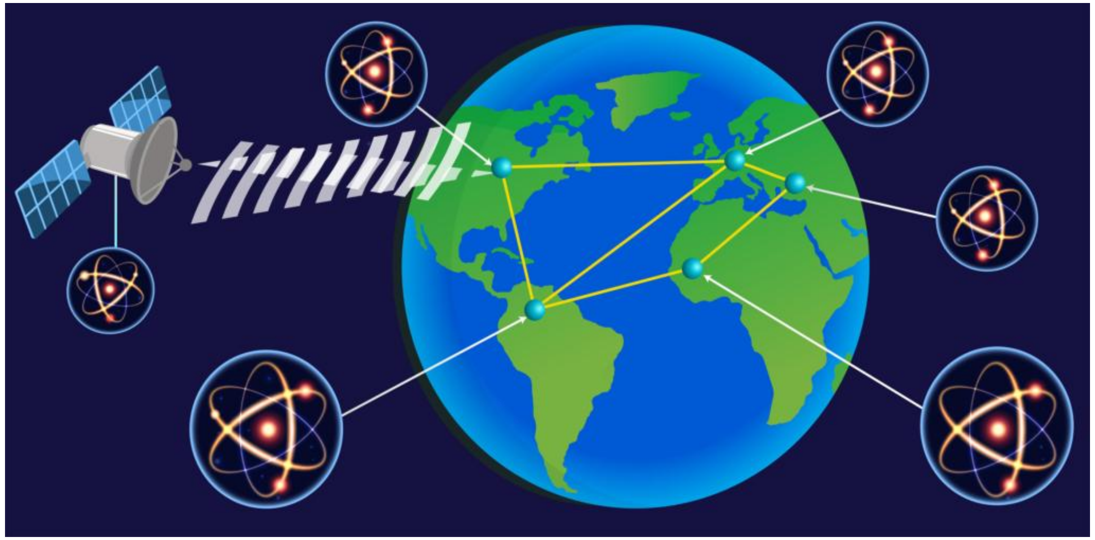
Disti-Mator: an entanglement distillation based state estimator

ICQIST, 2025, TCG Crest, Kolkata
10th February

Dr. Ananda G. Maity
Indian Institute of Technology Goa,
Farmagudi, Ponda 403401, Goa

Quantum network engineering



Quantum network and quantum Internet

Promised functionalities:
Quantum key distribution, higher-precision clock synchronization, more secure communication, distributed quantum computing

Requirement: Distribution of high-quality entanglement between the distant nodes

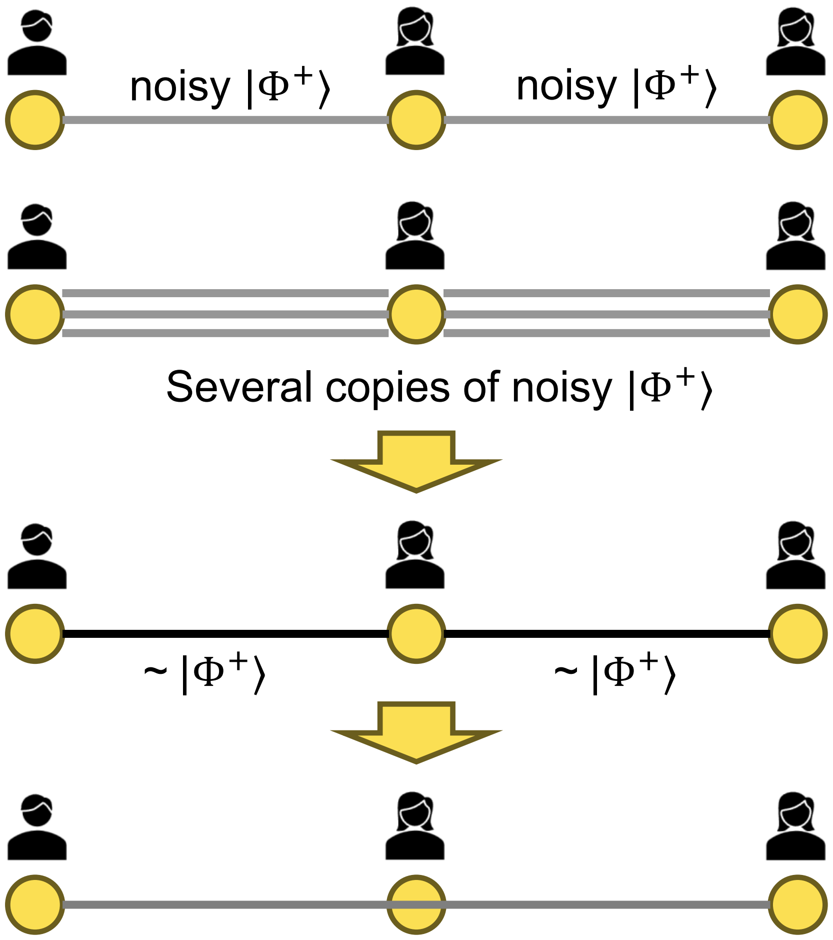
Challenges: Long-distance remote entanglement distribution remains a challenge–
* entanglement attenuates exponentially with distance
* cannot be amplified because of the no-cloning theorem;

Solution: **Entanglement distillation** can be used.

Wehner, Elkouss, Hanson, Science 362, 6412 (2018)

