Quantum singular value transformation without block encodings: Near-optimal complexity with minimal ancilla

Shantanav Chakraborty

CQST and CSTAR, IIIT Hyderabad

arXiv:2504.02385

Based on joint work with

Soumyabrata Hazra (IIIT Hyderabad)
Changpeng Shao and Yuxin Zhang (CAS, Beijing)
Tongyang Li and Xinzhao Wang (Peking University)

ICQIST, TCG Crest, 10 December 2025

Outline

Outline

QSVT

- Quantum singular value transformation (QSVT) is a unifying framework that encapsulates most known quantum algorithms and serves as the foundation for new ones. (e.g. Grover, quantum phase estimation, Hamiltonian simulation, solving linear systems, etc.) [Gilyen-Su-Low-Wiebe, STOC'19]

- It applies polynomial transformations on the singular values of an operator H, provided H is embedded in the top-left block of a unitary, known as block encoding. Namely, (Assume H is Hermitian and $\|H\| \leq 1$)

$$\underbrace{U := \begin{bmatrix} H & * \\ * & * \end{bmatrix}}_{\text{(unitary)}} \xrightarrow[\text{QSVT}]{} \underbrace{\widetilde{U} = \begin{bmatrix} P(H) & * \\ * & * \end{bmatrix}}_{\text{(unitary)}} \approx \begin{bmatrix} f(H) & * \\ * & * \end{bmatrix}$$

Previous works on QSVT

- The quantum circuit for QSVT: Assume $P(x) \in \mathbb{C}[x]$, degree d, even/odd, and $|P(x)| \leq 1$ for all $x \in [-1, 1]$, then there exists $\Phi := (\phi_1, \ldots, \phi_d) \in \mathbb{R}^d$, s.t.

$$\begin{bmatrix} P(H) & * \\ * & * \end{bmatrix} = U_\Phi = \begin{cases} e^{\mathbf{i}\phi_1 Z} U \prod_{j=1}^{(d-1)/2} \left(e^{\mathbf{i}\phi_{2j} Z} U^\dagger e^{\mathbf{i}\phi_{2j+1} Z} U \right), & \text{if } d \text{ is odd,} \\ \prod_{j=1}^{d/2} \left(e^{\mathbf{i}\phi_{2j-1} Z} U^\dagger e^{\mathbf{i}\phi_{2j} Z} U \right), & \text{if } d \text{ is even.} \end{cases}$$

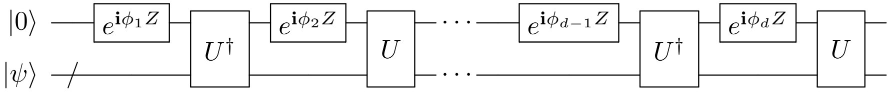


Figure 1: QSVT circuit for even degree d.

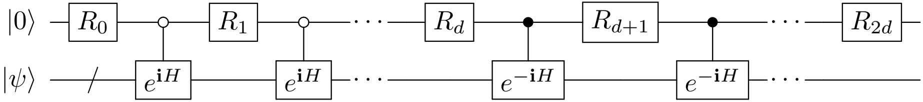
- Constructing the block encoding U is expensive

Generalized Quantum Signal Processing

Assuming access to U = controlled $e^{\mathbf{i}H}$ and $e^{-\mathbf{i}H}$, and extending Pauli-Z rotations in U_Φ to general single-qubit rotations with angles Φ, we can implement

$$U_\Phi = \begin{bmatrix} P(e^{\mathbf{i}H}) & * \\ * & * \end{bmatrix} \approx \begin{bmatrix} f(H) & * \\ * & * \end{bmatrix},$$

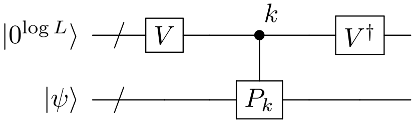
for any degree-d Laurent polynomial $P(z)$ satisfying $|P(z)| \leq 1$ for $|z| = 1$.
[Motlagh-Wiebe, PRX Quantum'24]



- Hence, if $f(x)$ can be approximated by $P(e^{\mathbf{i}x})$, we can implement $f(H)$.
- In order to obtain near-optimal complexity, constructing U is still expensive.

