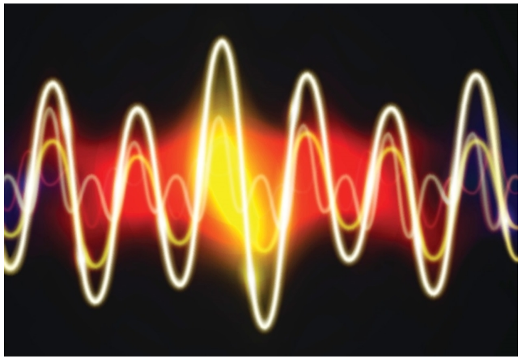
Enhancing wave-particle duality



Almut Beige
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ICQIST, Kolkata, December 2025

The Quantum Group

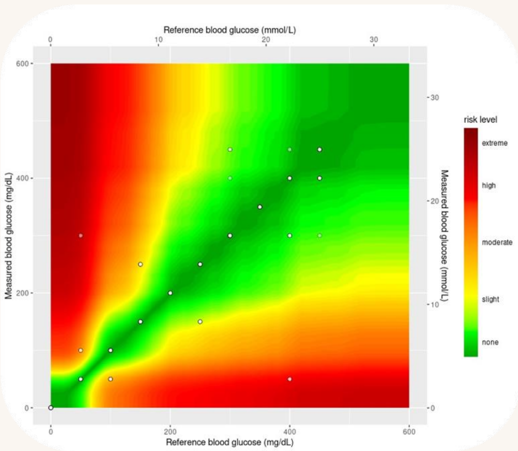
UNIVERSITY OF LEEDS

All members are involved in solving challenges of modern Theoretical Physics: from Quantum Physics to Computational Biophysics.

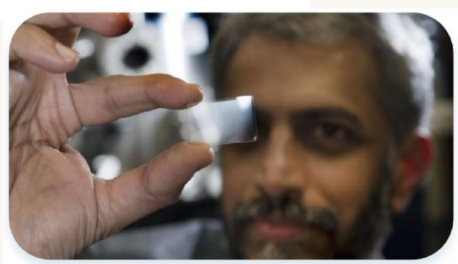
Example:

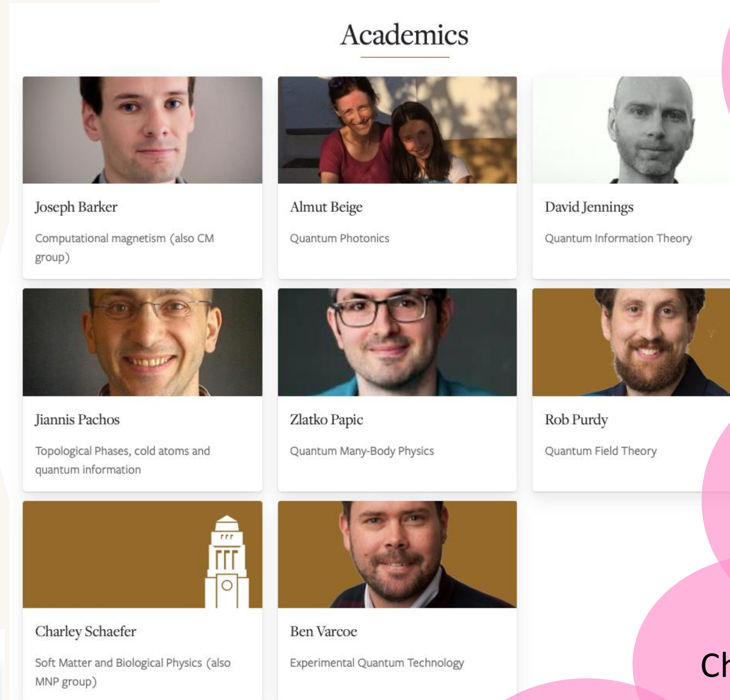
Quantum technologies for building a secure and resilient world:

- Quantum sensing
- Quantum simulations
- Quantum position, navigating and timing technology



NIQS





Mathe-matics

SCAPE

Chemistry

Physics and Astronomy

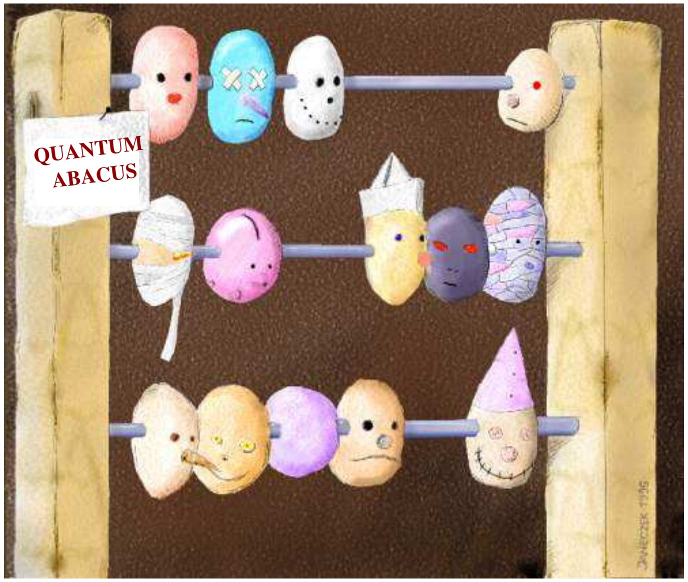
Overview

I Local photons

II The quantised EM field

III Modelling photon emission

IV Final comments



I

Local photons [1,2]

[1] Hodgson *et al.*, *Local photons*, Front. Photon. **3**, 978855 (2022).

