– Anomalous Landau levels –
From fundamentals to simulations

ICQIST, 2025

CQuERE, TCG CREST

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In collaboration with

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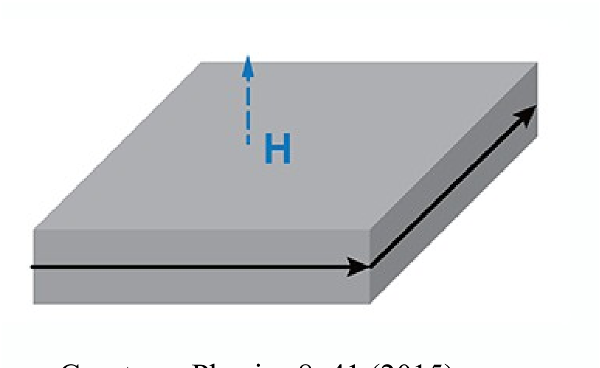
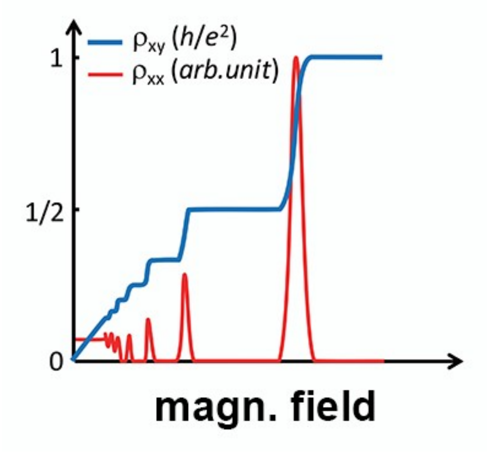
arXiv:2509.20462

The Quantum Hall effect is the first realized topological phase of matter, laying the foundation for all successive developments in modern topological physics

Resistance quantum → precision metrology

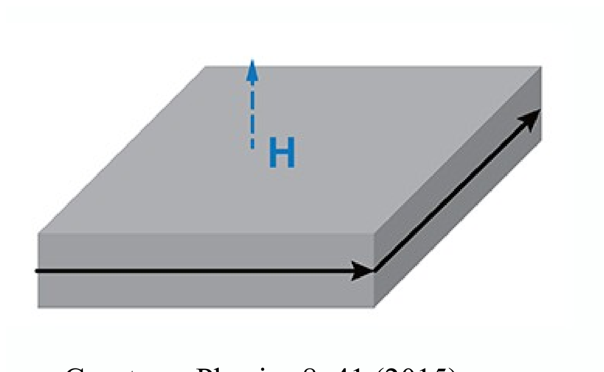


Courtesy: Physics 8, 41 (2015)



Landau quantization remains the cleanest way to understand how particles respond to magnetic flux and this physics reappears in many non-magnetic systems

Synthetic gauge fields → designer Landau spectra



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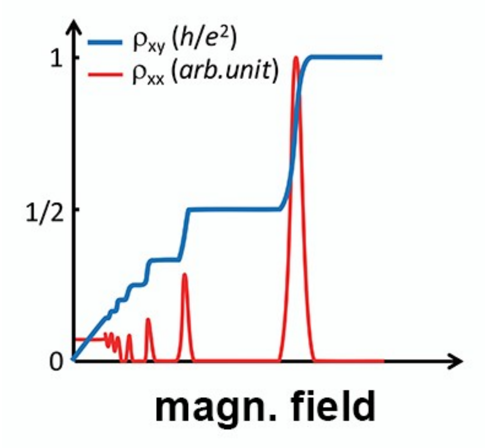

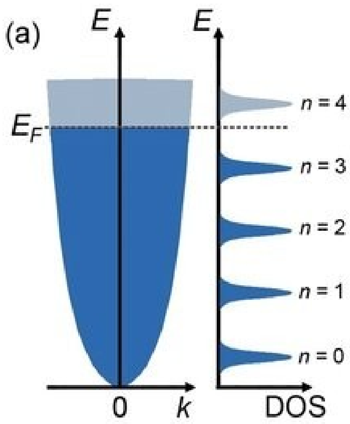

Courtesy: Fei et al. (2019)

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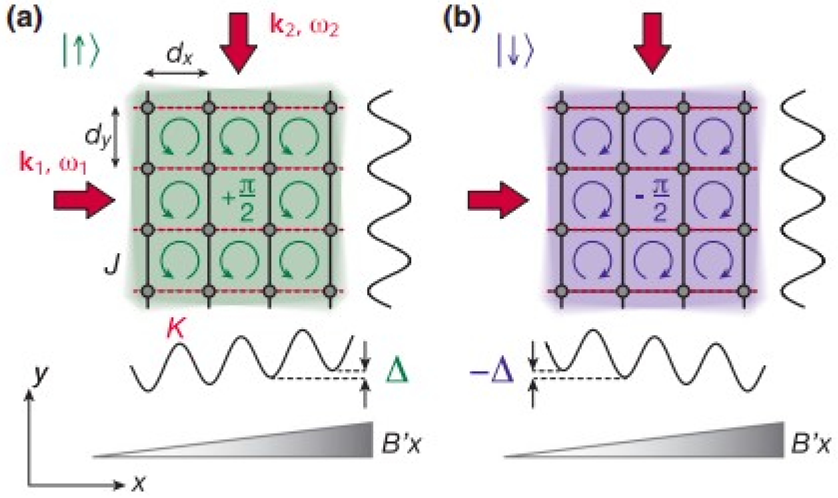