Network engineering & states estimation

Entanglement distillation and swapping

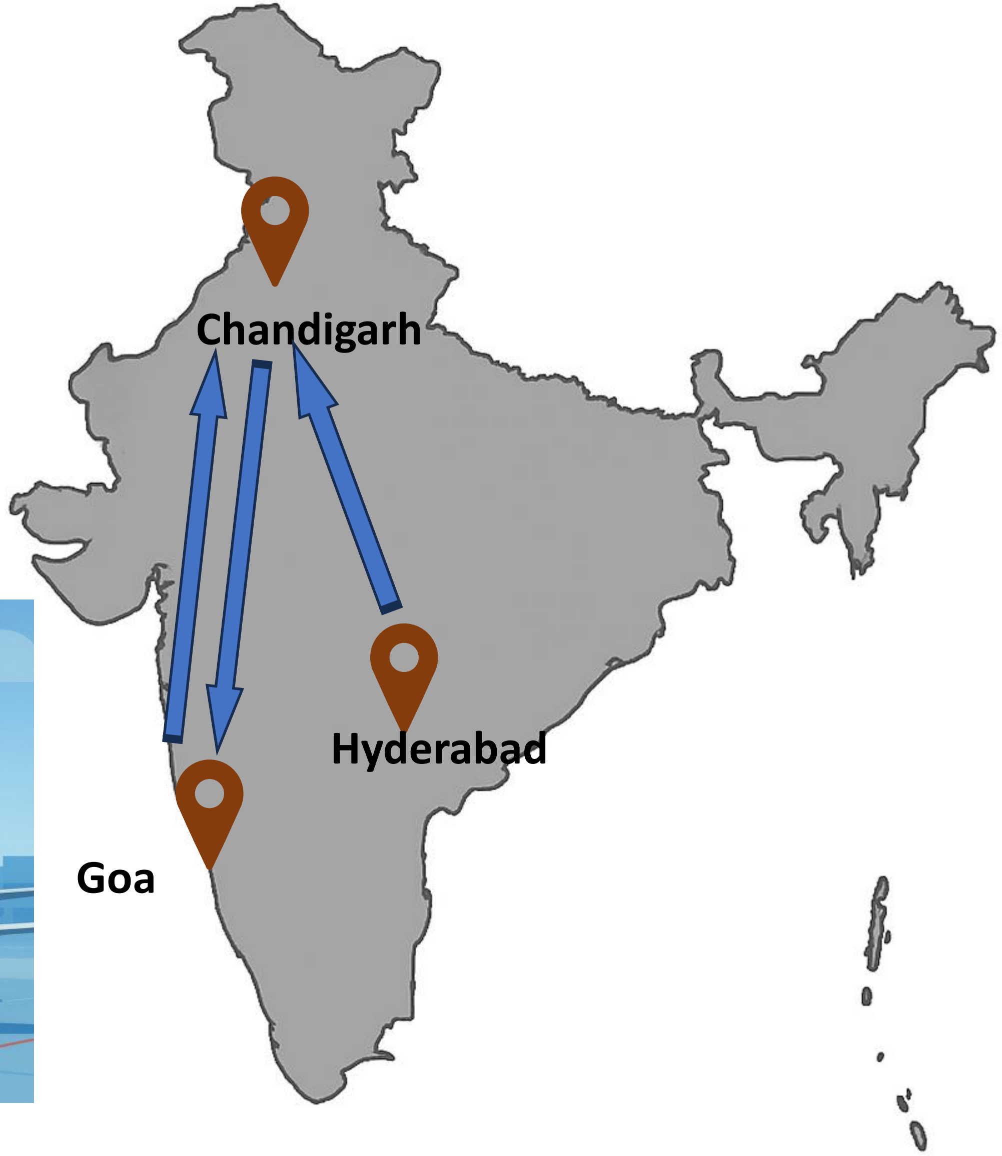


- To know if the desired state has been generated by the internet devices, we must characterize the links.
- Approaches include tomography, randomized benchmarking, self-testing, quantum gate set tomography.

*(Eisert et.al, Nature Reviews Physics **2**, 382 (2020))*

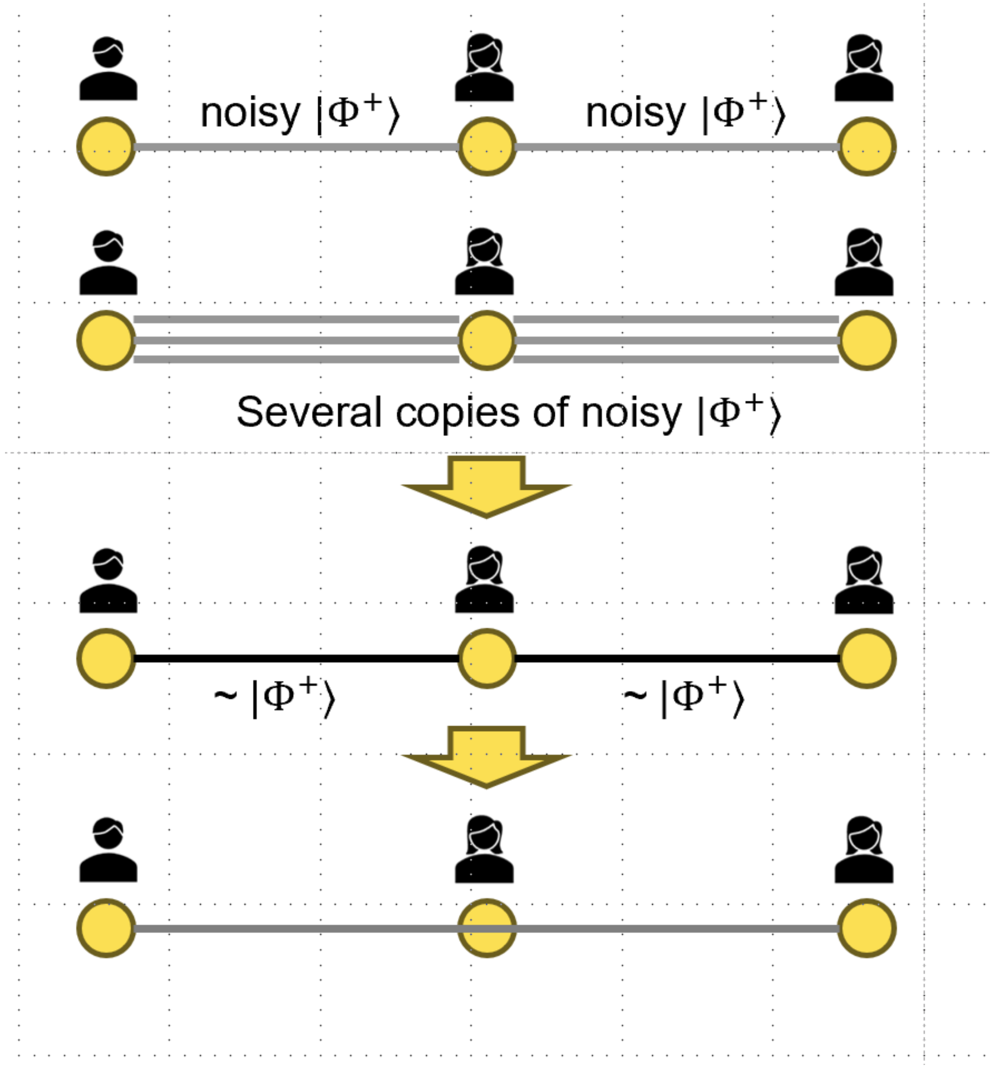
- However, most of these estimation methods fully measure the quantum resources, making them useless for further information processing tasks.

Network engineering & states estimation



Network engineering & states estimation

Entanglement distillation and swapping



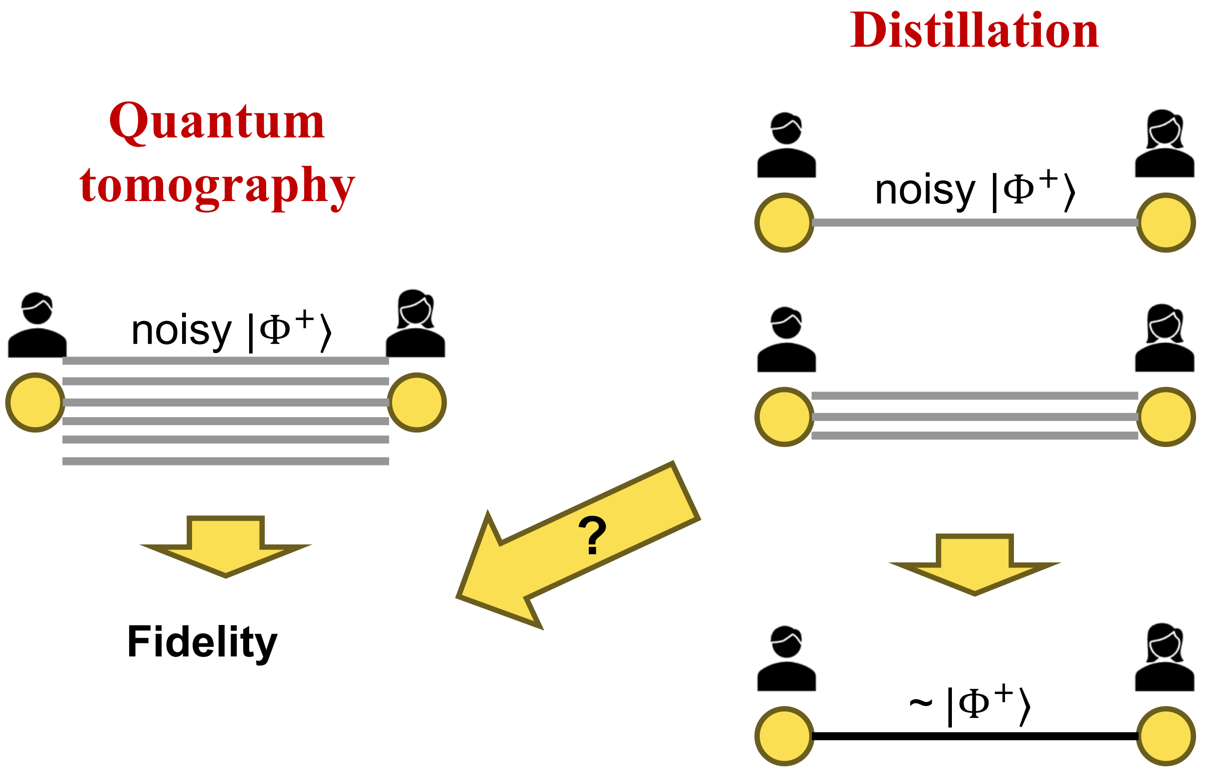
- To know if the desired state has been generated by the internet devices, we must characterize the links.