[2] Waite *et al.*, *Local photon model of the momentum of light*, Phys. Rev. A **111**, 023703 (2025).

Wave-particle duality

Wave-particle duality is the cornerstone of quantum physics.
Nevertheless, we treat photons and massive particles differently.

- **Massive particles:**
  Their wave packets have a position and a momentum representation.
  They also have a position operator.

- **Photons:**
  Wave packets of light are made up of monochromatic waves which cannot be localised?

Enhancing wave-particle duality

Canonical quantisation:

- We identify canonical variables and impose canonical commutator relations.

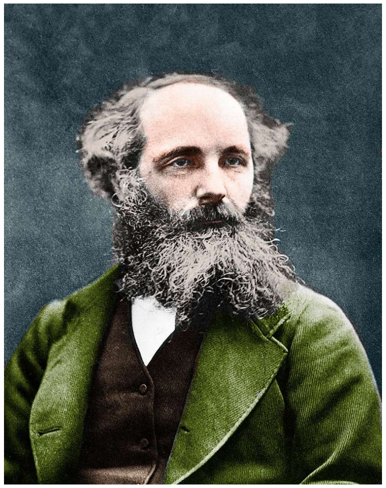
(Difficult task!)

A more physically-motivated approach:

- We identify the unique states of the classical system and map them onto pairwise orthogonal quantum states.
- The Hamiltonian is designed such that the most classical quantum states evolve as they would classically.

The resulting quantum theory is automatically consistent with classical physics.

Maxwell's equations



$$
\begin{aligned}
\vec{\nabla}\cdot\vec{E} &= 0\\
\vec{\nabla}\cdot\vec{B} &= 0\\
\vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\\
\vec{\nabla}\times\vec{B} &= \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}
\end{aligned}
$$

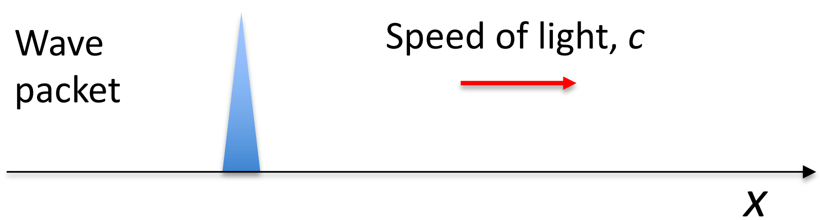
The basic solutions of MW's equations in free space are **waves**:

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{O}(x,t)=0 \quad \text{with } \boldsymbol{O}=\boldsymbol{E},\boldsymbol{B}$$

An alternative view

The basic solutions of MW's equations in free space are wave packets of any shape which travel at the speed of light either left or right:

$$\left(\frac{\partial}{\partial x}+\frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x}-\frac{1}{c}\frac{\partial}{\partial t}\right)\boldsymbol{O}(x,t)=0 \quad \text{with } \boldsymbol{O}=\boldsymbol{E},\boldsymbol{B}$$

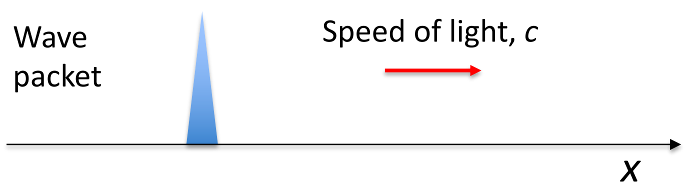


This includes highly-localised WPs which remain localised!

Local carriers of light

Suppose photons consists of **blips** ($=$ bosons localised in position) with creation operators $a^{\dagger}_{s\lambda}(x)$.

$$\begin{aligned} s = \pm 1 \quad &: \quad \text{direction of propagation} \\ \lambda = \mathsf{H}, \mathsf{V} \quad &: \quad \text{polarisation} \\ x \in (-\infty, \infty) \quad &: \quad \text{position} \end{aligned}$$



Commutator relations

The state of a single local excitation:
$|x_{s\lambda}\rangle = a^{\dagger}_{s\lambda}(x)|0\rangle$

These states are pairwise orthogonal for **bosonic** blips:

$$\begin{aligned}
\langle x_{s\lambda}|x'_{s'\lambda'}\rangle &= \langle 0|a_{s\lambda}(x)a^{\dagger}_{s'\lambda'}(x')|0\rangle \\
&= \left[a_{s\lambda}(x), a^{\dagger}_{s'\lambda'}(x')\right] \\
&= \delta_{s,s'}\,\delta_{\lambda,\lambda'}\,\delta(x-x')
\end{aligned}$$