Motivation

- Assume $H = \sum_{k=1}^{L} \lambda_k P_k$ is a Pauli decomposition such that $\lambda = \sum_{k=1}^{L} |\lambda_k|$. How expensive is the block encoding of H?
- Linear combination of unitaries (LCU): By LCU, we can construct a block encoding of H. Circuit depth is $O(L)$, number of ancilla qubits is $O(\log L)$.



$$\text{Here} \qquad V|0^{\log L}\rangle = \sum_{k=1}^{L} \sqrt{\lambda_k/\lambda}\,|k\rangle, \ \text{and } Q = \sum_{k=1}^{L} |k\rangle\langle k| \otimes P_k.$$

$$\text{Block encoding } \ U := (\langle\bar{0}| \otimes I)\,(V^\dagger \otimes I) Q (V \otimes I)\,(|\bar{0}\rangle \otimes I) = \begin{bmatrix} H/\lambda & * \\ * & * \end{bmatrix}.$$

Motivation

We show this to be a lower bound!

Theorem

Assume $H = \sum_{k=1}^{L} \lambda_k P_k$ *be a unitary decomposition of* H *then* $\ell = \Omega(\log L)$ *ancilla qubits are required for exactly block-encoding* H.

For any d-degree polynomial approximating $f(H)$ and $\lambda = \sum_{k=1}^{L} |\lambda_k|$, QSVT estimates

$$\langle \psi_0 | f(H) O f(H) | \psi_0 \rangle \pm \varepsilon,$$

in cost:

Algorithm	Ancilla	Depth per run	# Repetitions
Standard QSVT	$\lceil \log_2 L \rceil + 1$	$\widetilde{O}(L\, d\, \lambda)$	$\widetilde{O}(\|O\|^2/\varepsilon^2)$

We develop new algorithms for QSVT without block encodings!

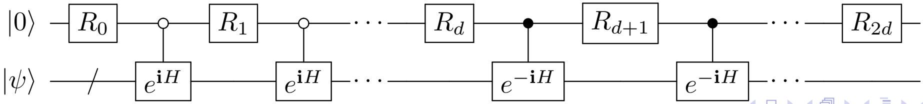
New QSVT algorithm without block encodings

For any function f approximated by a bounded d-degree polynomial, QSVT estimates $\langle\psi_0|f(H)Of(H)|\psi_0\rangle \pm \varepsilon$.

We develop new algorithms for QSVT without block encodings!

Algorithm	Ancilla	Depth per run	# Repetitions
Standard QSVT	$\lceil \log_2 L \rceil + 1$	$\widetilde{O}(L\, d\, \lambda)$	$\widetilde{O}(\|O\|^2/\varepsilon^2)$
QSVT with Trotterization (This work)	**1**	$\widetilde{O}(L\,(d\,\lambda_{\mathrm{comm}})^{1+o(1)})$	$\widetilde{O}(\|O\|^2/\varepsilon^2)$

Broad idea: Trotterization + Generalized QSP + **Classical extrapolation**



Outline

Interleaved sequences of Hamiltonian evolutions

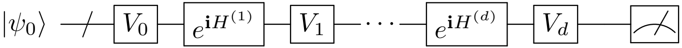
We propose a general framework that encompasses QSVT and extends to a broader class of problems.
This framework efficiently estimates

$$\langle\psi_0|W^\dagger O W|\psi_0\rangle \pm \varepsilon.$$

W is an interleaved sequence of unitary operations and Hamiltonian evolutions

$$W = V_0 \prod_{j=1}^{d} e^{\mathbf{i}H^{(j)}} V_j,$$

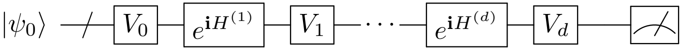
where $H^{(j)} = \sum_{\gamma=1}^{L} H_\gamma^{(j)}$ are Hermitian operators and $\{V_j\}_{j=0}^{d}$ are arbitrary unitaries.