- Approaches include tomography, randomized benchmarking, self-testing, quantum gate set tomography.

  *(Eisert et.al, Nature Reviews Physics **2**, 382 (2020))*

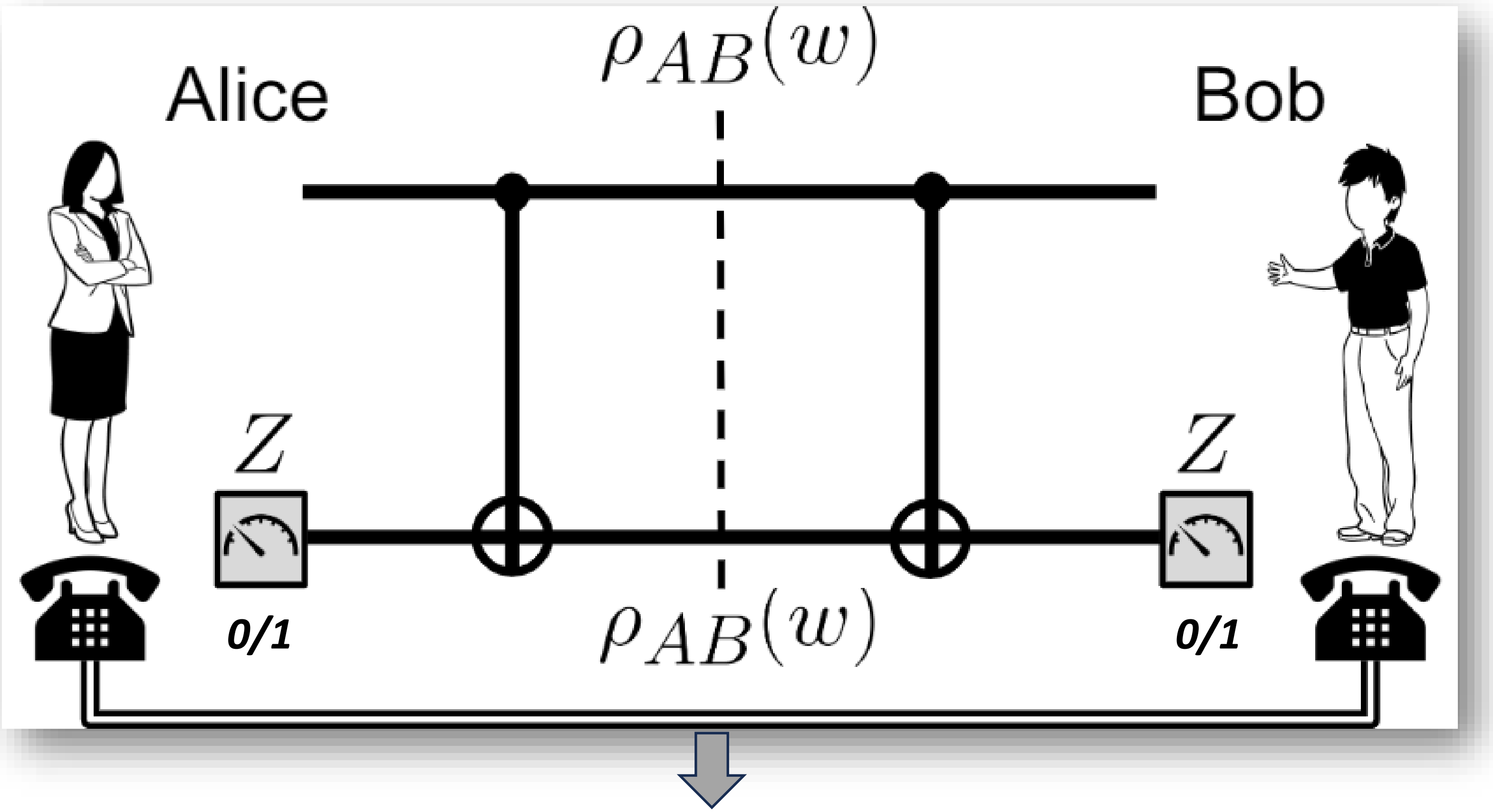
- However, most of these estimation methods fully measure the quantum resources, making them useless for further information processing tasks.

Network states estimation



Can the prepared states be efficiently characterized from the measurement statistics obtained during a distillation protocol?

Entanglement distillation protocol

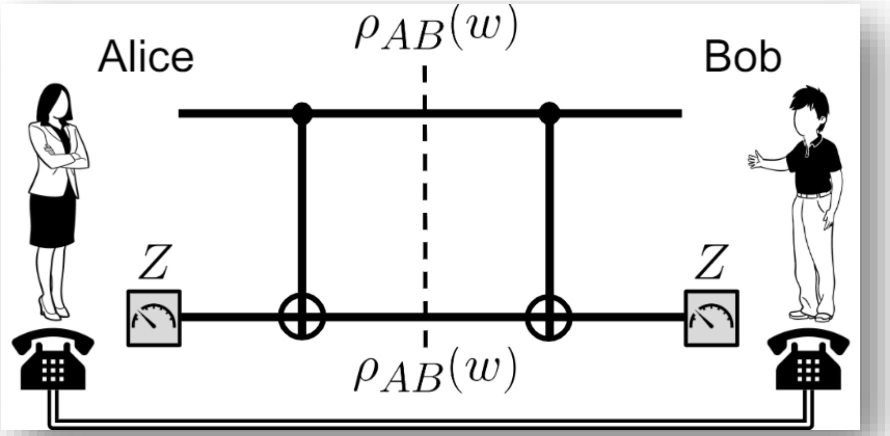


p_{00}

Can we obtain state information from this p_{00}?

Bennett Brassard, Popescu, Schumacher, Smolin, Wootters, Phys. Rev. Lett. 76, 722–725 (1996)

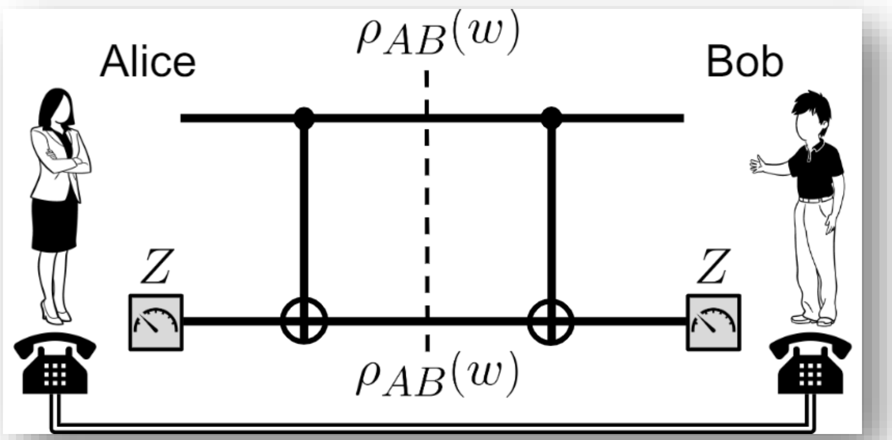
Entanglement distillation protocol and state estimation