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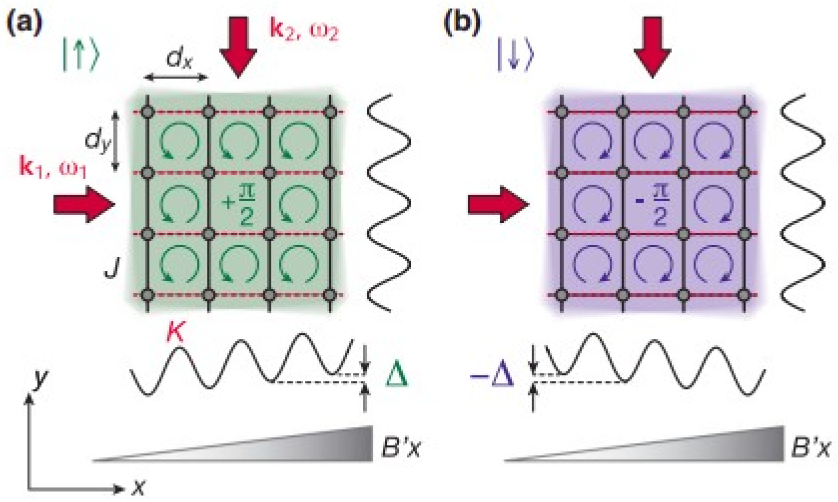

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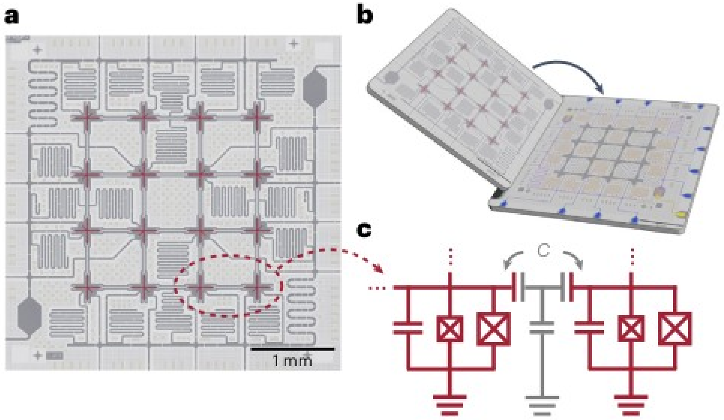
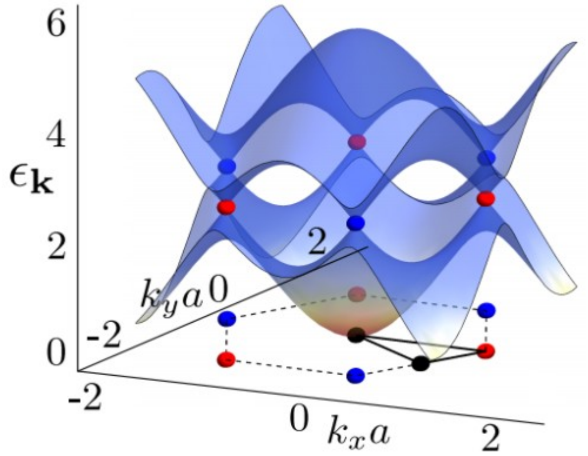
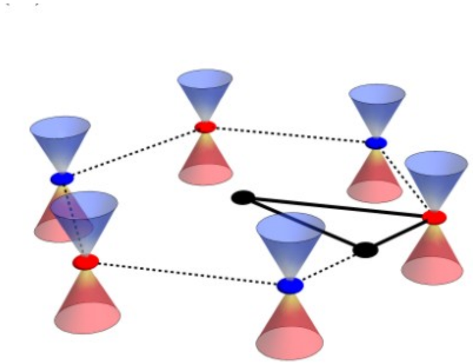


Rosen et al (2024)

Graphene realizes relativistic Landau quantization, producing a zero-energy Landau level and half-integer quantum Hall effect



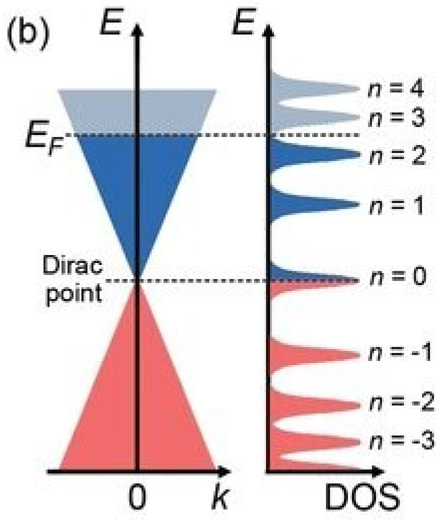
Courtesy: Balatsky



Relativistic dispersions around ***K*** and ***K'*** points

Graphene realizes relativistic Landau quantization, producing a zero-energy Landau level and half-integer quantum Hall effect

- Landau level quantization $E_n \sim \pm\sqrt{n}$
- Hall quantization $\sigma_{xy} = \nu\frac{e^2}{h}$; $\nu = 4(n + 1/2)$
- Enables practical quantum resistance standards.