A general framework: Interleaved Hamiltonian evolutions

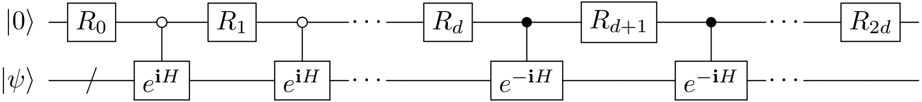
Estimate

$$\langle\psi_0|W^\dagger O W|\psi_0\rangle \pm \varepsilon,$$



- This framework is very general and encompasses problems beyond QSVT (e.g.: discrete adiabatic QC, simulation of time-dependent Hamiltonians, variational QAs, and linear combination of Hamiltonian simulation).

- Replace each $V_j \mapsto R_j \otimes I$ and $H^{(j)} = \mathrm{diag}(H, 0)$ or $\mathrm{diag}(0, -H)$ for all j, then W is simply the GQSP circuit.



A general framework: Interleaved Hamiltonian evolutions

Estimate

$$\langle\psi_0|W^\dagger O W|\psi_0\rangle \pm \varepsilon, \quad \text{where} \quad W = V_0 \prod_{j=1}^{d} e^{\mathbf{i}H^{(j)}} V_j .$$

Here, each $H^{(j)} = \sum_{\gamma=1}^{L} H_\gamma^{(j)}$, and $\{V_j\}_{j=0}^{d}$ are arbitrary unitaries.

Algorithm	Ancilla	Depth per coherent run	# Classical repetitions
Trotterization with extrapolation (This work)	**0**	$\widetilde{O}(L\,(d\,\lambda_{\mathrm{comm}})^{1+o(1)})$	$\widetilde{O}(\lVert O\rVert^2/\varepsilon^2)$

Algorithm design: High-order Trotterization + **Classical extrapolation**

Remark: By extrapolation, the depth of the circuit is reduced to $\mathrm{polylog}(1/\epsilon)$ from $\mathrm{poly}(1/\epsilon)$.

Hamiltonian simulation by Trotter methods

- Given Hamiltonian $H = \sum_{j=1}^{L} H_j$, aim to implement $e^{-\mathbf{i}Ht}$ with precision ε.
- Use p-th order Trotter formula $\mathcal{P}(s)$.
  - $\mathcal{P}(s) \approx e^{-\mathbf{i}Hs}$.
  - Apply $\mathcal{P}(s)$ repeatedly a total of $r = t/s$ times.
  - We have $P(s)^r = e^{-\mathbf{i}tH} + O(\varepsilon)$ for

$$r = O\left(\frac{\lambda_{\text{comm}}^{1/p} t^{1+1/p}}{\varepsilon^{1/p}}\right).$$

- **Problem:** Estimate $\langle\psi_0|e^{\mathbf{i}tH} O e^{-\mathbf{i}tH}|\psi_0\rangle \pm \varepsilon$.
- Using Richardson extrapolation, the circuit depth can be improved from $O(\text{poly}(1/\varepsilon))$ to $O(\text{polylog}(1/\varepsilon))$ [1].

[1] Watson and Watkins, PRX Quantum (2025).

Hamiltonian simulation by Trotterization and extrapolation

- **Richardson extrapolation:** For function $f(x) = f(0) + \sum_{k=1}^{\infty} c_k x^k$. We can choose m sample points $\{s_k\}_{k\in[m]}$ and appropriate coefficients b_k such that

$$\left|\sum_{k=1}^{m} b_k f(s_k) - f(0)\right| \leq O(s_0^m).$$

- We have

$$f(s) = \langle\psi_0|\mathcal{P}(s)^{-t/s} O \mathcal{P}(s)^{t/s}|\psi_0\rangle,$$

and $\lim_{s\to 0} f(s) = \langle\psi_0|e^{\mathbf{i}Ht} O e^{-\mathbf{i}Ht}|\psi_0\rangle$.

