Nature of correlation in a minimal no-input network

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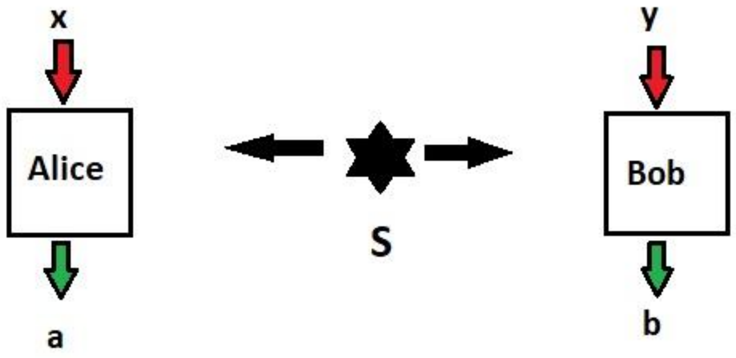
Correlation: Classical or Quantum?

BELL-CHSH SCENARIO



<u>Local Correlation</u>:

With the classical shared randomness λ, the distribution $p(a,b|x,y) = \int d\lambda \rho(\lambda) p(a|x,\lambda) p(b|y,\lambda)$ is local.

<u>Quantum correlation:</u>

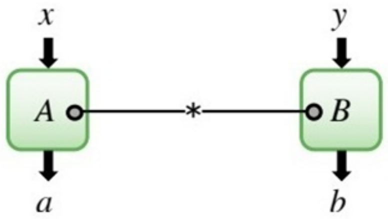
Quantum correlation for this scenario $p(ab|xy) = tr(\rho_{AB}\, M_{a|x} \otimes M_{b|y})$

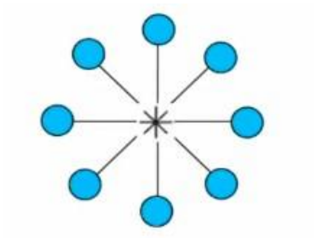
<u>Nonlocal Correlation:</u>

Any correlation, which can not be described by the Local description is said ***Nonlocal Correlation***.

From Bell to Network…

Bell:

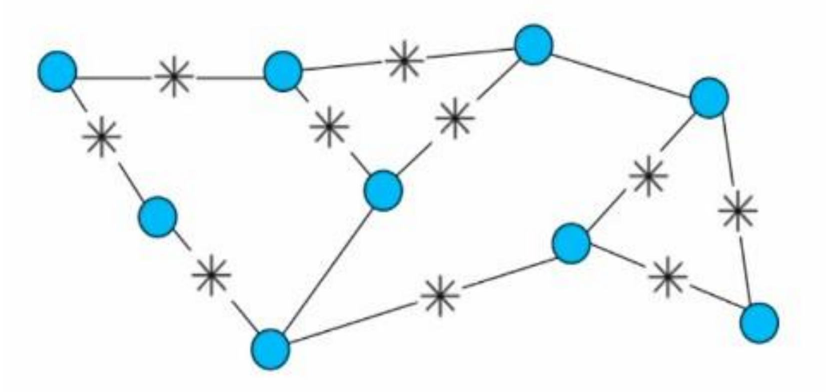




\# Convex Sets
\# Linear Bell Inequalities
\# One Common Source

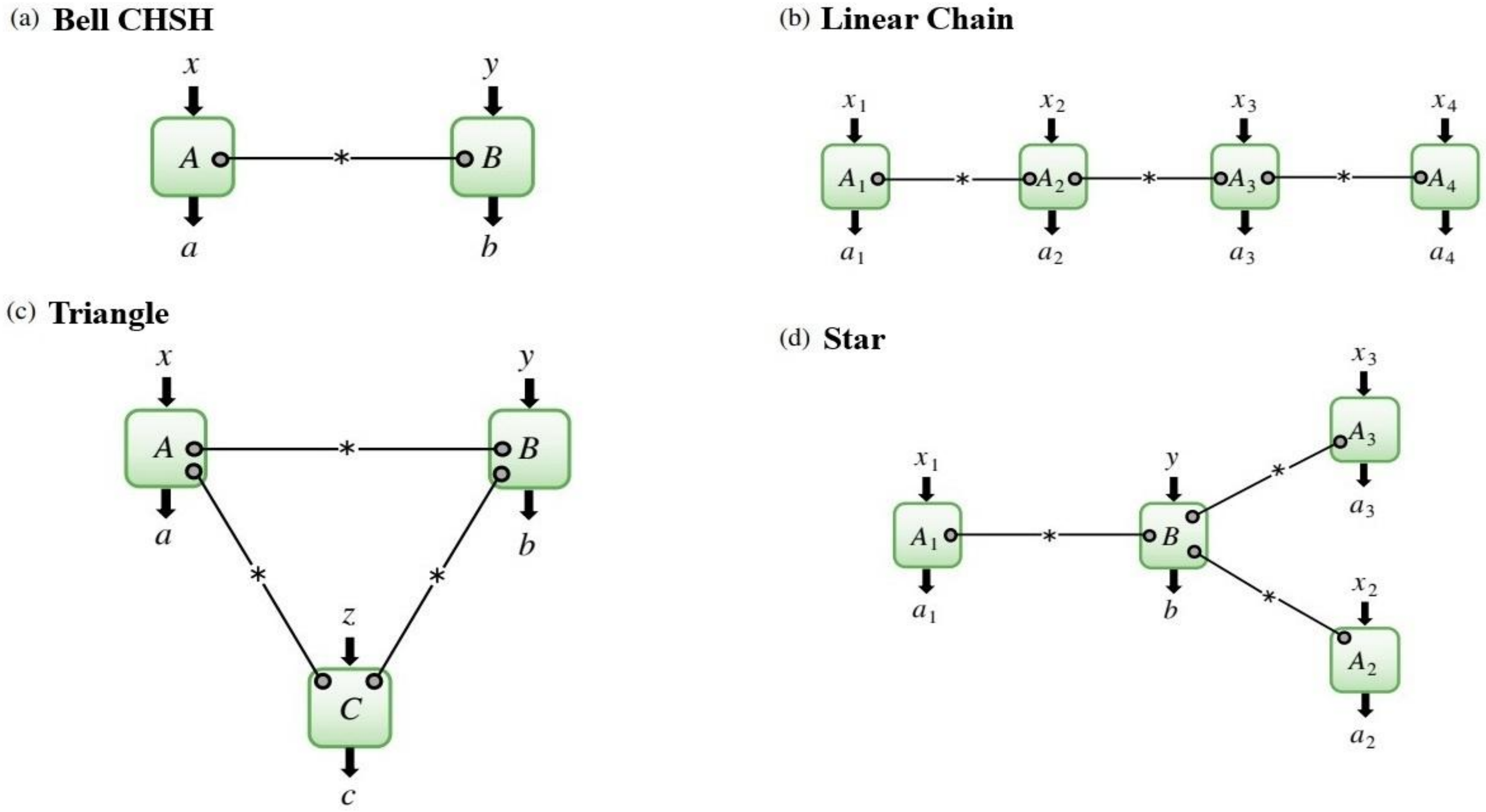
$$p(a,b|x,y) = \int d\lambda \mu(\lambda) p(a|x,\lambda) p(b|y,\lambda)$$

Network:

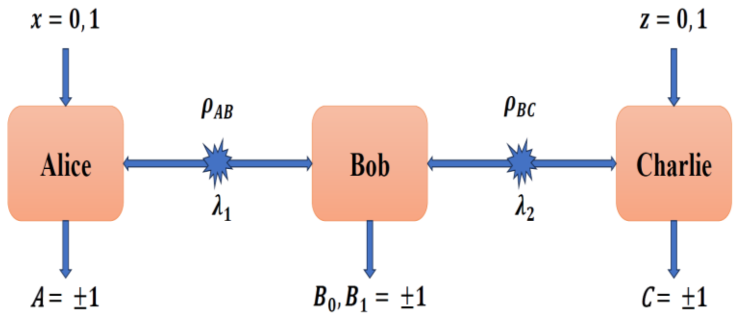


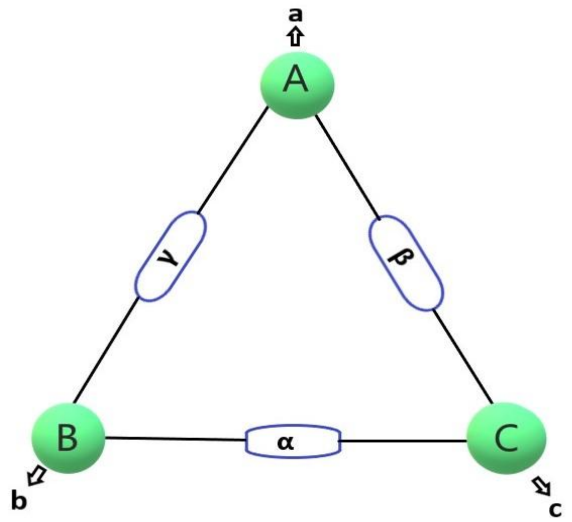
\# Non-Convex Sets
\# Non-linear Bell Inequalities
\# Many independent Sources

Various Network structures.



No-input network





Network Scenario: $p(a, b, c|x, y, z)$
$= \int d\lambda_1 d\lambda_2 \rho_1(\lambda_1)\rho_2(\lambda_2) p(a|x,\lambda_1) p(b|y,\lambda_1,\lambda_2) p(c|z,\lambda_2)$

No-input Network: For the minimal no-input network scenario, the Triangle scenario

$p(a, b, c) =$
$\iiint d\alpha d\beta d\gamma \mu(\alpha)\mu(\beta)\mu(\gamma) p(a|\beta\gamma) p(b|\gamma\alpha) p(c|\alpha\beta)$

Now if any quantum correlation $p_Q(a, b, c|x, y, z)$ from Bilocal or $p_Q(a, b, c)$ from triangle or from any other scenario can not be described by the local description it is Network-Nonlocal. But,

$p_Q(a, b, c)$ Local or Nonlocal? Hard to answer....

Why network?

Theoretical advancement:

Understanding the correlation more deeply.

Technological Advancement:

To go into Quantum Internet.

Triangle correlation...

The quantum correlation from the triangle network can be described by,

$$P_Q(a, b, c) = | < \varphi_a | < \varphi_b | < \varphi_c || \psi_\alpha > | \psi_\beta > | \psi_\gamma > |^2$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

→ Bell State Measurement Bases

↓

Local Correlation*

Nonlocality →

1. Joint Bases
2. Nonlocal States

Maximally Entangled state from each source

*Entropy **2019**, 21, 325

Elegant Joint Measurement(EJM)

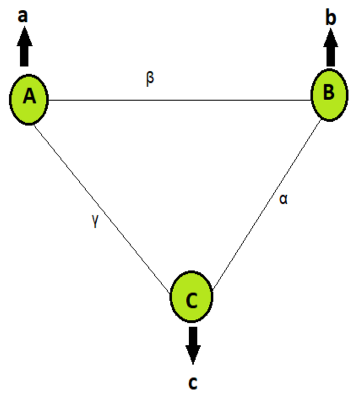
$$\vec{m}_1 = (+1, +1, +1)$$
$$\vec{m}_2 = (+1, -1, -1)$$
$$\vec{m}_3 = (-1, +1, -1)$$
$$\vec{m}_4 = (-1, -1, +1)$$

Four vertices of the tetrahedron inscribed in the Poincaré sphere.

$$|\Phi_j\rangle = \sqrt{\frac{3}{2}}|\vec{m}_j, -\vec{m}_j\rangle + i\frac{\sqrt{3}-1}{2}|\psi^-\rangle = \frac{\sqrt{3}+1}{2\sqrt{2}}|\vec{m}_j, -\vec{m}_j\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}|-\vec{m}_j, \vec{m}_j\rangle,$$

The elegant properties come from,

$$\langle\Phi_j|\vec{\sigma}\otimes\mathbb{1}|\Phi_j\rangle = \frac{1}{2}\vec{m}_j$$
$$\langle\Phi_j|\mathbb{1}\otimes\vec{\sigma}|\Phi_j\rangle = -\frac{1}{2}\vec{m}_j.$$



