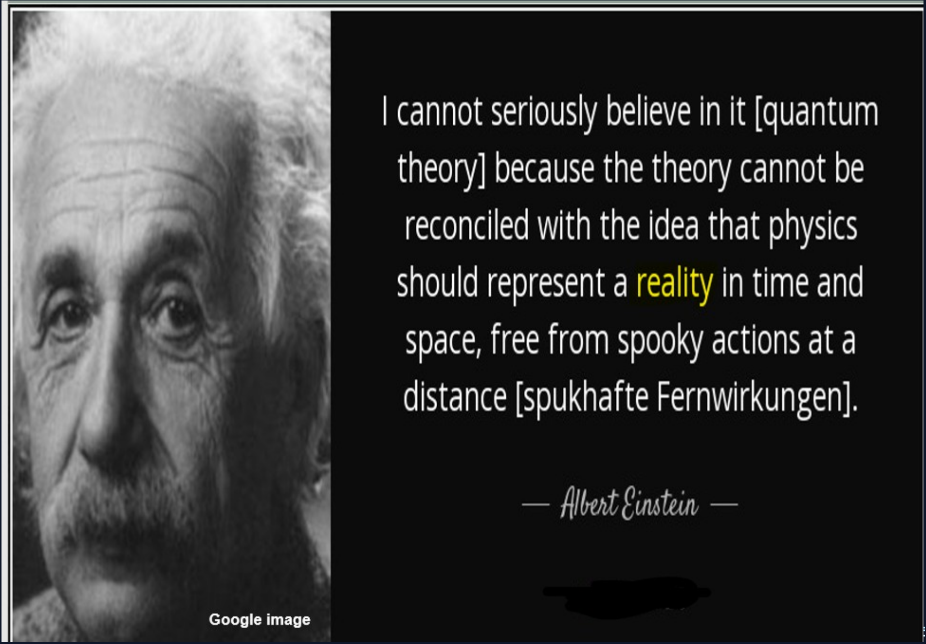
Pushing non-macrorealism to its extreme limits:
Temporal correlations beyond the quantum bound

A R Usha Devi

Bangalore University

December 12, 2025


I cannot seriously believe in it [quantum theory] because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance [spukhafte Fernwirkungen].
— Albert Einstein —
[redacted]
Google image

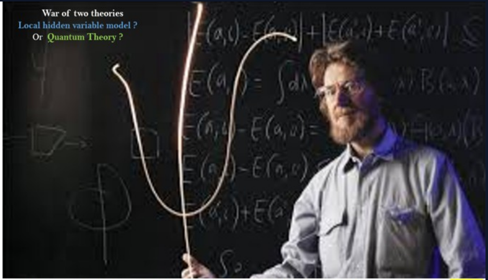
Einstein-Podolsky-Rosen Paradox and Non-locality

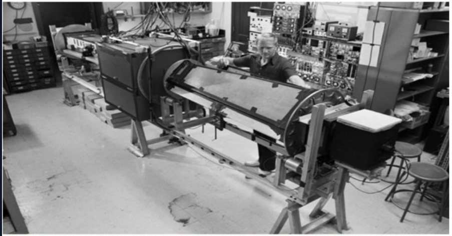
MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.





John Clauser works on his experiment to evaluate Bell inequalities. Credit: Steve Gerber, courtesy of Berkeley Lab

Physicists claim 'loophole-free' Bell-violation experiment 2015

02 Sep 2015 Hamish Johnston

Bell-CHSH Correlation Inequality:

$$|\langle A_1 B_1\rangle + \langle A_1 B_2\rangle + \langle A_2 B_1\rangle - \langle A_2 B_2\rangle| \leq 2$$

Quantum mechanical violation:

$$|\langle A_1 B_1\rangle + \langle A_1 B_2\rangle + \langle A_2 B_1\rangle - \langle A_2 B_2\rangle|^{\max}_{\mathrm{QM}} = 2\sqrt{2}.$$

Logically, Bell's inequality states that any physical theory that incorporates local realism cannot reproduce all the predictions of quantum mechanical theory.

Quantum Tsirelson Bound

Algebraic Maximum of CHSH: 4 → Generalized Probability Theory (beyond quantum theory

I cannot define the real problem,

Physics Today, April 1985, pp 38-47

therefore I suspect there's no real problem,
but I'm not sure
there's no real problem.