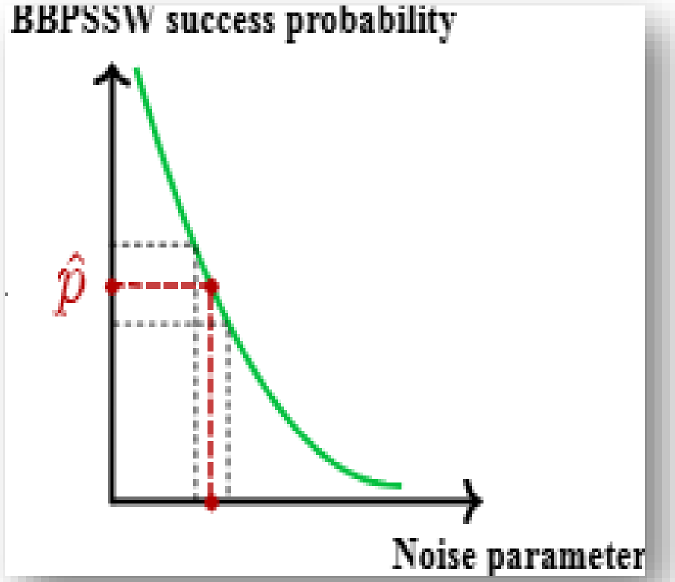


- For Werner states, $\rho_w = (1-w)|\Phi^+\rangle\langle\Phi^+| + \frac{w}{4}I$, the success probability (getting both "up" outcomes) is

$$p_{00} = \frac{1}{4}(2 - 2w + w^2).$$

Entanglement distillation protocol and state estimation

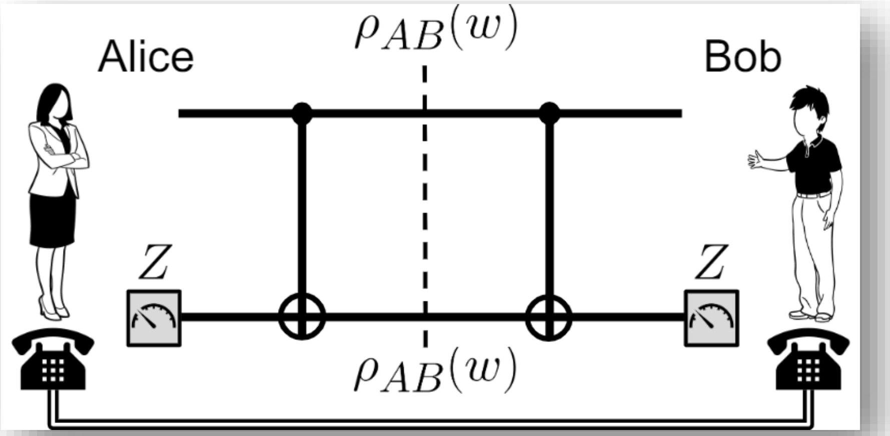


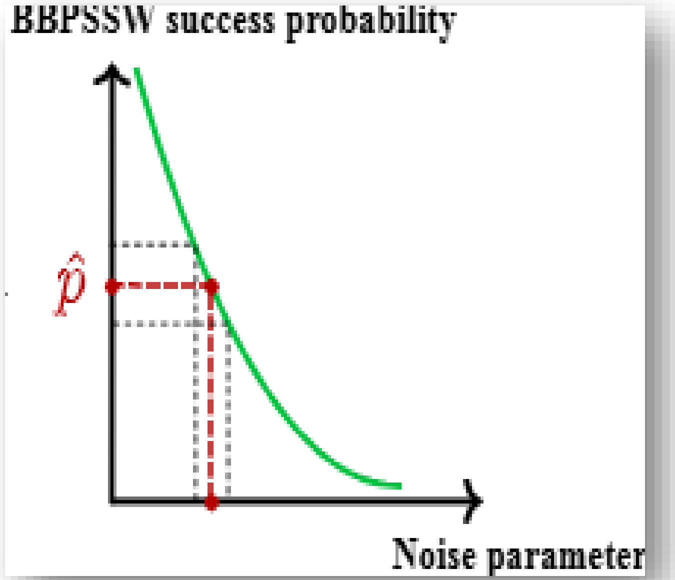


- For Werner states, $\rho_w = (1-w)|\Phi^+\rangle\langle\Phi^+| + \frac{w}{4}I$, the success probability (getting both "up" outcomes) is
$$p_{00} = \frac{1}{4}(2 - 2w + w^2).$$

- If $|\widehat{p}_{00} - \ p_{00}| \le \epsilon$ then what is $|\widehat{w} - w| \le \epsilon_w$?

Entanglement distillation protocol and state estimation



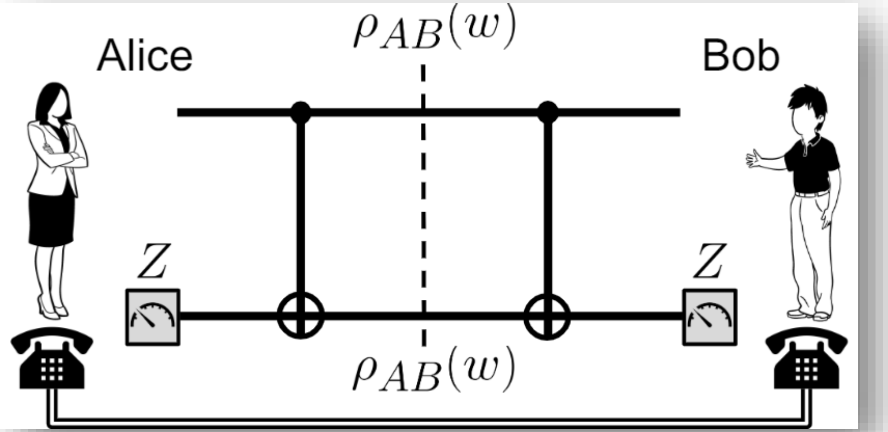


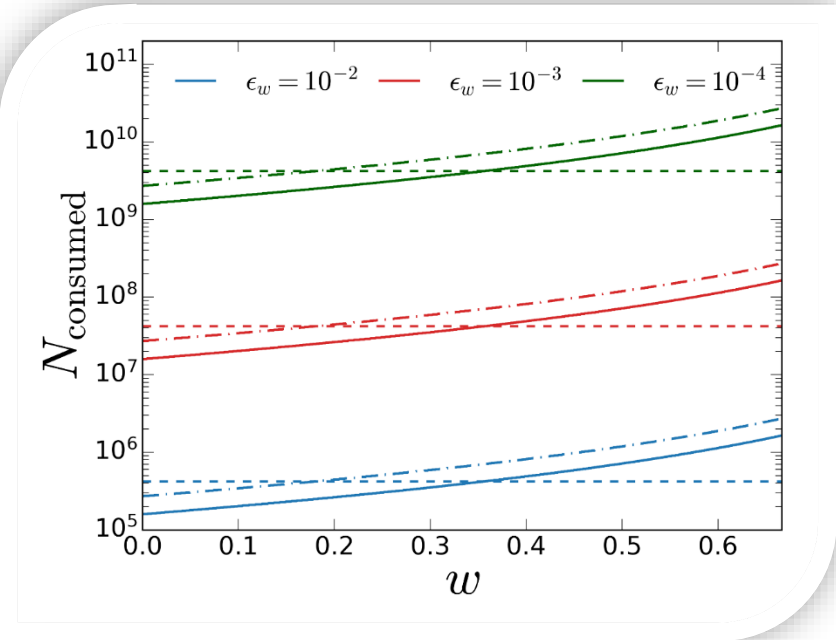
- For Werner states, $\rho_w = (1-w)|\Phi^+\rangle\langle\Phi^+| + \frac{w}{4}I$, the success probability (getting both "up" outcomes) is
$$p_{00} = \frac{1}{4}(2 - 2w + w^2).$$

- If $|\widehat{p}_{00} - \ p_{00}| \leq \epsilon$ then what is $|\widehat{w} - w| \leq \epsilon_w$?