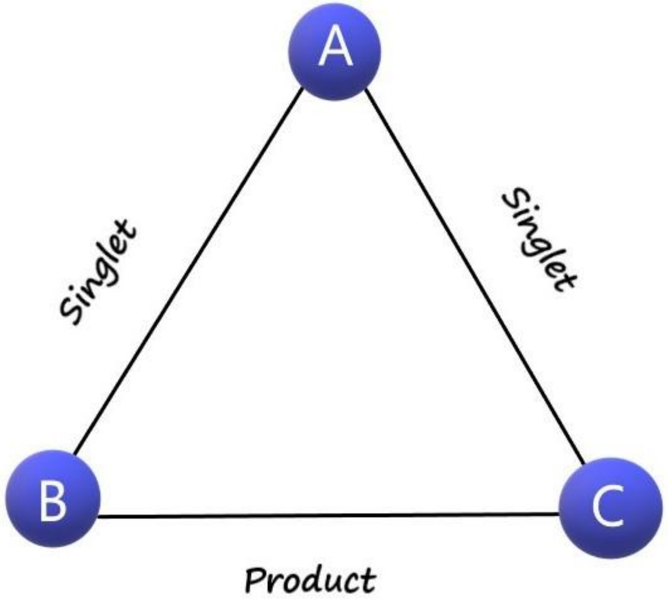
EJM as a basis and maximally entangled states from each source to the parties

Creates Non-Trilocal correlation(Gisin's conjecture).

Later, the neural network method supports Gisin's Conjecture numerically* that $P_{QEJM}(a, b, c)$ is nontrilocal.

*npj Quantum Information (2020) 6:70

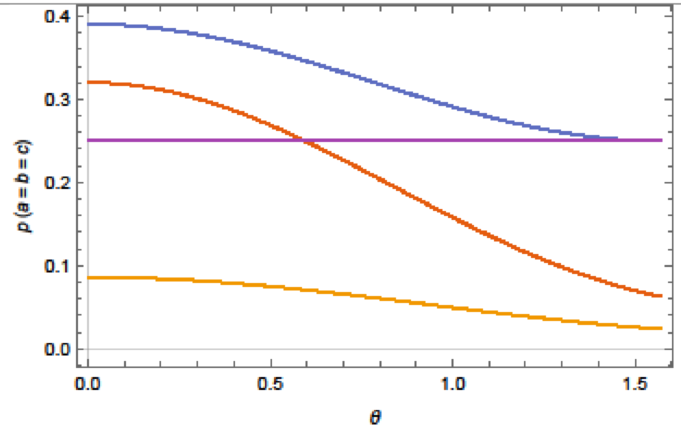
What if not all states from the sources are Entangled?



- Three maximally Entangled states ⟹ Nonlocal
- Two maximally entangled, one product state ⟹ Nonlocal
- Two products, one maximally entangled ⟹ Local

Generalised EJM*

$$|\Phi_b^\theta\rangle = \frac{\sqrt{3}+e^{i\theta}}{2\sqrt{2}}|\vec{m}_b, -\vec{m}_b\rangle + \frac{\sqrt{3}-e^{i\theta}}{2\sqrt{2}}|-\vec{m}_b, \vec{m}_b\rangle$$

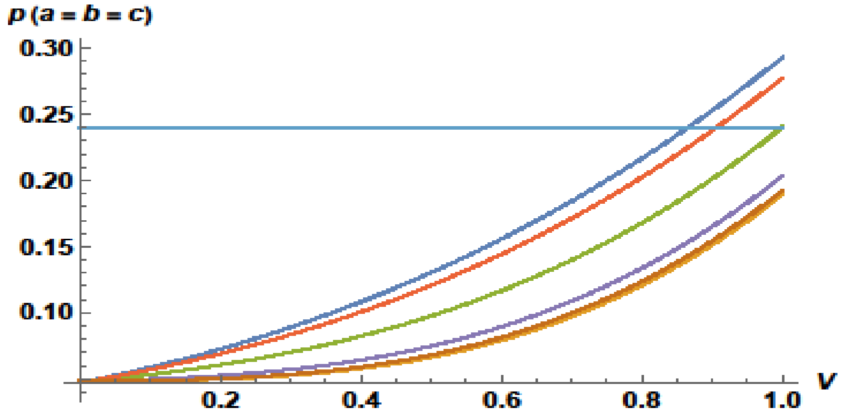


Generalised EJM with a variable θϵ{0, $^{\pi}/_{2}$}

θ=0 → EJM(Gisin's)

θ= $^{\pi}/_{2}$ → BSM

$$|\Psi\rangle = V|\psi^+\rangle\langle\psi^+| + \frac{1-V}{4}\mathbb{I}$$



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*Phys. Rev. Lett. **126**, 220401

#*ANNALEN DER PHYSIK* 2024, 536, 2300297

Nonlocality possible with separable bases?