These are a few lines of a poem
by David Mermin
Physics Today, April 1985

What is predicted by quantum formalism must occur in laboratory
- Asher Peres

ME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

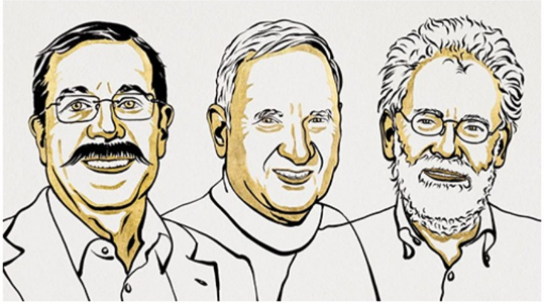
We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

Nature, 526 (7575), 682-6 2015 Oct 29

Loophole-free Bell Inequality Violation Using Electron Spins Separated by 1.3 Kilometres

B Hensen [1 2], H Bernien [1 2], A E Dréau [1 2], A Reiserer [1 2], N Kalb [1 2], M S Blok [1 2], J Ruitenberg [1 2], R F L Vermeulen [1 2], R N Schouten [1 2], C Abellán [3], W Amaya [3], V Pruneri [3 4], M W Mitchell [3 4], M Markham [5], D J Twitchen [5], D Elkouss [1], S Wehner [1], T H Taminiau [1 2], R Hanson [1 2]
Affiliations + expand
PMID: 26503041 DOI: 10.1038/nature15759

Nobel Prize in Physics 2022



Credit: Niklas Elmehed © Nobel Prize Outreach

Alain Aspect
Institut d'Optique Graduate School – Université Paris-Saclay and École Polytechnique, Palaiseau, France

John F. Clauser
J.F. Clauser & Assoc., Walnut Creek, CA, USA

Anton Zeilinger
University of Vienna, Austria

"For experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

- **Non-classical features play a key role in the modern field of quantum computation and quantum communication.**

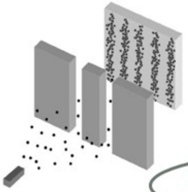
- **These early foundational conflicts/paradoxes of quantum theory paved way to new leaps of precision measurements, thus creating a firm platform for the modern interdisciplinary area of quantum information science.**

R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Quantum Entanglement*
REVIEWS OF MODERN PHYSICS, VOL. 81, pp. 865 – 942, 2009.

Macro-realism

When and how do physical systems stop behaving *quantumly* and begin to behave *classically*?

How do we *see macroscopic realm* emerging from the quantum world in an experimentally test?



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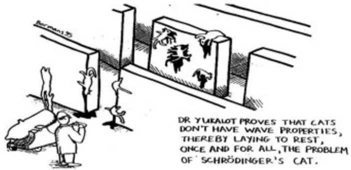
In the macroscopic realm do superpositions survive?



What is Macrorealism?

- **Realism:** Properties exist before measurement
- **Noninvasive Measurability:** Measurements do not disturb the system

Leggett-Garg (1985)

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985)

Sir Anthony James Leggett

Prof. Anupam Garg

Macrorealism

Macrorealism per se — ``Physical properties of a macroscopic object exist independent of the act of observation"

Non-invasive measurability — "*The measurement of an observable at any instant of time does not influence its subsequent evolution*"

The Leggett-Garg Framework

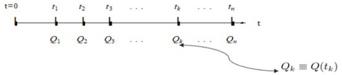
Tests macrorealism through **temporal correlations** in a single system measured at successive times.

- Analogous to Bell inequalities (but for **time**, not space)
- Places bounds on temporal correlations
- Violations indicate **non-macrorealistic behavior**

Leggett-Garg Correlation Inequality (Temporal Bell inequality)

Consider a *dynamic* system with a *dichotomic* quantity Q(t)

Dichotomic ⟹ $Q(t) = \pm 1$ at any given time



A. J. Leggett and A. Garg, PRL 54, 857 (1985)

Two-Time Correlation Coefficient

Temporal Correlation: $C_{ij} = \langle Q(t_i)Q(t_j)\rangle \equiv \langle Q_i Q_j\rangle$

$$\left.\begin{array}{lcl} C_{ij} & = & +1 \longrightarrow \text{perfect correlation} \\ C_{ij} & = & -1 \longrightarrow \text{perfect anticorrelation} \\ C_{ij} & = & 0 \longrightarrow \text{No correlation} \end{array}\right\} \Longrightarrow -1 \le C_{ij} \le +1$$

LG correlation inequality with 3 measurements

Define
$$\begin{aligned} K_3 &= C_{12} + C_{23} - C_{13} \\ &= \langle Q_1Q_2\rangle + \langle Q_2Q_3\rangle - \langle Q_1Q_3\rangle \end{aligned}$$

Notice that

When $Q_1 = Q_2$, $\quad Q_1Q_2 + (Q_2 - Q_1)Q_3 = +1$

When $Q_1 \neq Q_2$, $\quad Q_1Q_2 + (Q_2 - Q_1)Q_3 = -1 + (\pm 2) = +1 \text{ or } -3$

$$\Downarrow$$

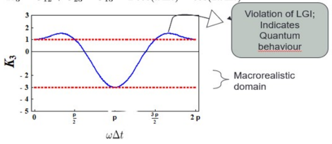
$$-3 \le \langle Q_1Q_2\rangle + \langle Q_2Q_3\rangle - \langle Q_1Q_3\rangle \le 1$$

$$\Downarrow$$

Leggett-Garg Inequality (LGI): $-3 \le K_3 \le 1$

LGI violation

$K_3 = C_{12} + C_{23} - C_{13} = 2\cos(\omega\Delta t) - \cos(2\omega\Delta t)$



Three-Time Measurement Protocol

Measure dichotomic observable $Q(t) = \pm 1$ at three successive times: t_1, t_2, t_3
$K_3 \equiv C_{21} + C_{32} - C_{31} = \langle Q_1 Q_2 \rangle + \langle Q_2 Q_3 \rangle - \langle Q_1 Q_3 \rangle$