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Landau levels for generic dispersions: EBK+Onsager prediction

Onsager semi-classical relation (EBK quantization): *Fermi surface area perp. to the applied magnetic field is quantized*

$$\boldsymbol{\int d^2k = 2\pi e B (n + \gamma)}$$

Berry phase

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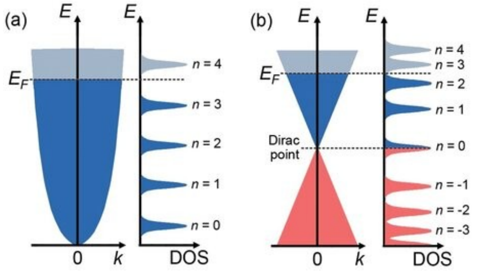

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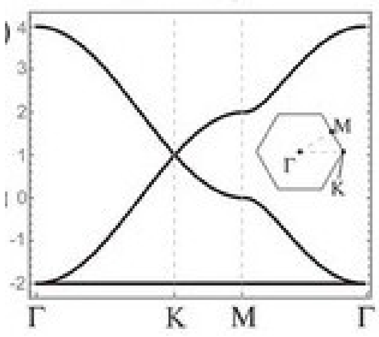
What happens in the absence of intrinsic scales?



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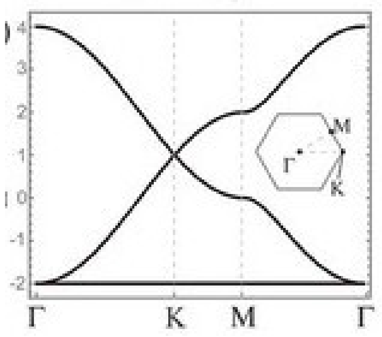
What happens in the absence of intrinsic scales?

Onsager relation breaks down for flat bands
— no semiclassical LLs.



Interesting physics happens for singular flat bands !

Flat band has zero dispersions $E(\mathbf{k}) = \mathbf{E_0}$

Trivial manifestation: the atomic limit in band theory

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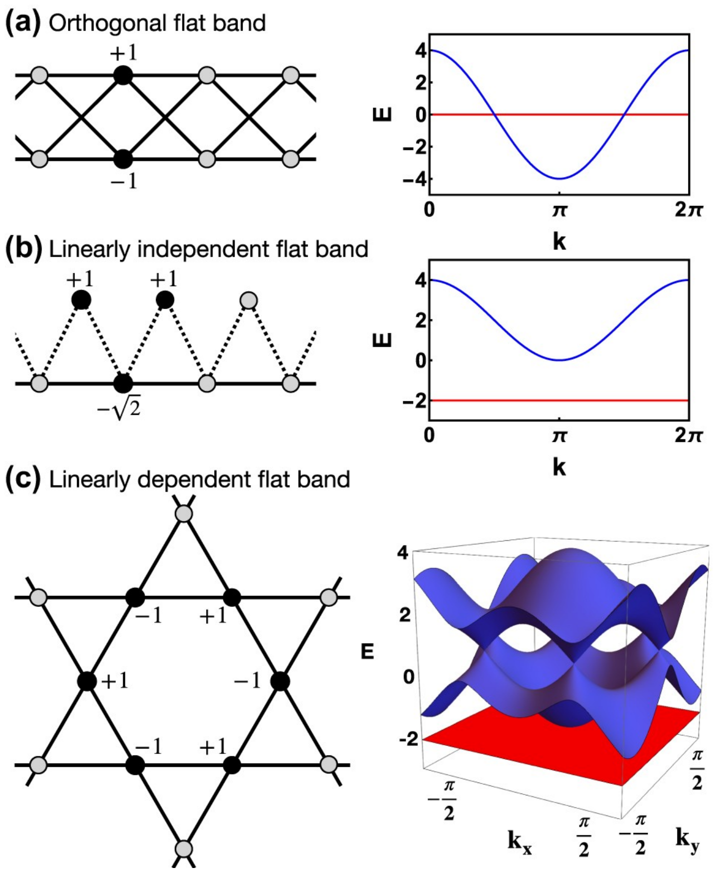

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As the kinetic energy is quenched, even weak interactions can have dramatic effects