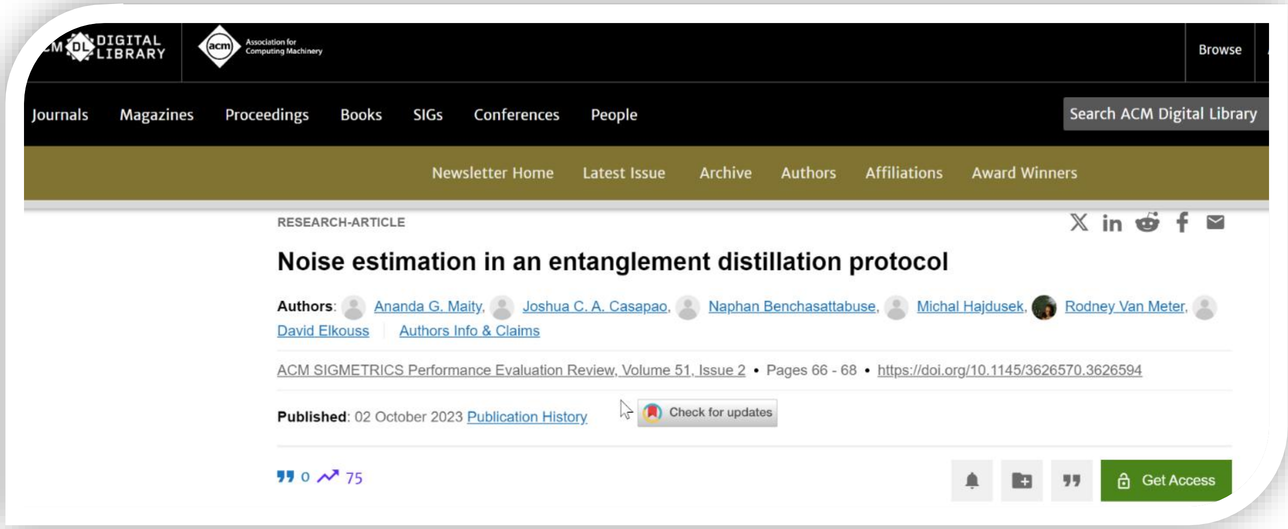
- Using Hoeffding's inequality, the estimator's failure probability is bounded by $\Pr(|\widehat{w} - w| \geq \epsilon_w) \leq \delta$ with $\delta = \sum_{m=L,R} \exp(-2N\epsilon_m^2)$.

Entanglement distillation protocol and state estimation



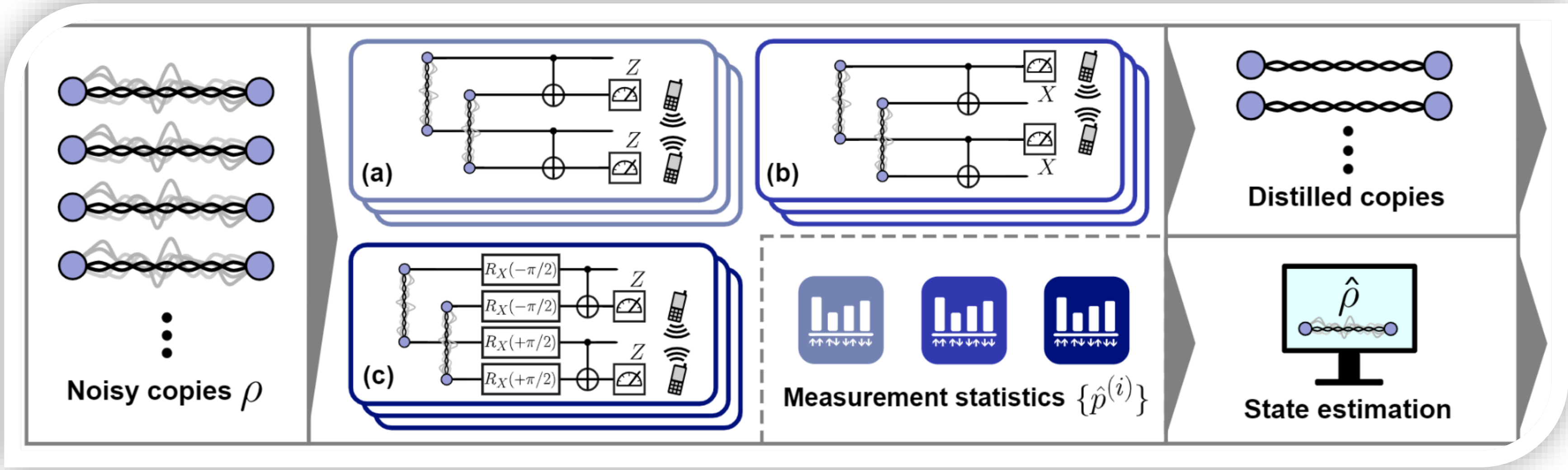


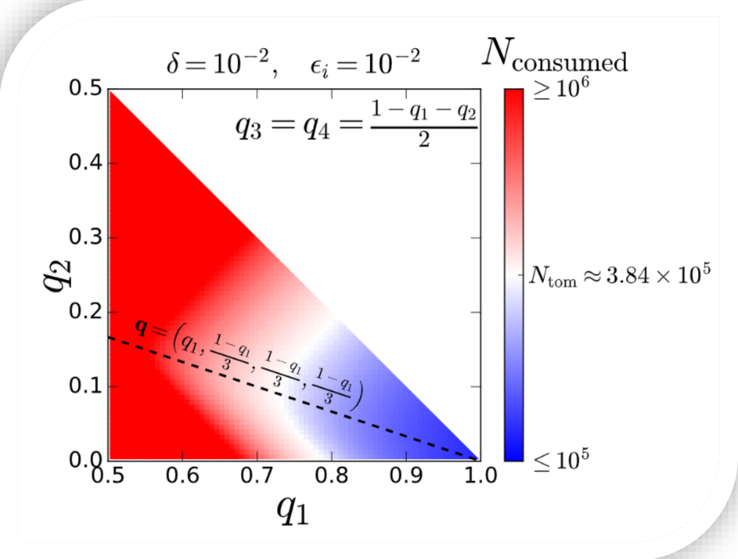
- For Werner states, $\rho_w = (1-w)|\Phi^+\rangle\langle\Phi^+| + \frac{w}{4}I$, the success probability (getting both "up" outcomes) is
$$p_{00} = \frac{1}{4}(2 - 2w + w^2).$$
- If $|\widehat{p}_{00} - \ p_{00}| \leq \epsilon$ then what is $|\widehat{w} - w| \leq \epsilon_w$?
- Using Hoeffding's inequality, the estimator's failure probability is bounded by $\Pr(|\widehat{w} - w| \geq \epsilon_w) \leq \delta$ with $\delta = \sum_{m=L,R} \exp(-2N\epsilon_m^2)$.



Bell-diagonal states estimation

- Bell diagonal state: $\rho = q_1|\Phi^+\rangle\langle\Phi^+| + q_2|\Phi^-\rangle\langle\Phi^-| + q_3|\Psi^+\rangle\langle\Psi^+| + q_4|\Psi^-\rangle\langle\Psi^-|$.

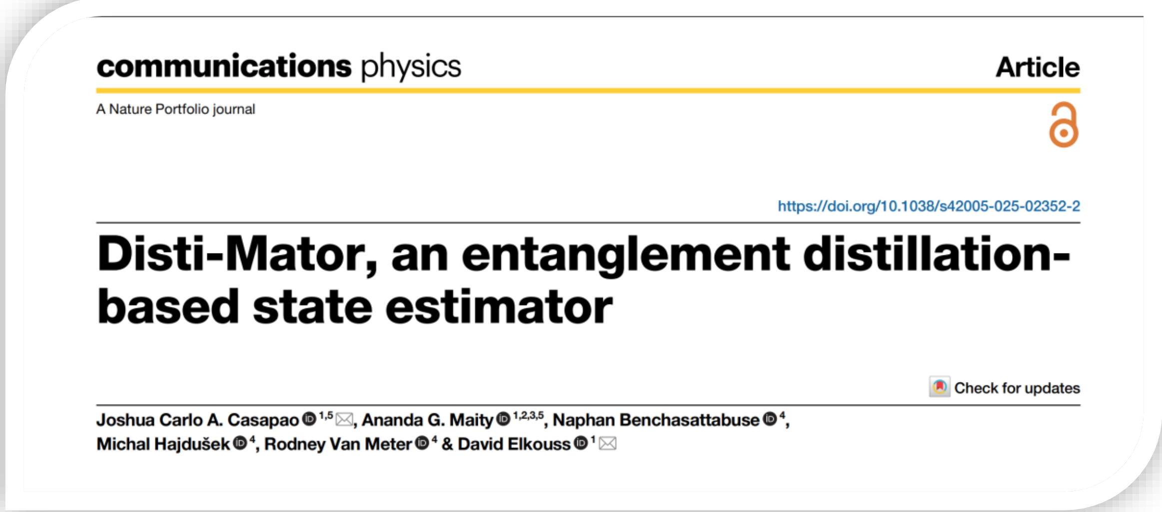




- The success probability of three distillation protocols are $p^{(i)}(x_i) \coloneqq p^{(i)}(q_i + q_{i+1})$
- The estimator's failure probability is bounded by $\Pr(D(\hat{\rho}(\hat{\boldsymbol{q}}), \bar{\rho}(\boldsymbol{q})) \geq \epsilon_T) \leq \delta$ with $\epsilon_T = \sum_{i=\{1,2,3\}} \epsilon_i$ and ϵ_i are error bound to x_i;