Considering the No-input minimal triangle network with cardinality 3-3-3.

- Each party have two particles from two different independent sources.
- Each party measure both the particle with a fixed measurement basis.
- This time, the basis is separable, and all the sources distribute maximally entangled states
- $P_Q(a, b, c)$ is Local or Not?

$$\pi_1 = |\psi_1\rangle\langle\psi_1|$$

$$\pi_2 = |\psi_2\rangle\langle\psi_2| + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$$

$$\pi_2 = |\psi_2\rangle\langle\psi_2| + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$$

Where*,

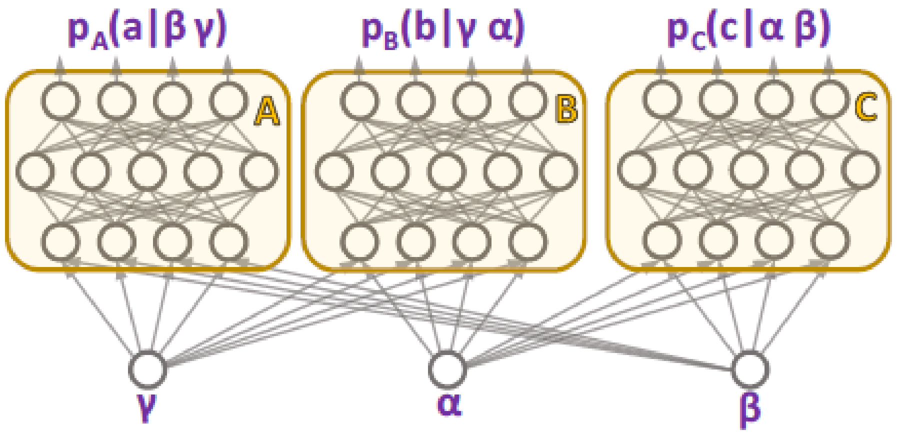
$\psi_1 = |01>$

$\psi_2 = (|\phi^+\rangle - |10\rangle)/\sqrt{2}$

$\psi_3 = (|\phi^+\rangle + |10\rangle)/\sqrt{2}$

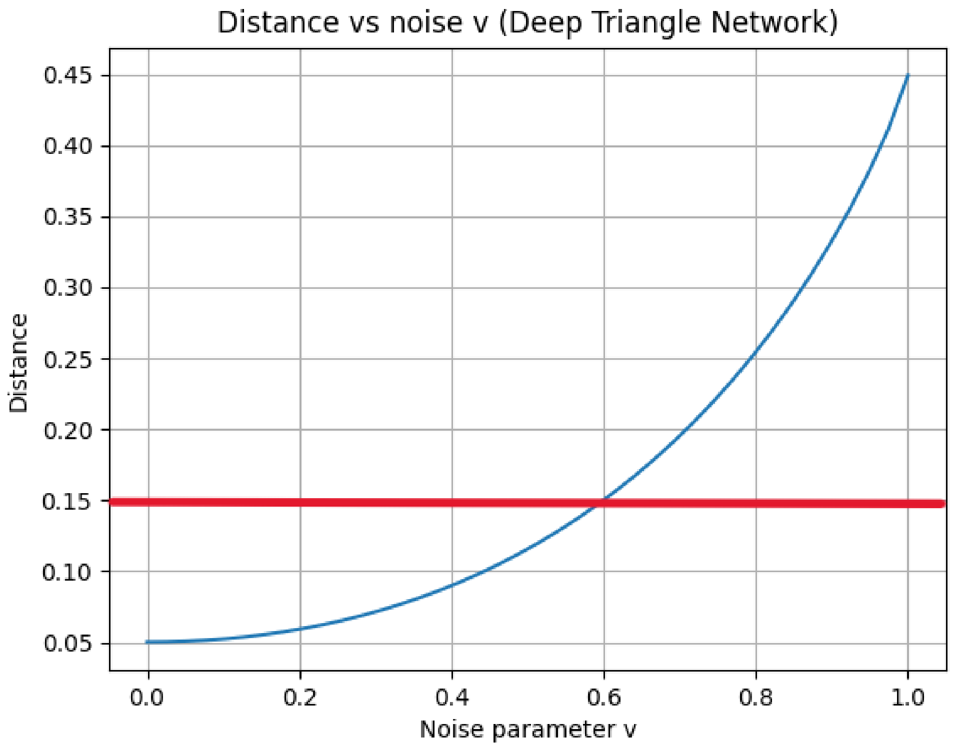