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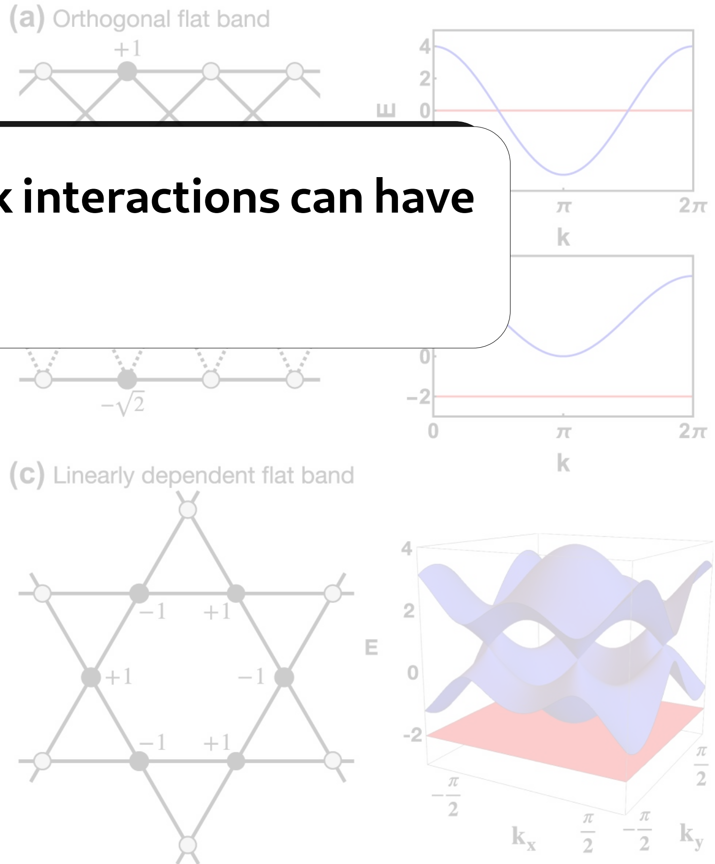

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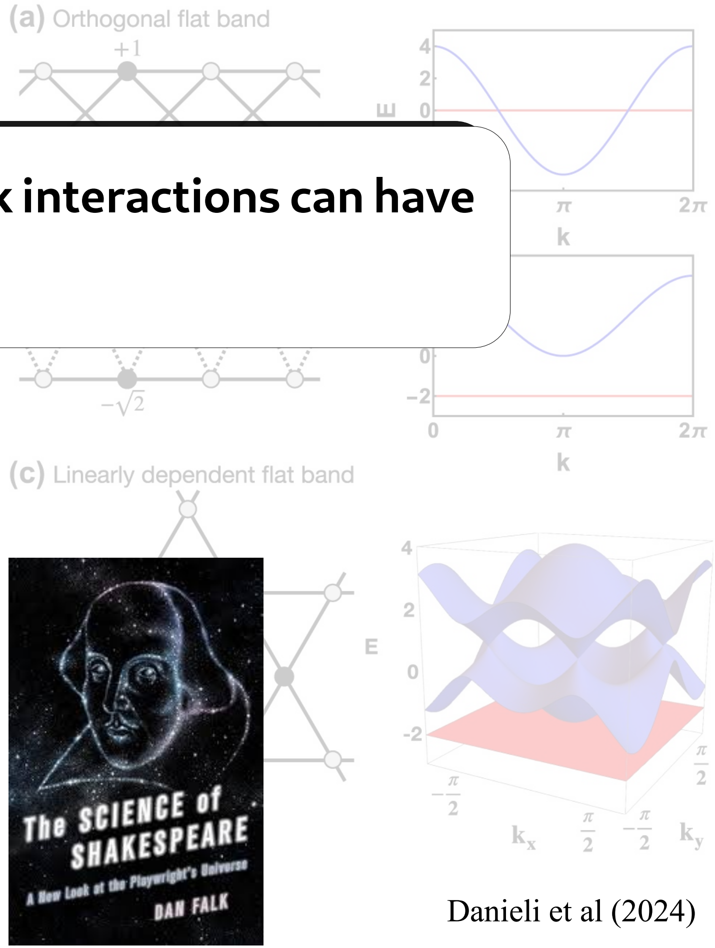
Trivial manifestation: the atomic limit in band theory

There can be more exotic physics in flat bands, Horatio, than is there with dispersions !



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Even the non-interacting setting is reach

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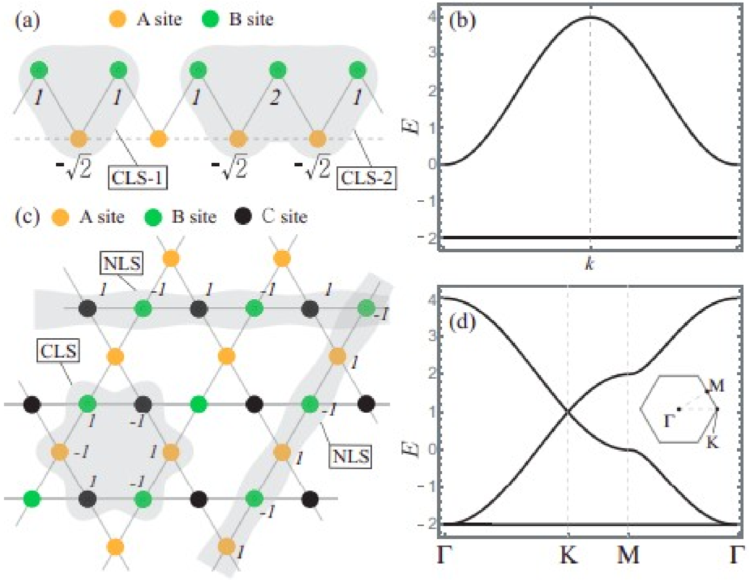
The single-particle states have special spatial structures

$$|\chi_{\mathbf{R}}\rangle \sim \sum_{\mathbf{k}\in \mathrm{BZ}} \alpha_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}} |\psi_{\mathbf{k}}\rangle \ \sim \sum_{\mathbf{R}'} \sum_{\mathbf{k}\in \mathrm{BZ}} \sum_{p} \alpha_{\mathbf{k}} v_{\mathbf{k},q} e^{-i\mathbf{k}\cdot(\mathbf{R}-\mathbf{R}')} a^{\dagger}_{\mathbf{R}',q} |0\rangle$$