$$\delta = 1 - \prod_{i=\{1,2,3\}} \left[1 - \sum_{m=L,R} \exp\left(-2N^{(i)} \epsilon_m^{(i)2}\right)\right]$$

$$D(\hat{\rho}(\hat{\mathbf{q}}), \bar{\rho}(\mathbf{q})) = \frac{1}{2}\sum_{k=1}^{4} |\hat{q}_k - q_k| = \frac{1}{2}\|\hat{\mathbf{q}} - \mathbf{q}\|_1,$$



communications physics
Article
A Nature Portfolio journal
https://doi.org/10.1038/s42005-025-02352-2

Disti-Mator, an entanglement distillation-based state estimator

Check for updates

Joshua Carlo A. Casapao [1,5], Ananda G. Maity [1,2,3,5], Naphan Benchasattabuse [4], Michal Hajdušek [4], Rodney Van Meter [4] & David Elkouss [1]

Validating Disti-Mator's performance via numerical experiment

❑ For Werner state

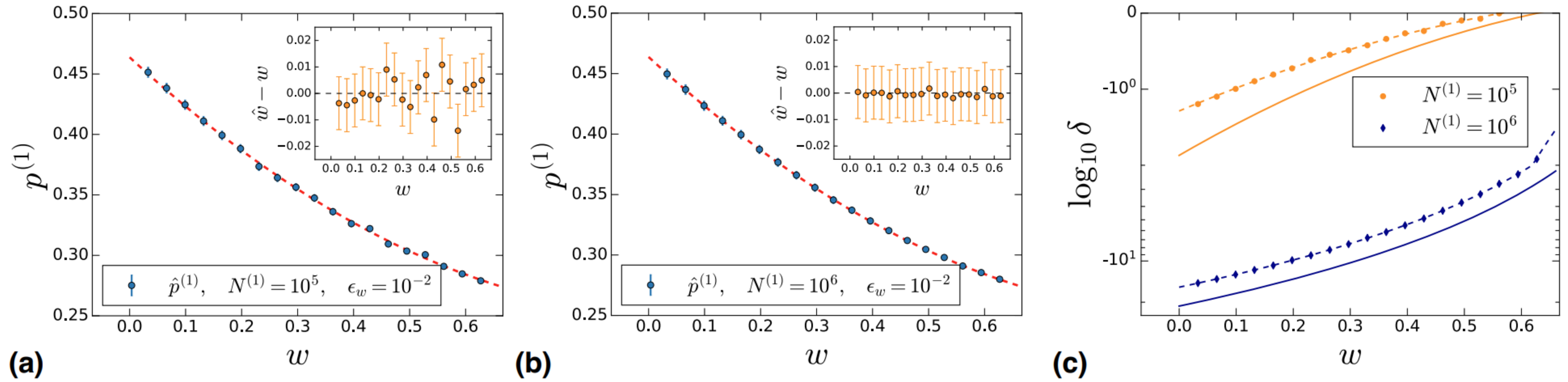


Fig. 4 | Parameter estimation for Werner states in a simulated noisy distillation experiment. In **a**, **b**, we perform estimation with $N^{(1)} = 10^5$ and $N^{(1)} = 10^6$ pairs of Werner states, respectively. We set the error bound as $\epsilon_w = 10^{-2}$ (shown in figure insets). The corresponding error bounds for $\hat{p}^{(1)}$ following the concentration bounds in the main text are shown. The red dashed curves show expected relationship between w and $p^{(1)}$. **c** shows the failure probability bound δ for each simulation. The solid curves indicate the expected bound given a noiseless distillation, while the dashed curves indicate the expected bound for a noisy distillation with the given noise parameters. The simulations use the following noise parameters (described in the main text): $\Delta t / T_{A,B}^{\text{dpo,dph}} = t_{\text{geom}}(p_g)/100$, with $p_g = 0.2$; $y_{A,B} = 0.01$; $\eta^Z_{A,B} = 0.99$.

Casapao, Maity, Benchasattabuse, Hajdusek, Van Meter, and Elkouss, Comm. Phys. 8, 461 (2025).