Macrorealistic bound on 3-term LGI:

$$-3 \leq K_3 \equiv C_{21} + C_{32} - C_{31} \leq 1$$

Temporal Tsirelson bound on a three term termporal correlations:

$$K_3^Q \leq 1.5$$

- Classical macrorealism: $-3 \leq K_3 \leq 1$
- Quantum evolution: $K_3^Q \leq 1.5$

PRL 107, 130402 (2011) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2011

Investigation of the Leggett-Garg Inequality for Precessing Nuclear Spins

Vikram Athalye,[1,*] Soumya Singha Roy,[2,†] and T. S. Mahesh[2,‡]
[1]*Department of Applied Physics, Cummins College of Engineering, Karvenagar, Pune 411052, India*
[2]*NMR Research Center, Indian Institute of Science Education and Research, Pune 411008, India*
(Received 12 February 2011; published 19 September 2011)

We report experimental implementation of a protocol for testing the Leggett-Garg inequality (LGI) for nuclear spins precessing in an external magnetic field. The implementation involves certain controlled operations, performed in parallel on pairs of spin-1/2 nuclei (target and probe) from molecules of a nuclear magnetic resonance ensemble, which enable evaluation of temporal correlations from an LG string. Our experiment demonstrates violation of the LGI for time intervals between successive measurements, over which the effects of relaxation on the quantum state of target spin are negligible. Further, it is observed that the temporal correlations decay, and the same target spin appears to display macrorealistic behavior consistent with LGI.

nature COMMUNICATIONS

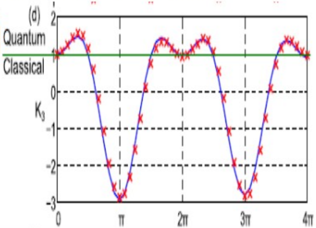
ARTICLE

Received 26 May 2011 | Accepted 24 Nov 2011 | Published 3 Jan 2012 DOI: 10.1038/ncomms1614

Violation of a Leggett–Garg inequality with ideal non-invasive measurements

George C. Knee[1], Stephanie Simmons[1], Erik M. Gauger[1,2], John J.L. Morton[1,3], Helge Riemann[4], Nikolai V. Abrosimov[4], Peter Becker[5], Hans-Joachim Pohl[6], Kohei M. Itoh[7], Mike L.W. Thewalt[8], G. Andrew D. Briggs[1] & Simon C. Benjamin[1,2]

The quantum superposition principle states that an entity can exist in two different states simultaneously, counter to our 'classical' intuition. Is it possible to understand a given system's behaviour without such a concept? A test designed by Leggett and Garg can rule out this possibility. The test, originally intended for macroscopic objects, has been implemented in various systems. However to date no experiment has employed the 'ideal negative result' measurements that are required for the most robust test. Here we introduce a general protocol for these special measurements using an ancillary system, which acts as a local measuring device but which need not be perfectly prepared. We report an experimental realization using spin-bearing phosphorus impurities in silicon. The results demonstrate the necessity of a non-classical picture for this class of microscopic system. Our procedure can be applied to systems of any size, whether individually controlled or in a spatial ensemble.



REVIEW ARTICLE

Leggett–Garg inequalities

Clive Emary[1], Neill Lambert[2] and Franco Nori[2,3]
Published 23 December 2013 • 2014 IOP Publishing Ltd
Reports on Progress in Physics, Volume 77, Number 1

Physical principles translate into limits on correlations

- Antonio Acin

- LGI imposes bounds on the linear combination of two-time correlations between consecutive measurements of a dichotomic observable of a system respecting macrorealism.
- Bounding the temporal correlations for LGI:
  - T. Fritz, New Journal of Physics 12, 083055 (2010).
  - C. Budroni, T. Moroder, M. Kleinmann, and O. Gühne, Phys. Rev. Lett. 111, 020403 (2013).

PRL 111, 020403 (2013) PHYSICAL REVIEW LETTERS week ending 12 JULY 2013

Bounding Temporal Quantum Correlations

Costantino Budroni, Tobias Moroder, Matthias Kleinmann, and Otfried Gühne
Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Walter-Flex-Straße 3, D-57068 Siegen, Germany
(Received 15 March 2013; published 10 July 2013)

Sequential measurements on a single particle play an important role in fundamental tests of quantum mechanics. We provide a general method to analyze temporal quantum correlations, which allows us to compute the maximal correlations for sequential measurements in quantum mechanics. As an application, we present the full characterization of temporal correlations in the simplest Leggett-Garg scenario and in the sequential measurement scenario associated with the most fundamental proof of the Kochen-Specker theorem.