Transformation into momentum space

x-space:

$x \in (-\infty, \infty)$
$\lambda = \mathsf{H}, \mathsf{V}$
$s = \pm 1$

k-space:

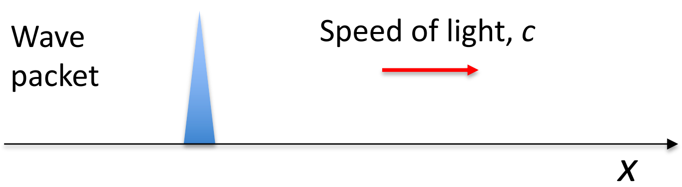
$k \in (-\infty, \infty)$
$\lambda = \mathsf{H}, \mathsf{V}$
$s = \pm 1$

When transferring the $a_{s\lambda}(x)$ via Fourier transforms, we obtain bosonic annihilation operators $\tilde{a}_{s\lambda}(k)$ for monochromatic field excitations:

$$\tilde{a}_{s\lambda}(k) \;\; = \;\; \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \mathrm{d}x \, \mathrm{e}^{-\mathrm{i}skx} \, a_{s\lambda}(x)$$

$$\text{with} \;\; \left[\tilde{a}_{s\lambda}(k), \tilde{a}^{\dagger}_{s'\lambda'}(k')\right] = \delta_{s,s'} \, \delta_{\lambda,\lambda'} \, \delta(k - k')$$

The Schrödinger equation



In the Heisenberg picture:

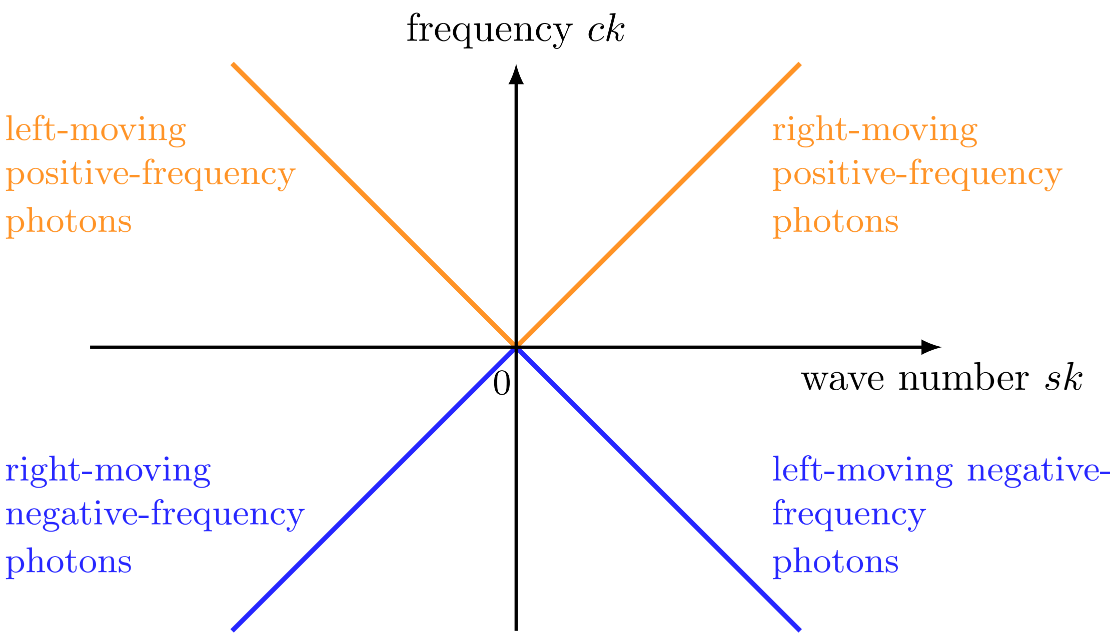
$$a_{s\lambda}(x,t) = a_{s\lambda}(x - sct, 0)$$

The dynamical Hamiltonian of blips:

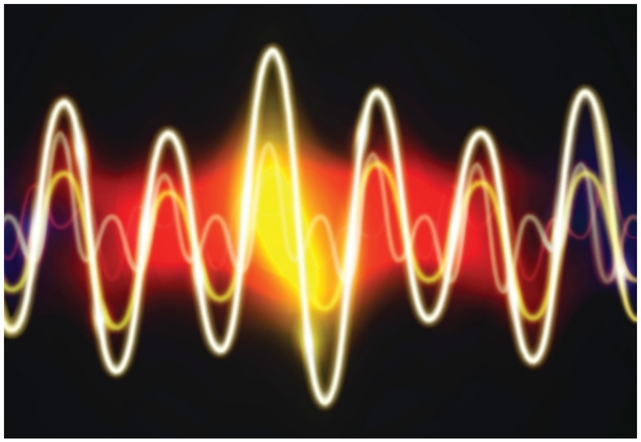
$$\begin{aligned} H_{\mathrm{dyn}} &= -\mathrm{i}\hbar \sum_{s=\pm 1} \sum_{\lambda=\mathrm{H,V}} \int_{-\infty}^{\infty} \mathrm{d}x \, sc \, a_{s\lambda}^{\dagger}(x') \, \frac{\partial}{\partial x} \, a_{s\lambda}(x) \\ &= \sum_{s=\pm 1} \sum_{\lambda=\mathrm{H,V}} \int_{-\infty}^{\infty} \mathrm{d}k \, \hbar ck \, \tilde{a}_{s\lambda}^{\dagger}(k) \tilde{a}_{s\lambda}(k) \end{aligned}$$

. . . has positive and negative eigenvalues

Frequencies and wave numbers



The basic building blocks of photons



There are different carriers of light, local and non-local:

1. **Monochromatic waves:**
   These correspond either to standing or to travelling waves.

