ed-----n-----------e----ed-------d---e--n
XZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJNZDIR
```

Cryptanalysis of Substitution Cipher

```
o-r-riend-ro--arise-a-inedhise--t---ass-it
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJ

hs-r-riseasi-e-a-orationhadta-en--ace-hi-e
NDIFEFMDZCDMQZKCEYFCJMYRNCWJCSZREXCHZUNMXZ

he-asnt-oo-in-i-o-redso-e-ore-ineandhesett
NZUCDRJXYYSMRTMEYIFZWDYVZVYFZUMRZCRWNZDZJJ

-ed-ac-inhischair-aceti-ted--to-ardsthes-n
XZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJNZDIR
```

Cryptanalysis of Substitution Cipher

Our friend from Paris examined his empty glass with surprise, as if evaporation had taken place while he wasn't looking. I poured some more wine and he settled back in his chair, face tilted up towards the sun.[1]

Vigenere Cipher

$\mathcal{M} = \mathcal{C} = \mathcal{K} = \mathbb{Z}_{26}^m$.

- Keygen: $K \leftarrow_\$ \mathbb{Z}_{26}^m$.
- Encryption: $C := (M + K) \mod 26$.
- Decryption: $M := (C + K) \mod 26$.

Poly-Alphabetic Cipher

Cryptanalysis of Vigenere Cipher

```
CHREEVOAHMAERATBIAXXWTNXBEEOPHBSBQMQEQERBW
RVXUOAKXAOSXXWEAHBWGJMMQMNKGRFVGXWTRZXWIAK
LXFPSKAUTEMNDCMGTSXMXBTUIADNGMGPSRELXNJELX
VRVPRTULHDNQWTWDTYGBPHXTFALJHASVBFXNGLLCHR
ZBWELEKMSJIKNBHWRJGNMGJSGLXFEYPHAGNRBIEQJT
AMRVLCRREMNDGLXRRIMGNSNRWCHRQHAEYEVTAQEBBI
PEEWEVKAKOEWADREMXMTBHHCHRTKDNVRZCHRCLQOHP
WQAIIWXNRMGWOIIFKEE
```

Cryptanalysis of Vigenere Cipher

- Given m, how can you decrypt?

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Can you use the following result?

- Index of coincidence: $IC = \Sigma_{i=1}^{n} \frac{f_i^2}{2^n}$, f_i freq of i-th letter.
- For a set of 26 random elements, $IC = \frac{1}{26} = 0.038$
- However, for English dictionary $IC = 0.065$

Cryptanalysis of Vigenere Cipher

- For the given ciphertext let $q_1, \ldots, q_{26}$ be the freq of i-th letter.
- Calculate $I_j = \Sigma_{i=1}^{26} p_i q_{i+j}$, for $j = 0, \ldots, 25$.
- Find k for which I_k is close to 0.065 and report that as the key.

Towards Modern Cryptography

Principles of Modern Cryptography

Ad-hoc Design

Provably Secure Design

Principles of Modern Cryptography

- Formal Definition (Power and Goal of the adversary)
- Precise Assumption
- Proofs of Security

Perfect Secrecy

Eve obtains no information about plaintext:

Definition

$$\Pr[\mathsf{M} = M \mid \mathsf{C} = C] = \Pr[\mathsf{M} = M], \ \forall M \in \mathcal{M}, \ C \in \mathcal{C}.$$

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An Alternate Definition

$$\Pr[Enc(M_0, K) = C] = \Pr[Enc(M_1, K) = C],\ \forall M_0, M_1 \in \mathcal{M},\ C \in \mathcal{C}.$$

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Perfect Indistinguishability

- Adv chooses two messages M_0 and M_1 of same length
- Challenger chooses b and returns $c = Enc(M_b, K)$
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Perfect Indistinguishable $\Rightarrow$ Pr[Adv wins] $= 1/2$.

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Theorem

Π achieves Perfect Secrecy $\Leftrightarrow$ Π is Perfectly Indistinguishable

Perfect Secrecy: Existence

One Time Pad

$\mathcal{M} = \mathcal{C} = \mathcal{K} = \{0,1\}^{\ell}$

$$Enc_K(M) = M \oplus K.$$

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If $\Pi :=$(KeyGen, Encryption, Decryption) is a perfectly secret encryption scheme with message space $\mathcal{M}$ and key space $\mathcal{K}$, then $|\mathcal{K}| \geq |\mathcal{M}|$.

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Proof Sketch

- $\mathcal{M}_C = \{M : Dec_K(C) = M, \text{ for some } K.\}$

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- $|\mathcal{M}_C| \leq |\mathcal{K}|$
- If $|\mathcal{K}| < |\mathcal{M}|$, there exists some M' such that $M' \in \mathcal{M}$, but $M' \notin \mathcal{M}(\mathcal{C})$ - Violates Perfect Security..!!

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Theorem

If $\Pi :=$(KeyGen, Encryption, Decryption) is a perfectly secret encryption scheme with message space $\mathcal{M}$ and key space $\mathcal{K}$, then $|\mathcal{K}| \geq |\mathcal{M}|$.

Computational Security

Perfect Security is too strong

- Absolutely no information about an encrypted message is leaked.
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How about a scheme that leaks information with

- probability at most 2^{-60} to adversaries
- investing up to 200 years of computational effort on the fastest available supercomputer?

Computational Security

A scheme is (t, ϵ)-secure if any adversary

- running for time at most t
- succeeds in breaking the scheme with probability at most ϵ.

Computational Indistinguishability

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IND-CPA Security

Adv is given the power to make encryption queries except M_0 and M_1.

Designing A Provably Secure Encryption Construction

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- Formal Definition (✓)
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  - Notions of Pseudo-Random Generator (PRG), Pseudo-Random Function (PRF)
  - Assume such function exists
- Proofs of Security
  - Encryption Scheme based on PRG G and PRF F.
  - Reduction-based Proofs.

References

- Introduction to Modern Cryptography by *Jonathan Katz* and *Yehuda Lindell* (✓)
- Cryptography - Theory and Practice by *Douglas Stinson*
- A Graduate Course in Applied Cryptography by *Dan Boneh* and *Victor Shoup*

Thank You..!!!