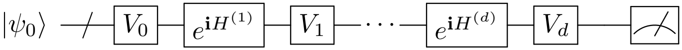
- Using $m = O(\log(1/\epsilon))$ sample points, reduce the circuit depth of Trotterization from $t^{1+1/p}\cdot(1/\epsilon)^{1/p}$ to $t^{1+1/p}\cdot\mathrm{polylog}(1/\epsilon)$.

General interleaved Hamiltonian evolutions

- Recall, we want to estimate

$$\langle\psi_0|W^\dagger O W|\psi_0\rangle \pm \varepsilon, \quad \text{where} \quad W = V_0 \prod_{j=1}^{d} e^{\mathbf{i}H^{(j)}} V_j.$$

  Here, each $H^{(j)} = \sum_{\gamma=1}^{L} H_\gamma^{(j)}$, and $\{V_j\}_{j=0}^{d}$ are arbitrary unitaries.



- Using the Trotter-Suzuki formula with step size s, we can approximate $\langle\psi_0|W^\dagger O W|\psi_0\rangle$ as

$$f(s) = \langle\psi_0|(V_0 \prod_{j=1}^{d} \mathcal{P}_j(s)^{t/s})^\dagger O (V_0 \prod_{j=1}^{d} \mathcal{P}_j(s)^{t/s})|\psi_0\rangle,$$

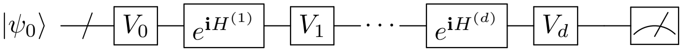
  where $\mathcal{P}_j(s)$ is the Trotter-Suzuki formula for $H^{(j)}$.

General interleaved Hamiltonian evolutions

Recall, we want to estimate

$$\langle\psi_0|W^\dagger O W|\psi_0\rangle \pm \varepsilon, \quad \text{where} \quad W = V_0 \prod_{j=1}^{d} e^{\mathbf{i}H^{(j)}} V_j .$$

Here, each $H^{(j)} = \sum_{\gamma=1}^{L} H_\gamma^{(j)}$, and $\{V_j\}_{j=0}^{d}$ are arbitrary unitaries.



Our contributions: We give a rigorous error expansion of the above $f(s) - f(0)$ to high order in s and bound the extrapolation error in terms of the commutators of $H_\gamma^{(j)}$.

Algorithm	Ancilla	Depth per coherent run	# Classical repetitions
Trotterization with extrapolation (This work)	**0**	$\widetilde{O}(L\,(d\,\lambda_{\mathrm{comm}})^{1+o(1)})$	$\widetilde{O}(\\|O\\|^2/\varepsilon^2)$

QSVT with Trotter

Theorem

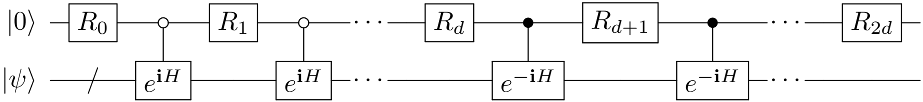
Let $H = \sum_{k=1}^{L} H_k$ be a Hermitian operator. Assume $f(x) \approx P(e^{\mathbf{i}x})$ for degree-d bounded Laurent polynomial P. Then there exists a quantum algorithm that computes $\langle\psi_0|f(H)^\dagger O f(H)|\psi_0\rangle \pm \varepsilon\|O\|$. The algorithm uses only 1 ancilla qubit and has maximum circuit depth

$$\widetilde{O}\Big(L\big(d\lambda_{\mathrm{comm}}\big)^{1+o(1)}\Big),$$

and total time complexity

$$\widetilde{O}\Big(L\big(d\lambda_{\mathrm{comm}}\big)^{1+o(1)}/\varepsilon^2\Big).$$

Here, λ_{comm} scales with the nested commutators of H_j such that $\lambda_{\mathrm{comm}} \le 4\lambda$.