2. **Localised wave packets:**
   blips = bosons localised in position

Single-photon wave packets of any shape can be obtained by superposing either waves or blips.

Enhanced wave-particle duality

- The position and momentum eigenstates of single photons:

$$|x\rangle = a^{\dagger}_{s\lambda}(x)\,|0\rangle\,, \quad |k\rangle = \tilde{a}^{\dagger}_{s\lambda}(k)\,|0\rangle$$

$$\text{with } \langle x|x'\rangle = \delta(x-x')\,, \quad \langle k|k'\rangle = \delta(k-k')$$

- The wave function of a single photon:

$$|\psi\rangle = \sum_{s\lambda}\int_{\mathcal{R}} \mathrm{d}x\,\psi_{s\lambda}(x)\,|x\rangle = \sum_{s\lambda}\int_{\mathcal{R}} \mathrm{d}k\,\widetilde{\psi}_{s\lambda}(k)\,|k\rangle$$

- Position and momentum operators: $\hat{x}$ and $\hat{p}$

"We can do quantum mechanics with photons."

II

The quantised EM field [1,2]

[1] Waite *et al.*, *Local photon model of the momentum of light*, Phys. Rev. A **111**, 023703 (2025).

[2] Hodgson *et al.*, *Picturing the Casimir effect without regularisation*, arXiv:2203.14385 (2025).

The energy observable

- By definition, H_{dyn} is the generator for time translations:

$$H_{\mathsf{dyn}}\,|\psi\rangle = -\mathrm{i}\hbar\,\frac{\partial}{\partial t}\,|\psi\rangle \quad \Rightarrow \quad H_{\mathsf{dyn}} = \sum_{s,\lambda}\int_{-\infty}^{\infty} \mathrm{d}k\,\hbar ck\,a^{\dagger}_{s\lambda}(k)a_{s\lambda}(k)$$

- The energy observable H_{eng} should always be positive:

$$H_{\mathsf{eng}} = \left\{ \begin{array}{ll} -H_{\mathsf{dyn}} & \text{for } k < 0 \\ +H_{\mathsf{dyn}} & \text{for } k \geq 0 \end{array} \right\} = \sum_{s,\lambda}\int_{-\infty}^{\infty} \mathrm{d}k\,\hbar c|k|\,a^{\dagger}_{s\lambda}(k)a_{s\lambda}(k)$$

- With respect to the complex field vectors $\boldsymbol{\mathcal{E}}^{\dagger}(x)$ and $\boldsymbol{\mathcal{B}}^{\dagger}(x)$:

$$H_{\mathsf{eng}} = \frac{A}{4}\int_{-\infty}^{\infty} \mathsf{d}x\left[\varepsilon\,\boldsymbol{\mathcal{E}}^{\dagger}(x)\cdot\boldsymbol{\mathcal{E}}(x) + \frac{1}{\mu}\boldsymbol{\mathcal{B}}^{\dagger}(x)\cdot\boldsymbol{\mathcal{B}}(x)\right]$$

Complex electric and magnetic field amplitudes

We want that an (s, λ, k) photon has the energy $\hbar c|k|$.

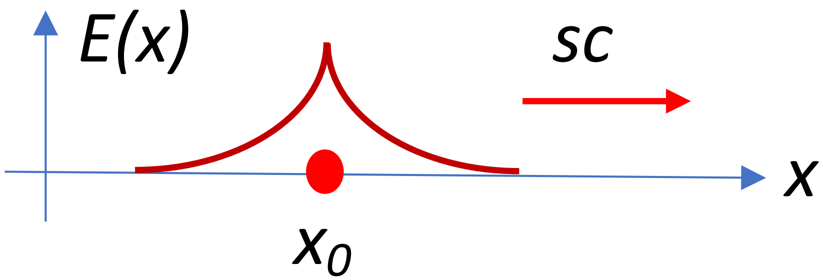
$$\Rightarrow \quad \mathcal{E}_{s\lambda}(x) = \int_{-\infty}^{\infty} \mathrm{d}x' \, g(x, x') \, a_{s\lambda}(x')$$

$$\mathcal{B}_{s\lambda}(x) = \frac{s}{c} \int_{-\infty}^{\infty} \mathrm{d}x' \, g(x, x') \, a_{s\lambda}(x')$$

$$\text{with} \quad g(x, x') = \left(\frac{\hbar c}{2\pi^2 \varepsilon_0 A} \right)^{1/2} \int_{-\infty}^{\infty} \mathrm{d}k \, \sqrt{|k|} \, \mathrm{e}^{\mathrm{i}k(x-x')}$$

$$= -\left(\frac{\hbar c}{4\pi \varepsilon_0 A} \right)^{1/2} \frac{1}{|x - x'|^{3/2}}$$

A physical picture of blips



- Local blips at x_0 can be felt everywhere.
- Localised fields can only be created by a non-local source.
- We now have positive and negative frequency photons.