Validating Disti-Mator's performance via numerical experiment

- For Bell diagonal state

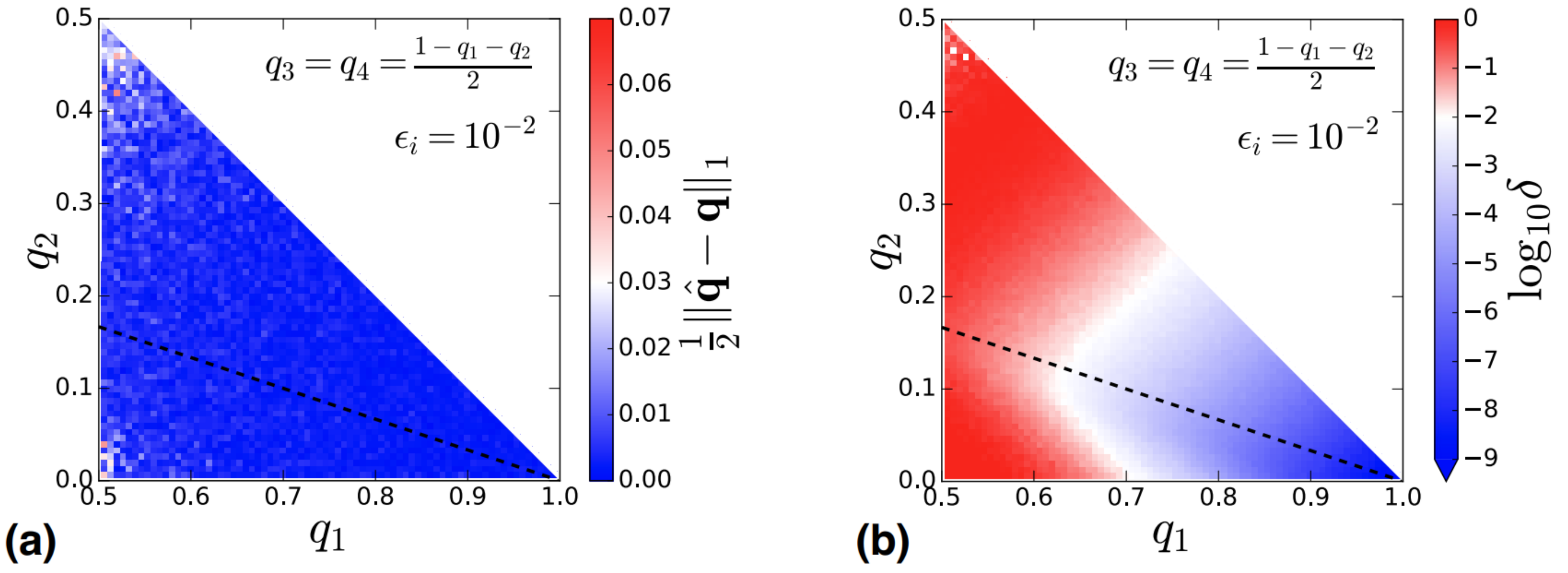


Fig. 5 | Parameter estimation for Bell-diagonal states in a simulated noisy distillation experiment. We perform estimation with $N^{(i)} = 2 \times 10^5$ pairs of Bell-diagonal states for each distillation protocol. We consider the set of states that satisfy $q_3 = q_4 = (1 - q_1 - q_2)/2$, and we set the error bound $\epsilon_i = 10^{-2}$ for all i (described in the main text). The dashed lines describe the collection of Werner states. **a** shows the trace distance $D(\hat{\rho}(\hat{\mathbf{q}}), \overline{\rho}(\mathbf{q})) = \frac{1}{2} \| \hat{\mathbf{q}} - \mathbf{q}\|_1$ between the estimation $\hat{\rho}(\hat{\mathbf{q}})$ and the expected state in Bell-diagonal form $\overline{\rho}(\mathbf{q})$. We observe that the estimation is close to the expected $\mathbf{q}$ whenever q_1 is close to one (as shown in blue). **b** shows the failure probability bound δ of the trace distance exceeding $\epsilon_T = 3 \times 10^{-2}$. The simulations use the following noise parameters (described in the main text): $\Delta t / T_{A,B}^{\mathrm{dpo,dph}} = t_{\mathrm{geom}}(p_g)/100$, with $p_g = 0.2$; $m_{A,B} = 0.01$; $y_{A,B} = 0.01$; $\eta_{A,B}^{Z,X} = 0.99$.

Casapao, Maity, Benchasattabuse, Hajdusek, Van Meter, and Elkouss, Comm. Phys. 8, 461 (2025).

A double selection entanglement distillation protocol

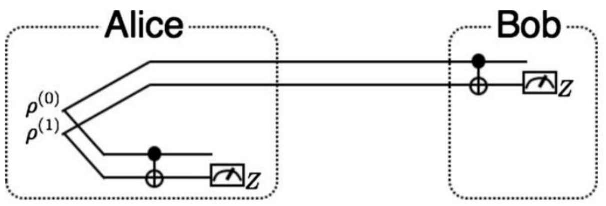


FIG. 1. Bipartite entanglement purification with single selection.

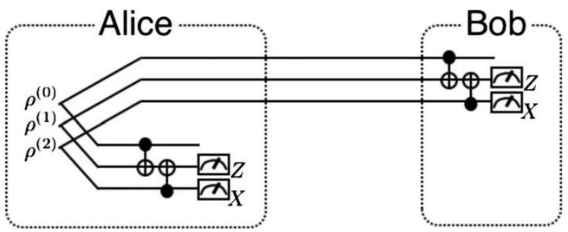
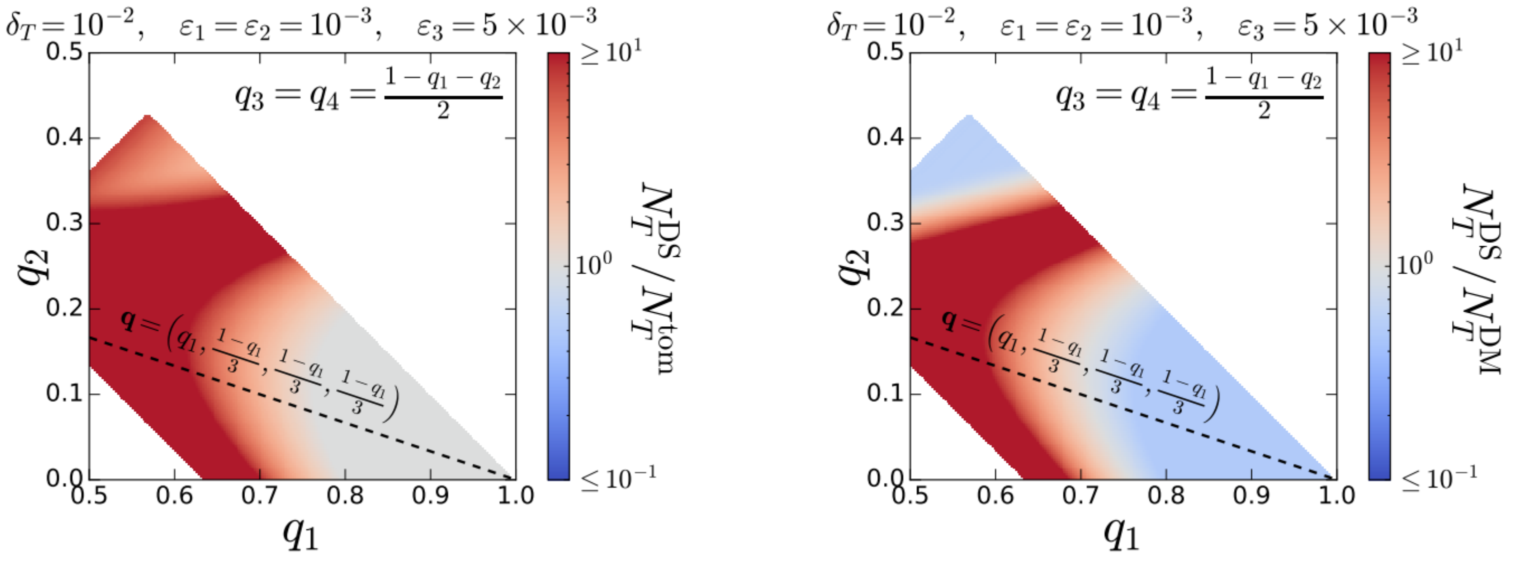


FIG. 2. Bipartite entanglement purification with double selection.

Fujii and Yamamoto, Phys. Rev. A, 80, 042308, (2009).