DOI: 10.1103/PhysRevLett.111.020403 PACS numbers: 03.65.Ud, 03.65.Ta

- **Macrorealistic bound on N-term LGI:**

$$S_N = \sum_{i=1}^{N-1} (C_{i+1\,i} - C_{N\,1}) \leq N-2,$$

where $C_{j\,i} = \langle Q_j\, Q_i\rangle$.

- **Quantum Tsirelson Bound:**

$$S_N^Q \leq N\,\cos\left(\frac{\pi}{N}\right)$$

The Central Question

How far can temporal correlations extend before they fundamentally overturn the notion of **macrorealism**?

- Classical physics assumes **definite properties** at all times
- Quantum mechanics violates this through **temporal superpositions**
- Can we reach the **algebraic maximum** violations?

Beyond temporal Tsirelson Bound (TTB)

The maximum allowed violation of 3-term LGI under **standard unitary evolution**:

$$K_3^{\text{max}} = 1.5$$

But can quantum mechanics exceed this? **Yes**— under unconventional scenarios!

Algebraic Maximum

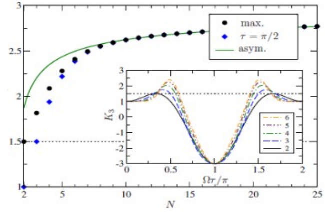
The **absolute upper limit** of the LGI parameter K_3 without any physical constraints:

$$K_3^{\mathrm{max}} = 3$$

Traditional quantum evolution reaches only 1.5. Can we get closer to 3?

Approaching algebraic maximum bound of LGI

Violation of 3-term LGI by temporal correlations in N-level systems.



PRL 113, 050401 (2014) PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2014

Temporal Quantum Correlations and Leggett-Garg Inequalities in Multilevel Systems

Costantino Budroni[1] and Clive Emary[2]
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[2]Department of Physics and Mathematics, University of Hull, Kingston-upon-Hull HU6 7RX, United Kingdom
(Received 14 September 2013; published 29 July 2014)

We show that the quantum bound for temporal correlations in a Leggett-Garg test, analogous to the Tsirelson bound for spatial correlations in a Bell test, strongly depends on the number of levels N that can be accessed by the measurement apparatus via projective measurements. We provide exact bounds for small N that exceed the known bound for the Leggett-Garg inequality, and we show that in the limit $N \to \infty$ the Leggett-Garg inequality can be violated up to its algebraic maximum.

SDP		Maximization								
M	K_3^{max}	M	N	K_3^{max}	M	N	K_3^{max}	M	N	K_3^{max}
2	$\frac{3}{2}$	3	3	2.1547	4	4	2.3693	5	5	2.5166
3	2.211507	3	4	2.1736	4	5	2.3877	5	6	2.5312
4	2.454629	3	5	2.2115	4	6	2.4181	5	7	2.5459
5	2.579333	3	6	2.2115	4	7	2.4315	5	8	2.5506
6	2.656005	3	7	2.2115	4	8	2.4545	5	9	2.5545

- Measurement state update rule: Use measurements that project the N level state into one of the $2 \leq M < N$ subspaces; retain dichotomic outcomes $Q = \pm 1$.
- Choice of $2 \leq M < N$ determines different state update rules.
- Enhanced violation of LGI is observed:

$$(K_3)_{\max} \to 3, \quad N \to \infty.$$

Pushing K_3 Beyond TTB in a two-level system

Classical Limit

$K_3 \leq 1$ *Macrorealism*

Standard Quantum Limit

$K_3 \leq 1.5$ *Temporal Tsirelsen Bound*

New Frontier: Non-unitary evolution and superposed unitaries can exceed TTB in a two-level system, approaching the algebraic maximum $K_3^{\max} = 3$

Two Different Scenarios Explored

(i) **PT-Symmetric Non-Hermitian Dynamics**

- Parity-Time symmetric Hamiltonian evolution (Existence of exceptional point)

(ii) **Superposition of Unitaries**

- Coherent superposition of different unitary operations (Time recorded by a melting clock)

Part I:
PT-Symmetric Non-Hermitian Dynamics

Parity-Time (PT) Symmetric Quantum Dynamics

Replace Hermiticity Requirement: $H = H^\dagger$

With the parity-time (PT) symmetry condition: $H = H^{\mathcal{PT}}$

- A non-Hermitian Hamiltonian has **real eigenvalues** if and only if it is **PT symmetric** i.e., if it commutes with the combined PT operation
- **PT symmetric Hamiltonian** shares common eigenvectors with PT operation up to an *exceptional point* or the PT symmetry breaking point

What is PT Symmetry?

- **P (Parity):** Spatial coordinate reversal ($x \to -x$)
- **T (Time):** Time reversal ($t \to -t$)
- A Hamiltonian is PT-symmetric if: $[H, \mathcal{PT}] = 0$
- Not necessarily Hermitian!