Similarities with gravitational fields

Like massive objects, local photons
are **local carriers** of **non-local fields**.

The origin of the Casimir effect

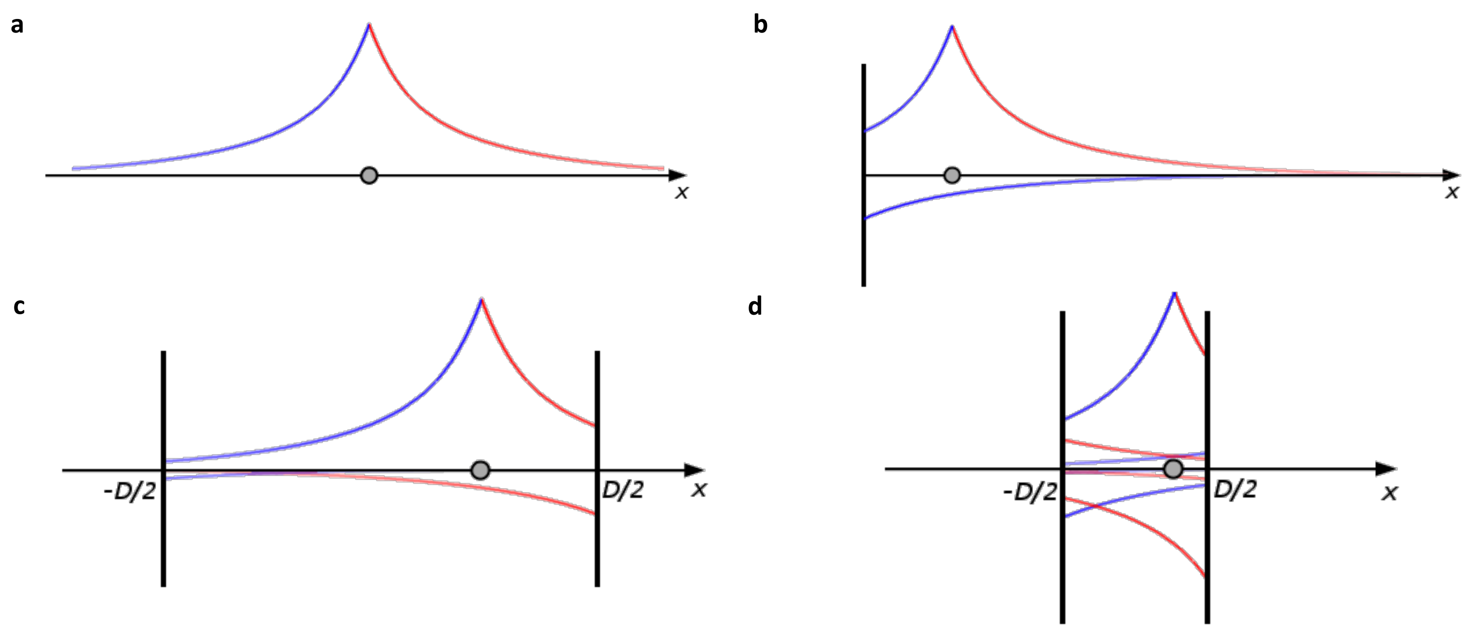


Figure 2: **a.** Because of the regularisation operator $\mathcal{R}$ in Eq. (6), local blip excitations contribute to local electric and magnetic field expectation values everywhere along the x axis (cf. Eq. (8)). **b.** Since a blip on one side of a highly reflecting mirror cannot contribute to the field expectation value on the other side, its field contribution must be folded back on itself. This effect alters the electric and magnetic field observables in the presence of a mirror. **c.** In the presence of two highly reflecting mirrors, blips outside the cavity cannot contribute to field expectation values on the inside. Moreover, the field contributions of blips on the inside need to be folded as in the case of one mirror. Now, however, the field contributions must be folded infinitely many times (cf. Eq. (18) in Methods). **d.** Comparing two cavities of different sizes, we see that the behaviour of the field contribution is now dependent on the cavity width.