Compact localization of the flat-band eigenstates guaranteed by the construction of $\alpha_{\mathbf{k}} v_{\mathbf{k},q}$

– is a polynomial of $e^{i\mathbf{k}\cdot\mathbf{a}_l}$ i.e. a finite sum of Bloch phases



Bergman et al (2008), Rhim et al (2019)

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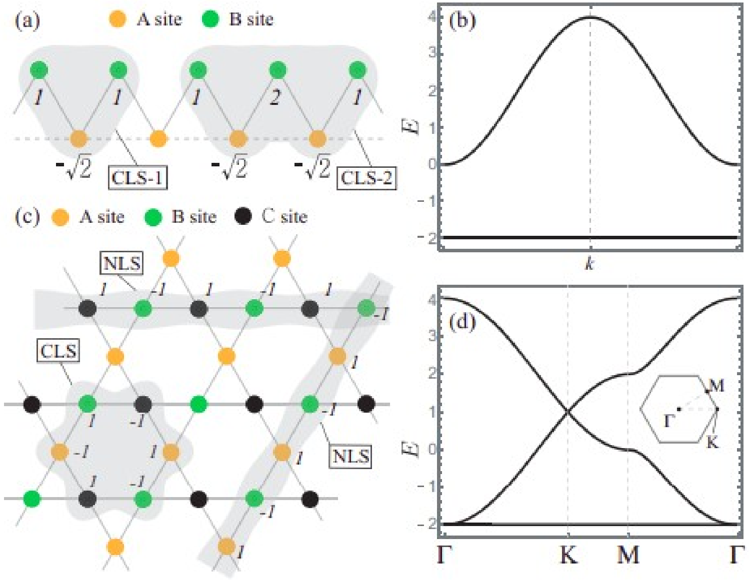
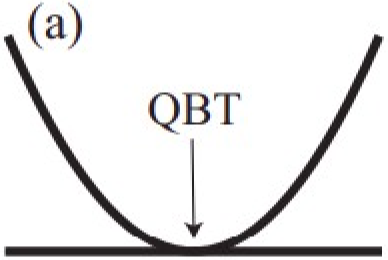


localization even in the absence of disorder !

Bergman et al (2008), Rhim et al (2019)

The classification of gapless flat bands

- Finding the complete set of compact localizates staes for gapless flat bands is tricky!
- Linear independence is not guaranteed when one of the $\alpha_{\mathbf{k}}$ is zero



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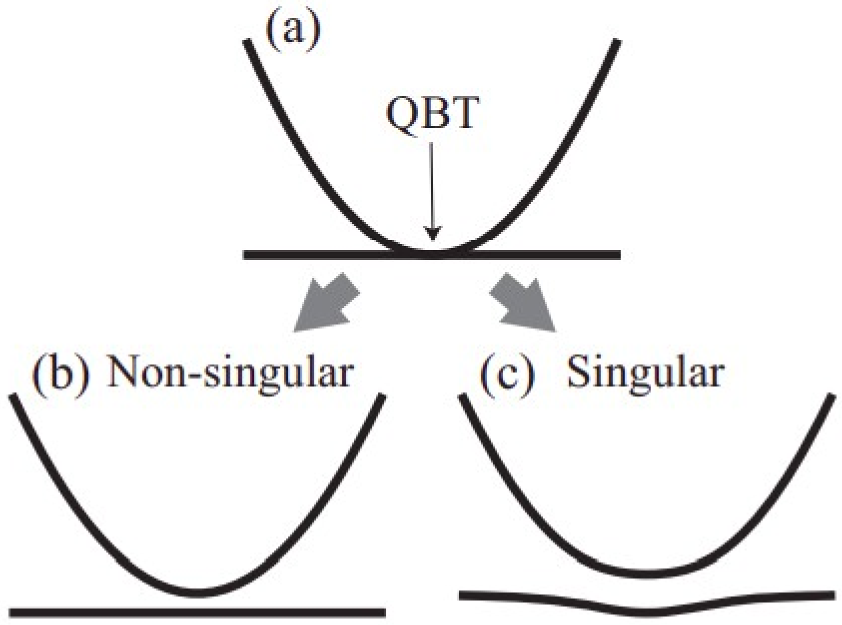
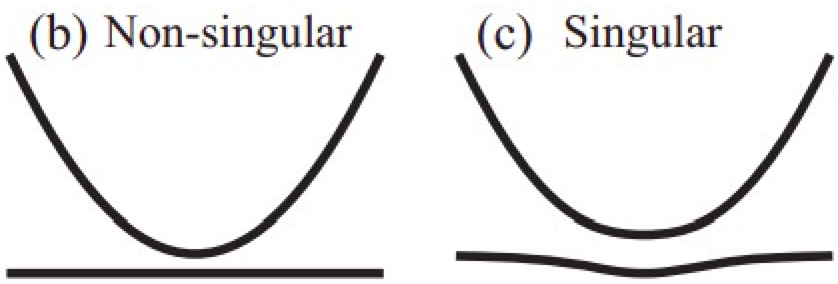