C. M. Bender and S. Boettcher, "Real Spectra in non-Hermitian Hamiltonians having PT symmetry," Phys. Rev. Lett. 80, 5243 (1998).

C. M. Bender, "Making sense of non-Hermitian Hamiltonians," Rep. Prog. Phys. 70, 9471018 (2007).

Non-Hermitian Hamiltonians

Traditionally, Hermiticity ensures real eigenvalues and conservation of probability.
PT-symmetric Hamiltonians can violate Hermiticity while maintaining real spectra!

- Real eigenvalues in **unbroken PT-symmetric phase**
- Complex eigenvalues at **exceptional points**
- Enable exotic quantum phenomena

C. M. Bender and S. Boettcher, "Real Spectra in non-Hermitian Hamiltonians having PT symmetry," Phys. Rev. Lett. 80, 5243 (1998).

C. M. Bender, "Making sense of non-Hermitian Hamiltonians," Rep. Prog. Phys. 70, 9471018 (2007).

PT Symmetric Dynamics of a Two-Level System

PT symmetric Hamiltonian:

$$H = s \begin{pmatrix} i \sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix} s, \quad \alpha \in \mathbb{R}, \quad s \neq 0$$

The non-Hermitian Hamiltonian commutes with the PT operator:

$$\mathcal{P} \to \sigma_x, \quad \mathcal{T} : i \to -i$$
$$\Rightarrow [H, \mathcal{PT}] = 0$$

Though the Hamiltonian is non-Hermitian for $\alpha \neq 0$, it possesses real eigenvalues (by virtue of PT symmetry):

$$E_{\pm} = \pm s \cos\alpha$$

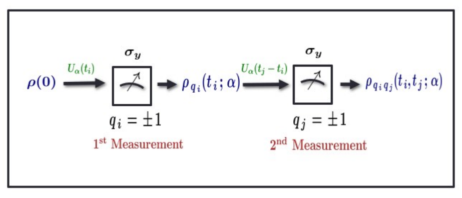
Non-Hermitian but real spectrum!

Qubit Undergoing PT Symmetric Dynamics

Density matrix evolution:

$$\rho(0) \xrightarrow{U_\alpha(t)} \rho(t;\alpha) = \frac{U_\alpha(t)\rho(0)U_\alpha^\dagger(t)}{\mathrm{Tr}\left[U_\alpha(t)\rho(0)U_\alpha^\dagger(t)\right]}$$

Two-time correlations of the observable $Q = \sigma_y$:



Temporal correlations under PT symmetric dynamics

Non-unitary time evolution generated by the PT symmetric Hamiltonian:

$$U_\alpha(t) = e^{-itH} = \frac{1}{\cos\alpha}\begin{pmatrix} \cos(t'-\alpha) & -i\sin t' \\ -i\sin t' & \cos(t'+\alpha) \end{pmatrix}$$

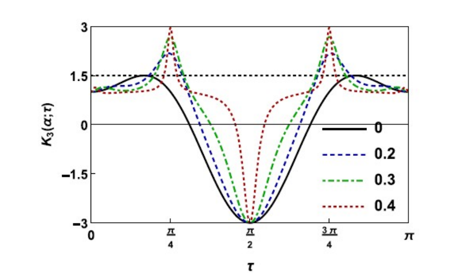
where $t' = \frac{\Delta E}{2} t$ and $\Delta E = E_+ - E_- = 2s\cos\alpha$ **Temporal correlations:**

$$\langle Q(t_i)Q(t_j)\rangle = \sum_{q_i,q_j} q_i q_j \cdot p(q_j, t_j | q_i, t_i; \alpha)$$

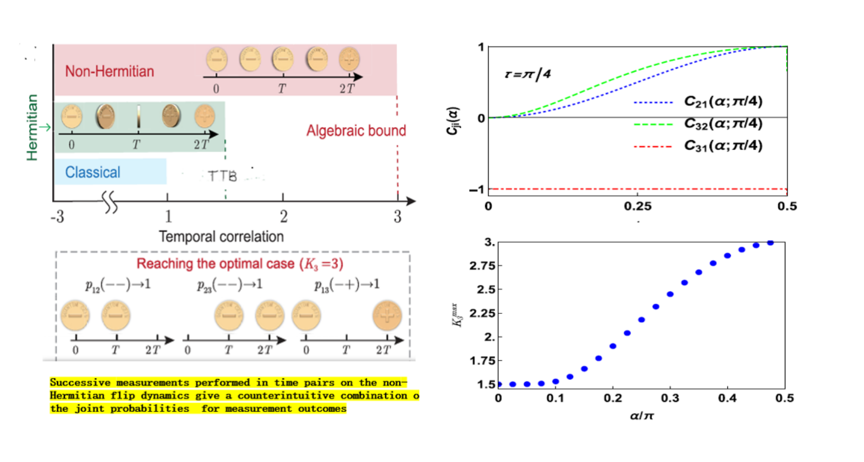
$$p(q_j, t_j | q_i, t_i; \alpha) = \frac{\mathrm{Tr}\left[U_\alpha(t_j - t_i)\rho_{q_i}(t_i;\alpha)U_\alpha^\dagger(t_j - t_i)\Pi_{q_j}\right]}{p(q_i, t_i; \alpha)}$$