QSVT with Trotter

Theorem

Let $H = \sum_{k=1}^{L} H_k$ be a Hermitian operator. Assume $f(x) \approx P(e^{\mathbf{i}x})$ for degree-d bounded Laurent polynomial P. Then there exists a quantum algorithm that computes $\langle\psi_0|f(H)^\dagger O f(H)|\psi_0\rangle \pm \varepsilon\|O\|$. The algorithm uses only 1 ancilla qubit and has maximum circuit depth

$$\widetilde{O}\big(L(d\lambda_{\mathrm{comm}})^{1+o(1)}\big),$$

and total time complexity

$$\widetilde{O}\big(L(d\lambda_{\mathrm{comm}})^{1+o(1)}/\varepsilon^2\big),$$

Key components: Generalized QSP + higher-order Trotter + classical extrapolation

Algorithm	Ancilla	Depth per run	Classical repetitions
Standard QSVT	$\lceil \log_2 L \rceil + 1$	$\widetilde{O}(L\,d\,\lambda)$	$\widetilde{O}(\|O\|^2/\varepsilon^2)$
QSVT with Trotter (This work)	**1**	$\widetilde{O}(L\,(d\,\lambda_{\mathrm{comm}})^{1+o(1)})$	$\widetilde{O}(\|O\|^2/\varepsilon^2)$

Outline

Quantum linear systems without block encodings

Theorem

Let $H = \sum_{j=1}^{L} \lambda_j P_j$ be an $N \times N$ matrix with condition number κ and $\lambda = \sum_{j=1}^{L} \|\lambda_j\|$. If we can efficiently prepare $|b\rangle$ and define $|x\rangle = H^{-1}|b\rangle / \|H^{-1}|b\rangle\|$, our algorithm estimates $\langle x|O|x\rangle \pm \varepsilon\|O\|$ using 4 ancilla qubits with complexity

$$\widetilde{O}(L(\lambda\kappa)^{1+o(1)}/\varepsilon^2).$$

Algorithm	Ancilla	Depth per run	Classical repetitions
State-of-the-art [Costa et al, PRX Quantum'22]	$\lceil \log_2 L \rceil + 6$	$\widetilde{O}(L\lambda\kappa)$	$\widetilde{O}(1/\varepsilon^2)$
This work	**4**	$\widetilde{O}(L(\lambda\kappa)^{1+o(1)})$	$\widetilde{O}(1/\varepsilon^2)$

Algorithm design: Discretized adiabatic evolution + Quantum Filtering + Classical extrapolation.

Quantum linear systems without block encodings

Theorem

Let $H = \sum_{j=1}^{L} \lambda_j P_j$ be an $N \times N$ matrix with condition number κ and $\lambda = \sum_{j=1}^{L} \|\lambda_j\|$. If we can efficiently prepare $|b\rangle$ and define $|x\rangle = H^{-1}|b\rangle / \|H^{-1}|b\rangle\|$, our algorithm estimates $\langle x|O|x\rangle \pm \varepsilon\|O\|$ using 4 ancilla qubits with complexity

$$\widetilde{O}(L(\lambda\kappa)^{1+o(1)}/\varepsilon^2).$$

Algorithm	Ancilla	Depth per run	Classical repetitions
State-of-the-art [Costa et al, PRX Quantum'22]	$\lceil \log_2 L \rceil + 6$	$\widetilde{O}(L\lambda\kappa)$	$\widetilde{O}(1/\varepsilon^2)$
This work	**4**	$\widetilde{O}(L(\lambda\kappa)^{1+o(1)})$	$\widetilde{O}(1/\varepsilon^2)$

First quantum algorithm for this problem to simultaneously satisfy: (i) constant ancilla, (ii) no oracular assumptions on H, (iii) near-optimal complexity.