Response to magnetic fields



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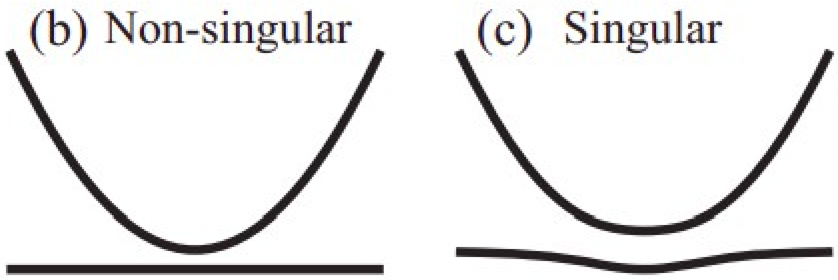

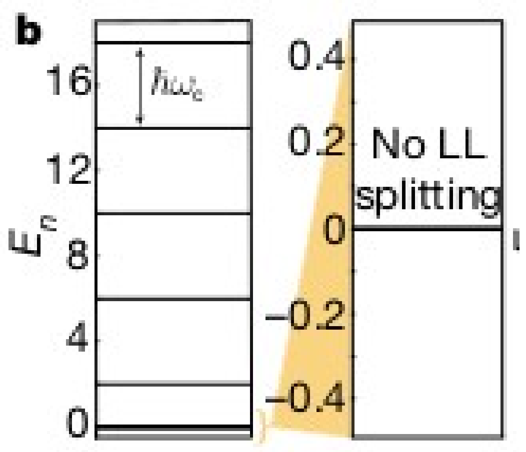

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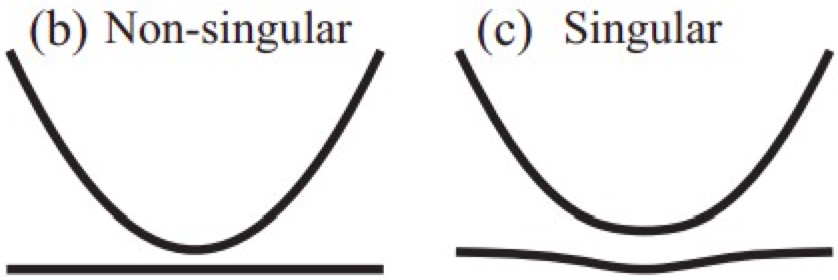

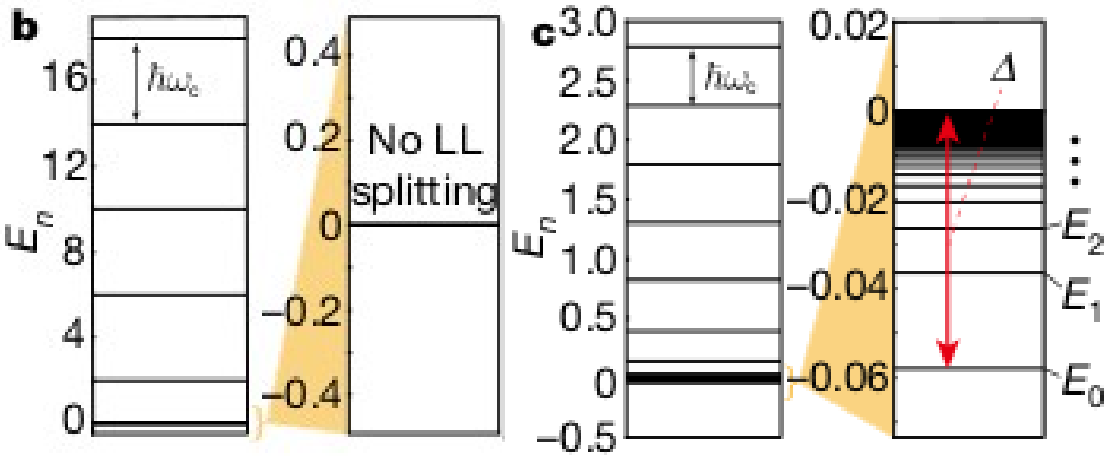

Rhim et al. (2020)

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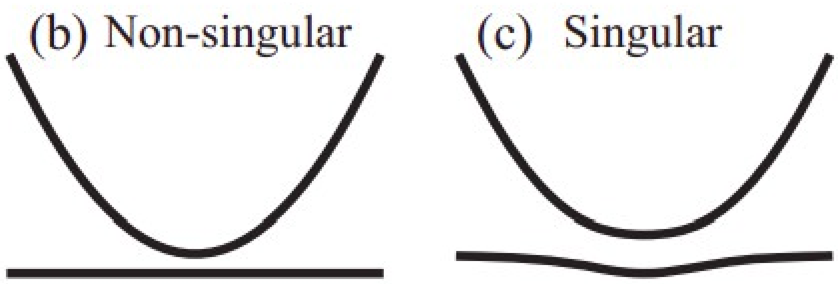