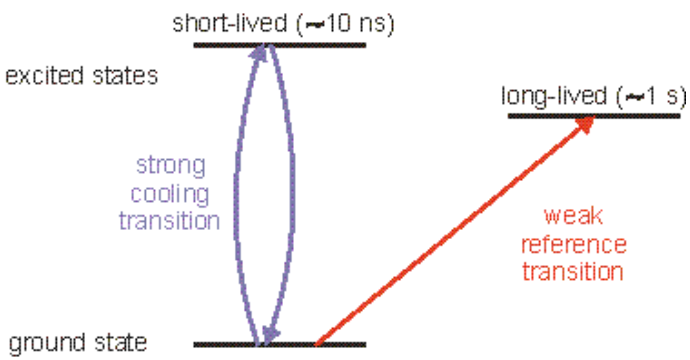
III

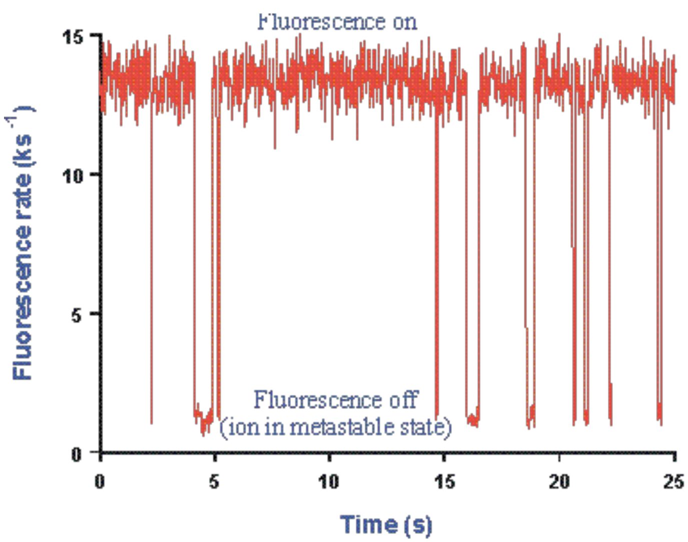
Modelling photon emission [1]

[1] Hartwell, *Photon emission without quantum jumps*, arXiv:2509.01702 (2025).

Macroscopic quantum jumps

The existence of a random telegraph signal in the fluorescence of single ions, was predicted as early as 1975 by Dehmelt.[1,2]

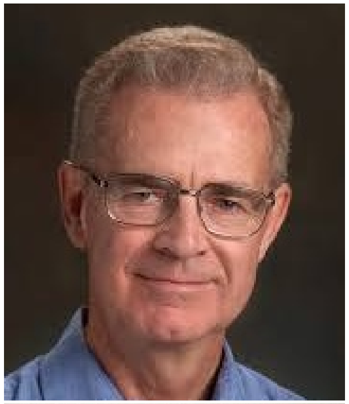




[1] Dehmelt, Bull. Am. Phys. Soc. **20**, 60 (1975).

[2] Nagourney *et al.*, PRL **56**, 2797 (1986); Sauter *et al.*, PRL **57**, 1696 (1986); Bergquist *et al.*, PRL **57**, 1699 (1986).

Ballentine's objection



The experimental observation of these macroscopic light and dark periods in the 1980s, despite some initial criticism of this interpretation, [1,2] eventually manifested the belief that spontaneous photon emission and quantum jumps are two closely related phenomena.

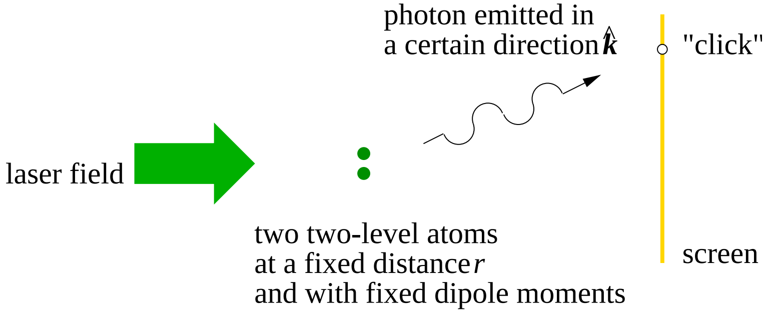
However:
Spontaneously emitted photons can interfere!

[1] Ballentine, L. E. Comment on "Quantum Zeno effect", Phys. Rev. A **43**, 5165 (1991).

[2] Itano *et al.*, Reply to "Comment on 'Quantum Zeno effect"' Phys. Rev. A **43**, 5168 (1991).

A two-atom double-slit experiment

Experiment proposed by Scully and Drühl: [1]



Experimental results by Eichmann *et al.*: [2]

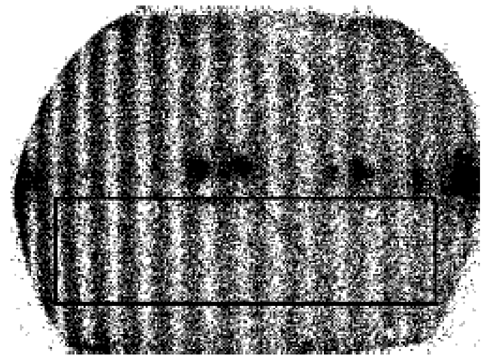


FIG. 5. Experimental fringe data for the case in which the detected light is polarized in the same direction as the incoming light (π case). The ion separation $d = 4.17$ μm. The angle ϕ (the deviation from the forward-scattering direction) increases to the right. The decrease in visibility with increasing ϕ is due to thermal motion of the ions. The dark spots are due to stray reflections of the laser beams. The data within the rectangle were summed along the vertical direction and least-squares fitted.