Beyond TTB violation of 3-term LGI under PT symmetric evolution

- H. S. Karthik, H. Akshata Shenoy, and A. R. Usha Devi, *Leggett-Garg inequalities and temporal correlations for a qubit under PT -symmetric dynamics*, Phys. Rev. A **103**, 032420 (2021)
- A. V. Varma, I. Mohanty, and S. Das, Temporal correlation beyond quantum bounds in non-Hermitian PT-symmetric dynamics of a two level system, J. Phys. A **54**, 115301 (2021).



Strange combination of joint probabilities



PT symmetry goes quantum

Stefan Rotter, May 30, 2018, Physics **11**, 54

- A proposed microwave circuit would allow exploration of the quantum side of parity-time symmetry, which, in classical devices, gives rise to effects like one-way or stopped light (Phys. Rev. A 97, 053846 (2018)).

Physics

VIEWPOINT

PT Symmetry Goes Quantum

A proposed microwave circuit would allow exploration of the quantum side of parity-time symmetry, which, in classical devices, gives rise to effects like one-way or stopped light.

by Stefan Rotter*

No physicist would tamper with the conservation of energy—the fundamental law that says energy cannot be created or destroyed. Researchers have, however, taken an interest in devices whose energy is conserved somewhat artificially. Known as *PT*-symmetric systems [1], these devices are engineered to have a balance of energy flowing in and out. And they feature many unusual properties, which have, for example, been harnessed to make optical components that only allow light

the components feeding energy in (the gain) with components allowing energy to escape (the loss). The second is time reversal, which is akin to running a "movie" of the device backwards. A simple example of a *PT*-symmetric system is a stick that's being cooled on one end and heated on the other, both at exactly the same rate (Fig. 1, left). If you swap the sources performing the heating and cooling and then time reverse these processes, the system looks exactly the same.

Researchers were initially interested in whether *PT* sym-

Experimental PT Implementation

- **Photonic systems with gain and loss** (L. Xiao, K. Wang, X. Zhan, Z. Bian, K. Kawabata, M. Ueda, W. Yi, , and P. Xue, Phys. Rev. Lett. **123**, 190401 (2019))
- **Trapped ion systems** (L. Ding et al. Experimental determination of PT-symmetric exceptional points in a single trapped ion. Phys. Rev. Lett. **126**, 083604 (2021))
- **Nitrogen vacancy centers in diamond** (Y. Wu et al. Observation of parity-time symmetry breaking in a single-spin system. Science **364**, 878–880 (2019))
- **Superconducting circuits** (M. Naghiloo, M. Abbasi, Y. N. Joglekar, & K. W. Murch, Quantum state tomography across the exceptional point in a single dissipative qubit. Nat. Phys.**15**, 1232–1236 (2019))
- **Cavity QED with engineered dissipation** (F. Quijandría, U. Naether, S. K. Ozdemir, F. Nori, & D. Zueco, PT-symmetric circuit QED. Phys. Rev. A **97**, 053846 (2018))

PT symmetric dynamics of a trapped ion

Phys. Rev. Res. **7**, 013058 (2025)

PHYSICAL REVIEW RESEARCH **7**, 013058 (2025)

Observation of quantum temporal correlations well beyond Lüders bound

Chun-Wang Wu,[1,2] Man-Chao Zhang,[1,2] Yan-Li Zhou,[1,2] Ting Chen,[1,2] Ran Huang,[3] Yi Xie,[1,2] Wen-bo Su,[1,2] Bao-Quan Ou,[1,2] Wei Wu,[1,2,4] Adam Miranowicz,[3,5] Franco Nori,[3] Jie Zhang,[1,2,*] Hui Jing,[6,†] and Ping-Xing Chen[1,2,4,‡]