Ground State Property Estimation

Theorem

Let $H = \sum_{j=1}^{L} H_j$ *with spectrum in* $[-1,1]$*. If we can prepare* $|\phi_0\rangle$ *with* $|\langle\phi_0|\psi_0\rangle| \geq \gamma$*, our algorithm estimates* $\langle\psi_0|O|\psi_0\rangle \pm \varepsilon\|O\|$ *using two ancilla qubits with complexity* $\widetilde{O}(L(\lambda/(\Delta\gamma))^{1+o(1)}/\varepsilon^2)$*.*

	Ancilla	Depth per run	Classical repetitions
[Lin and Tong, Quantum'20]	$\lceil \log_2 L \rceil + 6$	$\widetilde{O}(L\lambda/(\Delta\gamma))$	$\widetilde{O}(1/\varepsilon^2)$.
This work	**2**	$\widetilde{O}(L(\lambda/(\Delta\gamma))^{1+o(1)})$	$\widetilde{O}(1/\varepsilon^2)$

Algorithm design: Shifted sign function + Classical extrapolation

Ground state property estimation

Theorem

Let $H = \sum_{j=1}^{L} H_j$ *with spectrum in* $[-1, 1]$*. If we can prepare* $|\phi_0\rangle$ *with* $|\langle\phi_0|\psi_0\rangle| \geq \gamma$*, our algorithm estimates* $\langle\psi_0|O|\psi_0\rangle \pm \varepsilon\|O\|$ *using two ancilla qubits with complexity* $\widetilde{O}(L\big(\lambda/(\Delta\gamma)\big)^{1+o(1)}/\varepsilon^2)$*.*

	Ancilla	Depth per run	Classical repetitions
[Lin and Tong, Quantum'20]	$\lceil\log_2 L\rceil + 6$	$\widetilde{O}(L\lambda/(\Delta\gamma))$	$\widetilde{O}(1/\varepsilon^2)$.
This work	**2**	$\widetilde{O}(L\big(\lambda_{\mathrm{comm}}/(\Delta\gamma)\big)^{1+o(1)})$	$\widetilde{O}(1/\varepsilon^2)$

First quantum algorithm for this problem to simultaneously satisfy: (i) constant ancilla, (ii) no oracular assumptions on H, (iii) near-optimal complexity.

Outline

Open questions

- Can we achieve circuit depth $\widetilde{O}(d)$ instead of $O(d^{1+o(1)})$?
- Can we generalize our results for interleaved Hamiltonian evolutions to incorporate quantum channels?
- Can we jointly mitigate the algorithmic error (Trotterization error) and physical error (hardware noise)?

Randomized QSVT

- We considered $H = \sum_{j=1}^{L} \lambda_j P_j$ such that $\lambda = \sum_j |\lambda_j|$.
- The complexity of our algorithm depended linearly in L. For many Hamiltonians in quantum chemistry and condensed matter physics, L is prohibitively large.
- In a separate work, we develop randomized quantum algorithms for QSVT (still without block encodings, and with only one ancilla).
- We assume *sampling access* to H: each P_j is sampled according to $|\lambda_j|/\lambda$.
- Overall complexity $\widetilde{O}(\lambda^2 d^2)$, is independent of L. The quadratic dependence on d is optimal within this access model.
- We numerically benchmark the gate complexity of the ground state property estimation algorithm for quantum chemistry Hamiltonians.
- Our method requires shallower circuits by several orders of magnitude. For more details, check out: arXiv:2510.06851 and **Soumyabrata's poster**!

Thank You!

"The first time it was reported that our friends were being butchered there was a cry of horror. Then a hundred were butchered. But when a thousand were butchered and there was no end to the butchery, a blanket of silence spread. When evil-doing comes like falling rain, nobody calls out *stop*!

When crimes begin to pile up they become invisible. When sufferings become unendurable the cries are no longer heard. The cries, too, fall like rain in summer."

Bertolt Brecht (When Evil-Doing Comes Like Falling Rain)