[1] Scully and Drühl, PRA **25**, 2208 (1982).

[2] Eichmann, Berquist, Bollinger, Gilligan, Itano, Wineland, and Raizen, PRL **70**, 2359 (1993).

A locally-acting Hamiltonian

Locality, thermodynamics, particle conservation and other observations suggest that the Hamiltonian of emitter and field can be written as

$$\begin{aligned} H &= H_{\mathrm{E}} + H_{\mathrm{F}} + H_{\mathrm{int}} \\ H_{\mathrm{int}} &= \hbar g \, a^{\dagger}(0)\sigma^{-} + \mathrm{H.c.} \end{aligned}$$

$$\text{with} \quad a^{\dagger}(0) = \frac{1}{g} \sum_{\lambda=\mathsf{H},\mathsf{V}} \int_{\mathcal{S}} \mathrm{d}^2 s \, g_{\boldsymbol{s}\lambda} \, a^{\dagger}_{\boldsymbol{s}\lambda}(0)$$

The effect of this Hamiltonian H can be analysed without approximations.

Quantised light from a point source

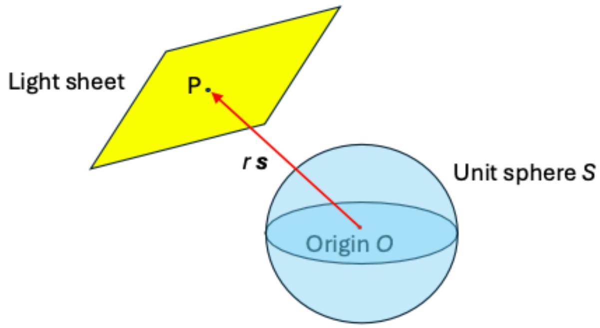


FIG. 2. To find a convenient parametrisation of the local excitations of the quantised electromagnetic field coming from a point-like source at the origin O, we consider light sheets (yellow planes). Each plane can be characterised by a unit vector $\boldsymbol{s}$ and its $r \in (-\infty, \infty)$ from the source. Here $\boldsymbol{s}$ indicates the direction of propagation and negative and positive distances r imply that the light sheet is moving towards and away from the origin, respectively. Here we are only interested in the local field excitations with annihilation operators $a_{\boldsymbol{s}\lambda}(r)$ placed at the point P which is closest to the origin, since we are only modelling light originating from $\mathcal{O}$.

- The blips coming from a point source can be parametrised by a set of parameters $(\boldsymbol{s}, \lambda, r)$.

- These correspond to annihilation operators $a_{\boldsymbol{s}\lambda}(r)$ with

$$\left[a_{\boldsymbol{s}\lambda}(r), a^{\dagger}_{\boldsymbol{s}'\lambda'}(r')\right] = \frac{1}{r^2}\,\delta^2(\boldsymbol{s}-\boldsymbol{s}')\,\delta_{\lambda\lambda'}\,\delta(r-r')\,.$$

The solution of Schrödinger's equation

Suppose, emitter and field have been prepared in $|\psi(0)\rangle = |0_{\mathrm{F}}, 1_{\mathrm{E}}\rangle$.

$$|\psi(t)\rangle \quad = \quad c_0(t)\,|0_{\mathrm{F}}, 1_{\mathrm{E}}\rangle + \int_0^\infty \mathrm{d}r\, c_r(t)\,|r_{\mathrm{F}}, 0_{\mathrm{E}}\rangle$$

$$c_0(t) = \mathrm{e}^{-\frac{1}{2}\Gamma t}\,\mathrm{e}^{-\mathrm{i}\omega_0 t}$$

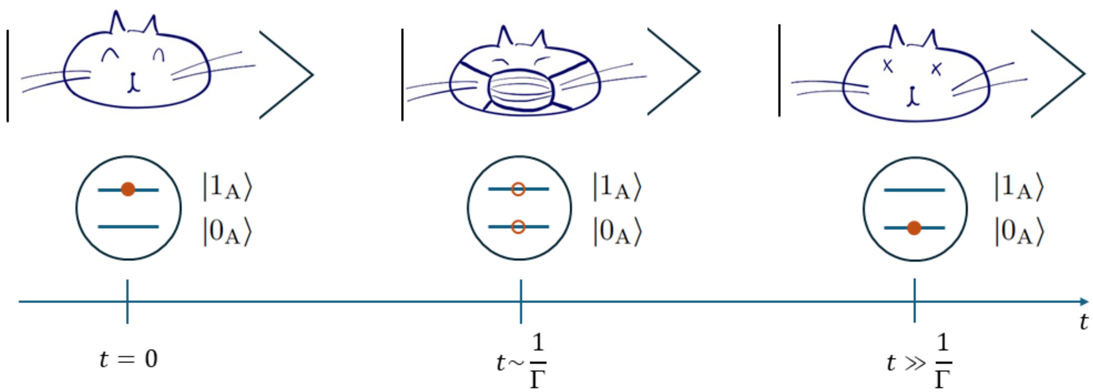
$$c_r(t) = \left\{ \begin{array}{cl} -\mathrm{i}\,\left(\frac{\Gamma}{c}\right)^{1/2}\mathrm{e}^{-\frac{1}{2}\Gamma(t-r/c)}\,\mathrm{e}^{-\mathrm{i}\omega_0(t-r/c)} & \text{for} \quad t < rc \\ 0 & \text{for} \quad t > rc \end{array} \right\}$$