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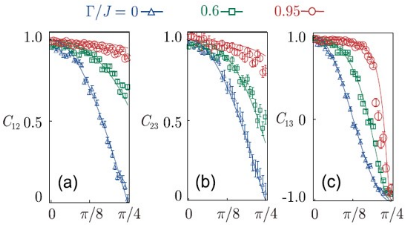
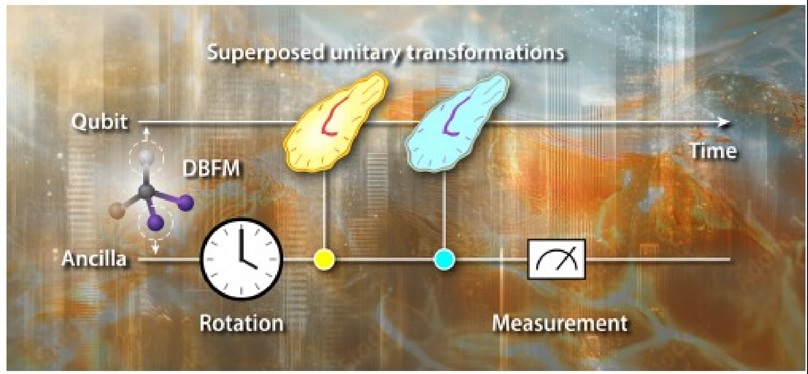
The ability of achieving strong quantum spatial correlations has helped the emergence of quantum information science. In contrast, how to achieve strong quantum temporal correlations (QTCs) has remained as a long-standing challenge, thus hindering their applications in time-domain quantum control. Here we experimentally demonstrate that by using a parity-time ($\mathcal{PT}$)-symmetric single ion, the conventional QTC limit known as the Lüders bound can be well surpassed within a standard measurement scenario, approaching the predicted maximum QTC value. Our work, as a step toward quantum engineering of $\mathcal{PT}$ devices in the time domain, can stimulate more efforts on operating various quantum devices with the aid of strong QTC resources.

$$\hat{H}_{\rm PT} = J\hat{\sigma}_x + \mathrm{i}\Gamma\hat{\sigma}_z$$

$J > \Gamma$ PT-symmetric region



Part II:
Superposition of Unitaries



Extending Superposition to Dynamics

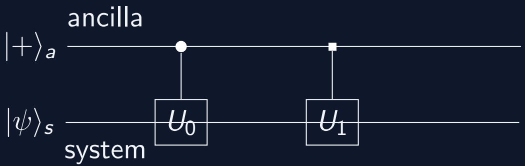
If superposition of **states** leads to non-classical behavior, what about superposition of **unitary operations**?

System evolves under:

$$U = \alpha U_0 + \beta U_1$$

How to Realize It?

Introduce an ancillary qubit in superposition to conditionally apply different unitaries.



Realizing Superposition of Unitaries

Initial state:

$$|\Psi_0\rangle = |+\rangle_a \otimes |\psi\rangle_s = \frac{1}{\sqrt{2}}\left(|0\rangle_a + |1\rangle_a\right) \otimes |\psi\rangle_s$$

Evolution: $\mathcal{U} = |0\rangle\langle 0| \otimes U_0 + |1\rangle\langle 1| \otimes U_1$

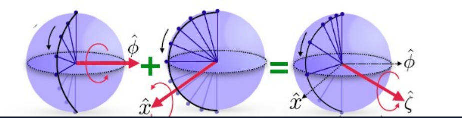
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle_a \otimes U_0|\psi\rangle_s + |1\rangle_a \otimes U_1|\psi\rangle_s\Big)$$

Projecting the ancilla back onto $|+\rangle_a$ (and discarding its outcome) yields the effective system evolution

$$|\psi\rangle_s \longrightarrow \frac{1}{\sqrt{2}}\left(U_1 + U_2\right)|\psi\rangle_s\,.$$

Superposition of rotating frames of reference

Consider superposition of two rotating frames of reference:

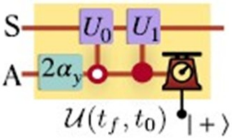


Rotation by angle θ about axis $\mathbf{n}_1$

SUPERPOSED WITH

Rotation by angle φ about axis $\mathbf{n}_2$

Physical Insight:
By coherently combining different dynamical paths, the system explores an enlarged space of temporal correlations that ordinary evolution cannot reach.



$$|\alpha\rangle_A = \cos\alpha|0\rangle + \sin\alpha|1\rangle$$

$$|\alpha\rangle\langle\alpha|_A \otimes \rho_S$$

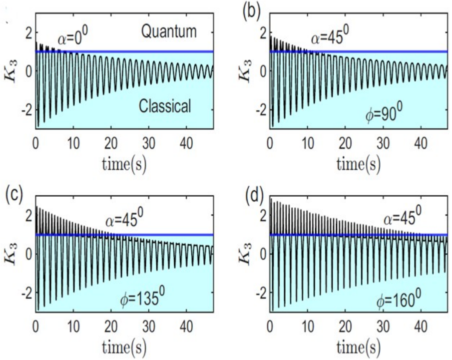
$$\downarrow\ |0\rangle\langle 0|_A \otimes \underbrace{e^{-i\omega\sigma_x t/2}}_{U_0} + |1\rangle\langle 1|_A \otimes \underbrace{e^{-i\omega\sigma_\phi t/2}}_{U_1}$$

$$\cos^2\alpha\, |0\rangle\langle 0|_A \otimes U_0\rho_S U_0^\dagger + \sin^2\alpha\, |0\rangle\langle 0|_A \otimes U_1\rho_S U_1^\dagger$$

$$+\cos\alpha\sin\alpha\left(|0\rangle\langle 1|_A \otimes U_0\rho_S U_1^\dagger + |1\rangle\langle 0|_A \otimes U_1\rho_S U_0^\dagger\right)$$

$$\downarrow\ \text{(post-selecting A in } |+\rangle \text{ state)}$$

$$\frac{\cos^2\alpha\, U_0\rho_S U_0^\dagger + \sin^2\alpha\, U_1\rho_S U_1^\dagger + \cos\alpha\,\sin\alpha\left(U_0\rho_S U_1^\dagger + U_1\rho_S U_0^\dagger\right)}{\mathrm{tr}\left[\cos^2\alpha\, U_0\rho_S U_0^\dagger + \sin^2\alpha\, U_1\rho_S U_1^\dagger + \cos\alpha\,\sin\alpha\left(U_0\rho_S U_1^\dagger + U_1\rho_S U_0^\dagger\right)\right]}$$