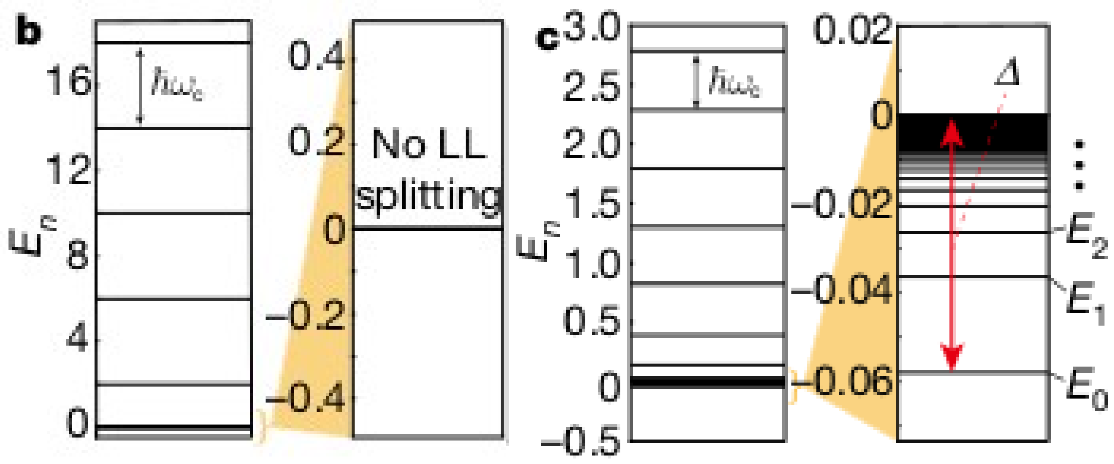

Anomalous LLs

$$\mathbf{E_n \sim -1/n}$$

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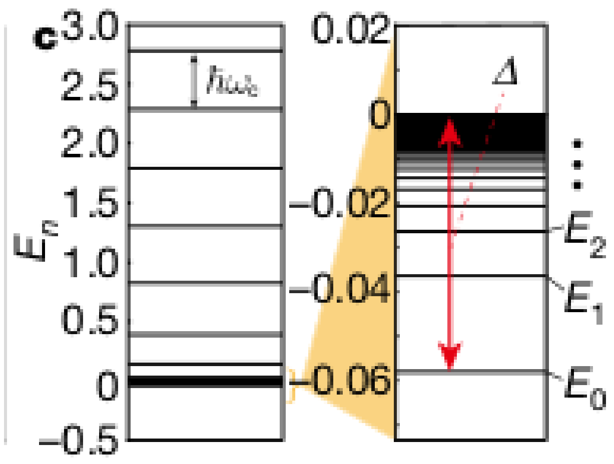

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The prefactor is dictated by the quantum distance around the gap closing point

- Dispersions do not fully characterize quantum dynamics.
- The Bloch wavefunctions encode additional **geometric information**.

Quantum geometry: The key element in the puzzle

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- Quantum geometric tensor (QGT)

$$\mathrm{Q}_{ij}(\mathbf{k}) = \langle \partial_{\mathbf{i}} \mathbf{u}(\mathbf{k}) | \big(\mathbf{1} - |\mathbf{u}(\mathbf{k})\rangle\langle \mathbf{u}(\mathbf{k})| \big) | \partial_{\mathbf{j}} \mathbf{u}(\mathbf{k}) \rangle$$

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Real part: quantum metric controls the distance d^2_{HS}

$$d(\mathbf{k}_1, \mathbf{k}_2) = 1 - |\langle u_{\mathbf{k}_1} | u_{\mathbf{k}_2} \rangle|^2$$

Imag part: Berry curvature

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quantum metric in polar coordinate

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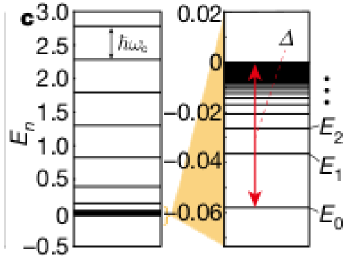
quantum metric in polar coordinate

A finite quantum distance implies band-mixing between the LLs of the dispersive band and the flat band
→ level repulsion



A glimpse of the model

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A two-band Hamiltonian supporting a flat band touching a quadratic dispersive band

$$\mathcal{H}(\mathbf{k}) = \sum_\mu d_\mu(\mathbf{k})\sigma_\mu$$

The parameters specify the *d* vector

$$d_0(\mathbf{k}) = \frac{1}{2}\left[a_1^2k_x^2 + \left(a_2^2 + a_3^2 + a_4^2\right)k_y^2 + 2a_1a_2k_xk_y\right]$$

$$d_1(\mathbf{k}) = a_3a_4k_y^2 \ , \quad d_2(\mathbf{k}) = a_2a_4k_y^2 + a_1a_4k_xk_y,$$

$$d_3(\mathbf{k}) = \frac{1}{2}\left[a_1^2k_x^2 + \left(a_2^2 + a_3^2 - a_4^2\right)k_y^2 + 2a_1a_2k_xk_y\right]$$

Datta, **KR** (2025)