ω_0: transition frequency of the emitter

$\Gamma = g^2/c$: spontaneous decay rate

The dynamics of the emitter

Interpretation:



Relation to **quantum optical master equations**:
These describe the dynamics of $\rho_E(t) = \mathrm{Tr}_F(\,|\psi(t)\rangle\langle\psi(t)|\,)$:

$$\dot{\rho}_E \quad = \quad -\frac{i}{\hbar}[H_E, \rho_E] + \frac{\Gamma}{2}\left(2\,\sigma^- \rho_E \sigma^+ - \sigma^+\sigma^- \rho_E - \rho_E \sigma^+ \sigma^-\right)$$

The properties of the emitted light

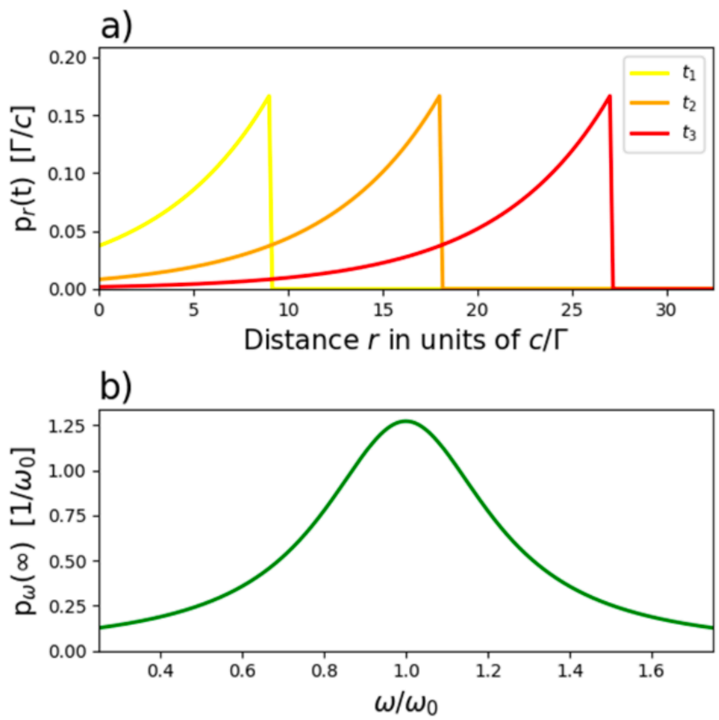


FIG. 3. (a) Probability density $p_r(t)$ in Eq. (15) to detect a photon at time t a distance r away from the emitter. (b) Probability density $p_\omega(\infty)$ in Eq. (18) for the emitted photon to have the frequency ω after a sufficiently long time, i.e. when all energy has left the emitter, confirming that the spectrum for emitted light is indeed Lorentzian.

A more intuitive picture of photon emission for applications, like quantum sensing

- The phase of the emitted light is the phase of the source.
- The spatial distribution of the emitted light reflects the internal structure of the emitter.
- Complementary information in fluorescence emission profiles and in the spectrum of the emitted light.
- We can now model more complex scenarios, including the presence of partially-transparent mirror surfaces, inhomogeneous media, far-field interference, ...

IV

Final remarks

Comments

We quantised the EM field in 1D **in position space.**
Previous **no-go theorems** can be overcome by doubling the Hilbert space and allowing for positive- and negative-frequency photons.

This approach allows us to:

- better understand quantum effects (Casimir, Doppler, Unruh)
- model photon emission in a more intuitive way
- introduce locally-acting mirror Hamiltonians
- ...

Enhanced wave-particle duality

There are still differences in how quantum physics treats photons and QM particles:

- **Photons:**
  Wave packets of light of any shape travel at only one speed.

- **Massive particles:**
  Momentum and velocity are synonymous. Hence we cannot choose the velocity of a QM particle independent of the shape of its wave packet.

Some references

1. **Local photons**,
   D. Hodgson *et al.*, Front. Photon. **3**, 978855 (2022).

2. **Local photon model of the momentum of light**,
   G. Waite *et al.*, Phys. Rev. A **111**, 023703 (2025).
   (Editor's Suggestion)

3. **An intuitive picture of the Casimir effect**,
   D. Hodgson *et al.*, arXiv:2203.14385 (2025).

4. **Photon emission without quantum jumps**,
   T. Hartwell *et al.*, arXiv:2509.01702 (2025).

5. **Enhancing wave-particle duality**,
   A. Bukhari *et al.*, New J. Phys. **27**, 084501 (2025).

See also related work by Dirac, R. J. Cook, Hawton, Mostafazadeh, Pendry and others.