The superposed unitary slows down this decoherence process

NMR experiment

- 3 qubits: Three spin-1/2 nuclei of ^{13}C-dibromofluoromethane (DBFM) molecule. System S: ^{13}C, Ancilla A: ^{19}F, Measurement M: ^{1}H.

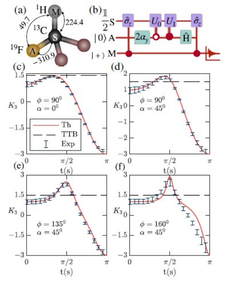


Figure: (a) The molecular structure of DBFM (b) Quantum circuit for determining C_{ij} (c-f) Experimentally measured values of K_3 vs theoretical curves (solid curves) for different values of α and ϕ.

Artistic Intuition: Dalí's Melting Clocks

Salvador Dalí's "The Persistence of Memory" (1931):

Soft, melting clocks draped across an surreal landscape...

Classical Time (Rigid Clocks):

- Time flows uniformly in a **single absolute frame**
- Events happen in a **definite sequence**: $t_1 < t_2 < t_3$
- Each clock measures time the *same way*
- Properties of events are *fixed* before measurement (Macrorealism)

Quantum Time recorded from superposed "clocks" (Melting Clocks):

- Time becomes **fluid**
- Events exist in **superposed temporal order**

From Rigid Clocks to Melting Time

The Analogy:

Rigid Clock:

One temporal frame $\leftrightarrow$ One unitary U

A *single* way to measure time, a *single* way to describe evolution

$$|\psi_{\text{rigid}}\rangle = U|\psi\rangle, \quad K_3 \leq 1.5$$

Melting Clocks (Quantum):

Superposed temporal frames $\leftrightarrow$ Superposed unitaries $(U_0 + U_1)$

Time itself becomes a superposition—events "blur" between different temporal orderings

$$|\psi_{\text{melting}}\rangle = \frac{1}{\sqrt{2}}(U_0 + U_1)|\psi\rangle, \quad K_3 > 1.5$$

Art Meets Physics: Dalí's Insight

"Time is not rigid. Time is not universal. Time can melt."

In Quantum Mechanics:

- **Standard evolution** treats time rigidly (unitary U)
- **Non-unitary evolution** (PT systems) allows temporal fluidity
- **Superposed frames** create interference in time itself
- **LGI violations** are signatures of this melting

The algebraic maximum $K_3 = 3$ represents the complete melting of temporal boundaries—a state where time has become maximally fluid and deformable.

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Leggett-Garg inequalities and temporal correlations for a qubit under $\mathcal{PT}$-symmetric dynamics

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Macrorealistic description governing the classical realm is known to get uprooted in the quantum domain. The Leggett-Garg inequality [A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985)] proposes to probe the macrorealistic limit emerging from the quantum scenario. It places a bound on the linear combinations of temporal correlations of observables measured sequentially at different time intervals. Violation of the Leggett-Garg inequality, up to the so-called temporal Tsirelson bound (TTB), has been realized in the quantum domain.

PHYSICAL REVIEW LETTERS **135**, 220202 (2025)

Featured in Physics

Extreme Violations of Leggett-Garg Inequalities for a System Evolving under Superposition of Unitaries

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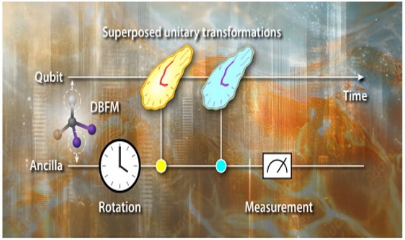
The violation of Leggett-Garg inequality (LGI) indicates general temporal correlations in quantum systems that cannot be explained classically. Under unitary dynamics and projective measurements, the violation of LGI is restricted up to the temporal Tsirelson's bound (TTB). Here, we consider superposition of unitary time evolutions and find them to produce an enhancement in the violation of LGI beyond the TTB, growing monotonically with increasing superposition. We experimentally realize superposition

Physics — VIEWPOINT

Melting Temporal Limits with Quantum Dynamics

Extending the superposition principle to the time domain gives rise to enhanced correlations that exceed a theoretical quantum limit—a result that could inspire new forms of quantum control.

By Fernando J. Gómez-Ruiz



Thanks for your kind attention

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