*arXiv:2501.15807

Neural Network Method…

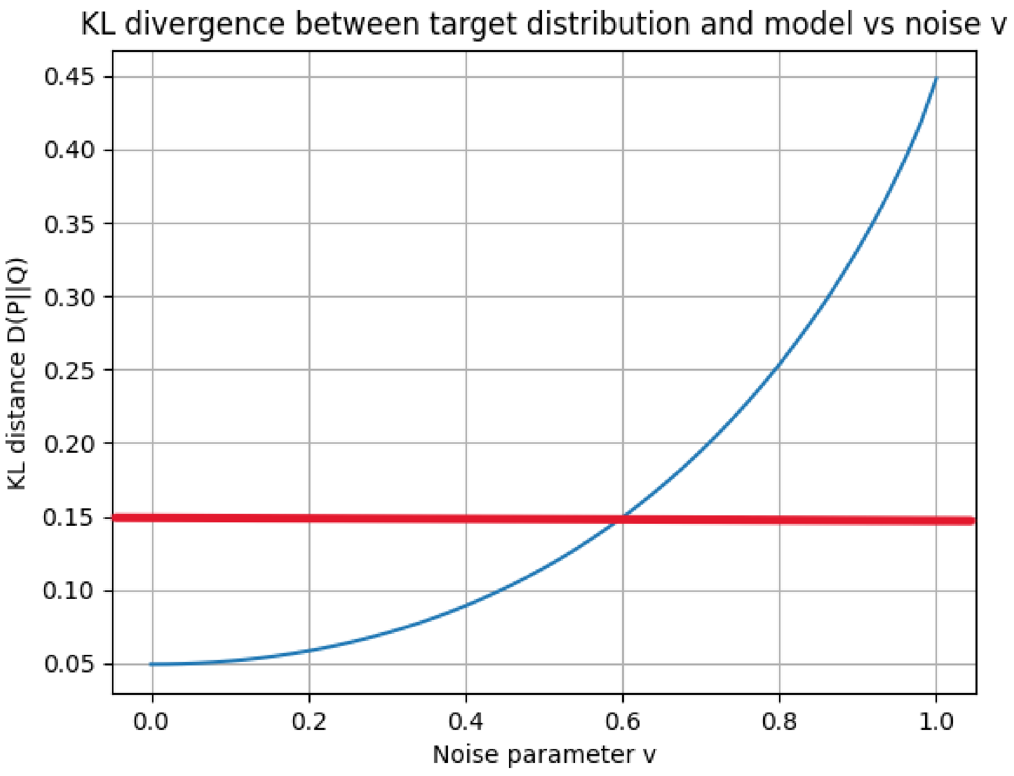


the distance between the target and the learned distribution,

$$d(p_t, p_M) = \sqrt{\sum_{abc} [p_t(abc) - p_M(abc)]^2},$$



Multiple Layer Neural Network



Single Layer Neural Network

Conclusions

- Source Independence is the key point here.
- For an open network, there is a way to capture the network nonlocal correlation.
- No-Input Network is interesting and way more complex.
- A triangle is the simplest No-Input network.
- The general extension is the Square, Pentagon, etc.
- Network correlation will help to build a Quantum internet, multi-party cryptography, Long-Distance communication, etc.

Thank You

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