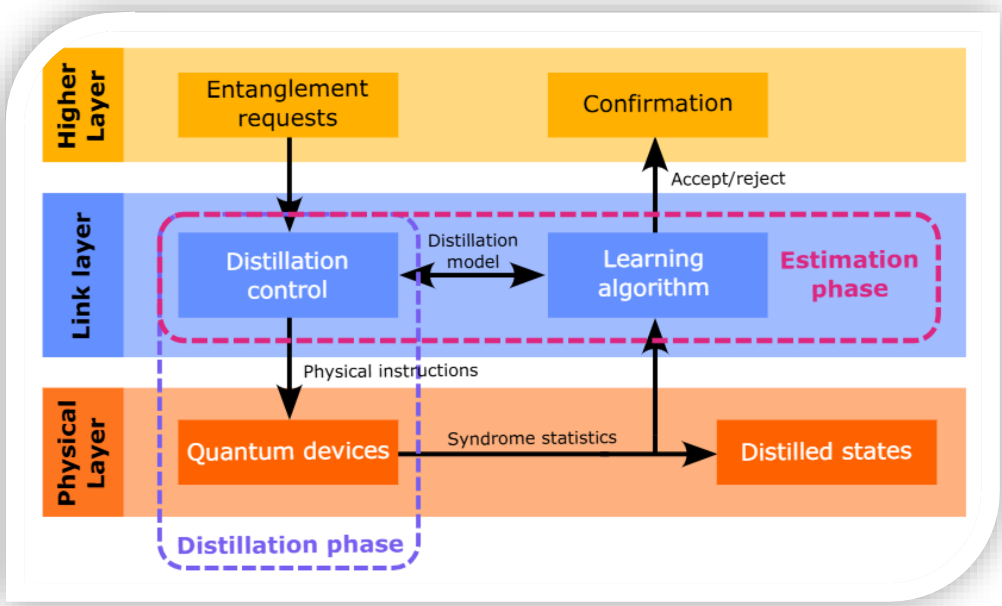


(a) Tomography vs. Double selection Disti-Mator

(b) Original Disti-Mator [15] vs. Double selection Disti-Mator

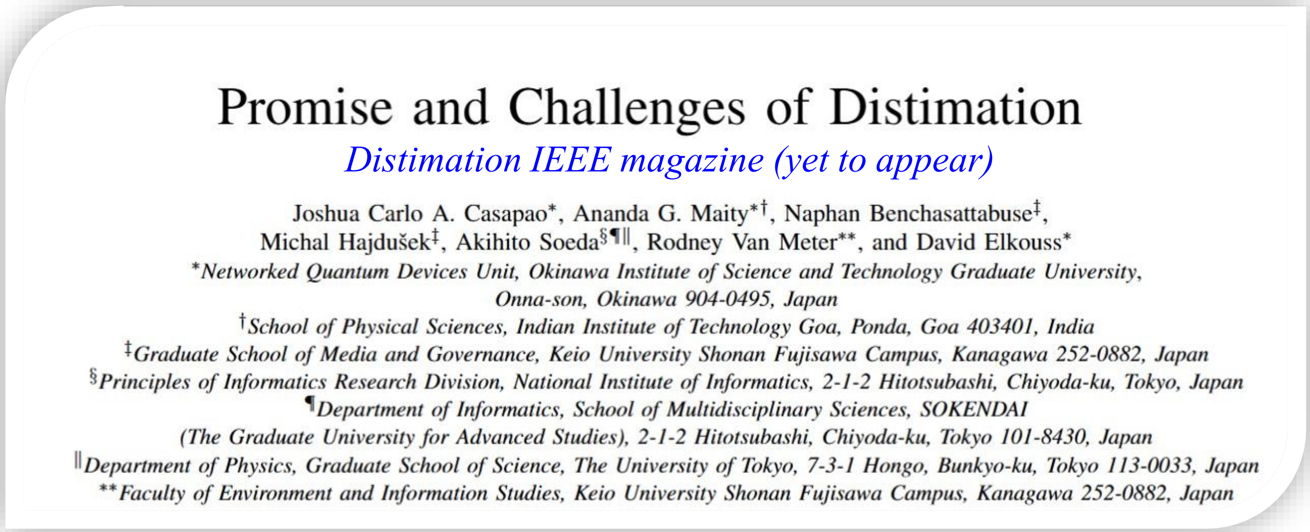
Casapao, Maity, Benchasattabuse, Hajdusek, Soeda, Van Meter, and Elkouss, In proceeding of IEEE QCE (2025).

Integrating distimation in a network protocol stack



- Higher layers send entanglement requests to the bottom layers with explicit quality requirements such as entanglement fidelity.
- The distillation control mechanism at the link layer determines the amount of distillation necessary to assure the entanglement quality.
- In the distillation phase, instructions to perform distillation are sent by the link layer to the quantum hardware platform (physical layer).
- In the estimation phase, the distillation model together with the syndrome statistics from distillation are given as input to the learning algorithm.

A Quantum Internet Architecture by *VanMeter et. al., IEEE International Conference on Quantum Computing and Engineering (QCE) (2022).*

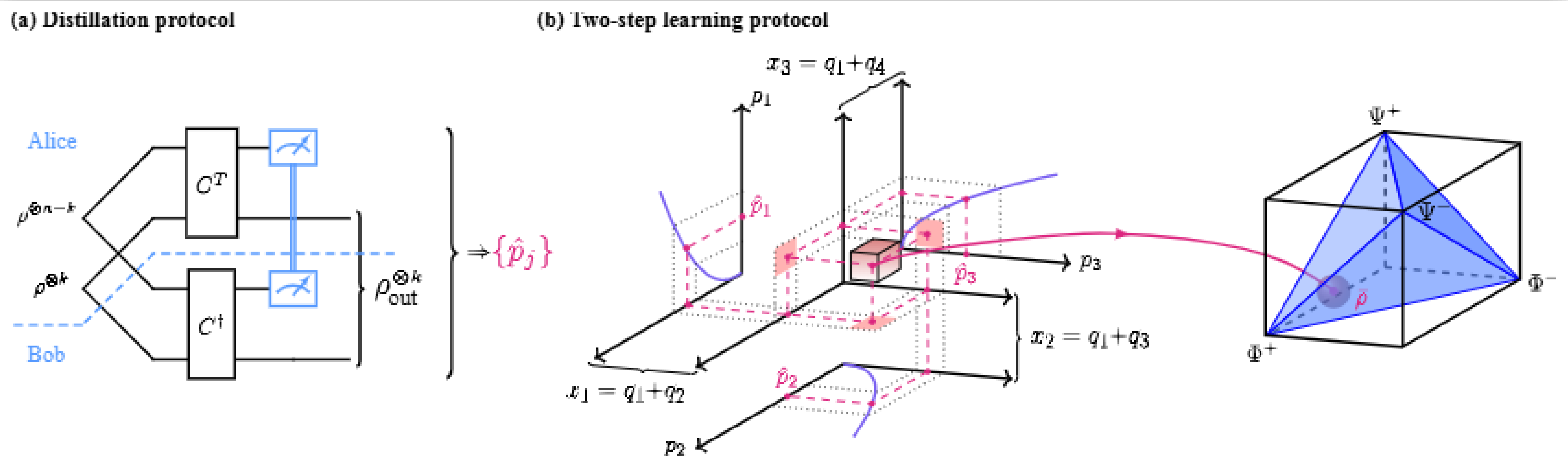


Promise and Challenges of Distimation

Distimation IEEE magazine (yet to appear)

Joshua Carlo A. Casapao[*], Ananda G. Maity[*†], Naphan Benchasattabuse[‡],
Michal Hajdušek[‡], Akihito Soeda[§¶‖], Rodney Van Meter[**], and David Elkouss[*]
[*]*Networked Quantum Devices Unit, Okinawa Institute of Science and Technology Graduate University, Onna-son, Okinawa 904-0495, Japan*
[†]*School of Physical Sciences, Indian Institute of Technology Goa, Ponda, Goa 403401, India*
[‡]*Graduate School of Media and Governance, Keio University Shonan Fujisawa Campus, Kanagawa 252-0882, Japan*
[§]*Principles of Informatics Research Division, National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo, Japan*
[¶]*Department of Informatics, School of Multidisciplinary Sciences, SOKENDAI (The Graduate University for Advanced Studies), 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*
[‖]*Department of Physics, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*
[**]*Faculty of Environment and Information Studies, Keio University Shonan Fujisawa Campus, Kanagawa 252-0882, Japan*

General recipe for Distimation



Summary

- We show that one can robustly and efficiently characterize entangled states using the measurement statistics obtained from probabilistic two-way entanglement distillation protocols.

- Whenever distillation is a necessary step, our method offers an efficient way to certify the Bell-diagonal parameters of an arbitrary state from the distillation statistics without consuming further quantum resources beyond what is needed for the distillation itself.

- Moreover, if the initial entangled states are of high fidelity, our method can have a sample complexity that is favorable than a tomographic protocol.

- We provide several state estimation toolboxes based on entanglement distillation protocols that is robust even in the presence of SPAM errors and noisy distillation gates (see the github repository) and exhibit its robustness through numerical simulations.

- Incorporating these technique in a quantum network will simply the network management task since an additional protocol of benchmarking the network state can be bypassed.

Collaborators

Joshua Carlo A. Casapao

David Elkouss

(Okinawa Institute of Science and Technology, Okinawa, Japan)

Akhito Soeda

(NII & SOKENDAI, Tokyo, Japan)

Naphan Benchasattabuse

Michal Hajdusek

Rodney VanMeter

(Keio University, Tokyo, Japan)

References:

- *Comm. Phys. 8, 461 (2025).*
- *ACM SIGMETRICS Per. Eval. Rev. 51, 66 (2023).*
- *IEEE QCE proceedings (2025).*
- *Distimation Magazine (yet to appear).*

I am looking for motivated Ph.D. students to join my new **Quantum COMMunication and Information Theory (Q-COMMIT) Lab** at the **School of Physical Sciences, IIT Goa**.

THANK YOU