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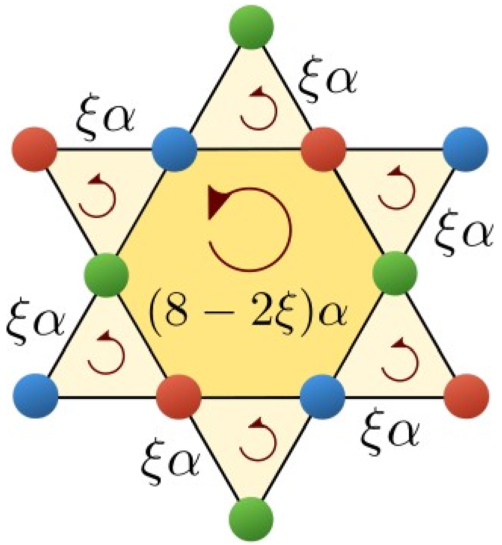
Straightforward calculations solving the Schroedinger equations yield

$$E_n = -\frac{1}{8}\left|\frac{3a_1 a_4^2/l^2}{(2n+3)\sqrt{a_2^2+a_3^2+a_4^2}+a_3}\right|$$

Datta, **KR** (2025)

Lattice realization

A kagome network subjected to uniform magnetic fluxes (hoppings admit Peierls phases)



α controls the flux density, ξ controls the distribution

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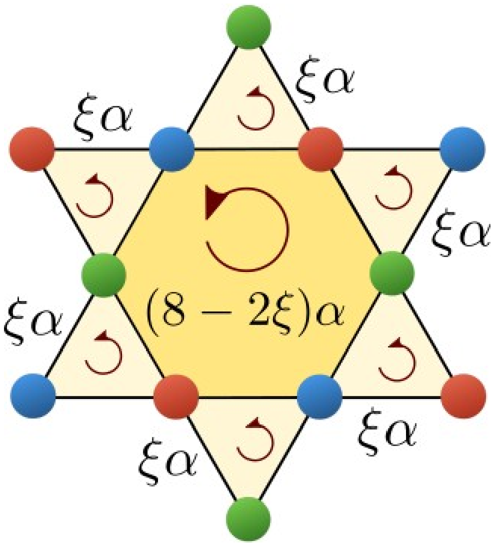

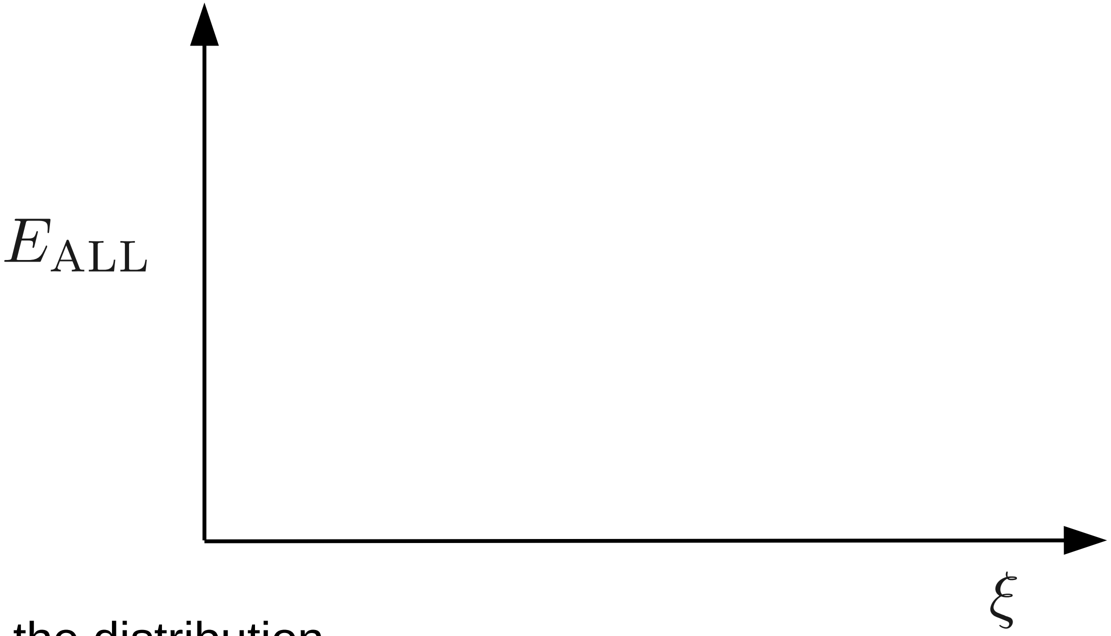

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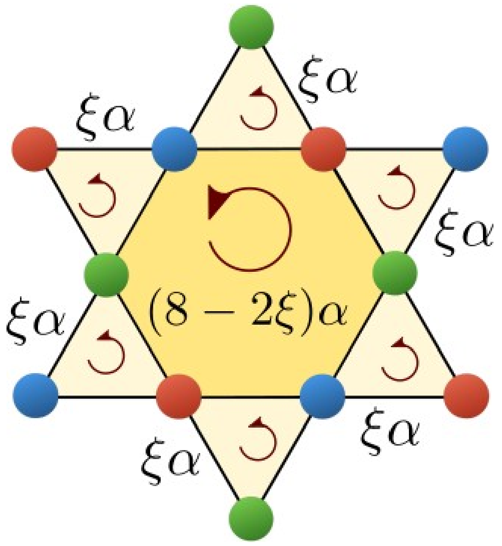

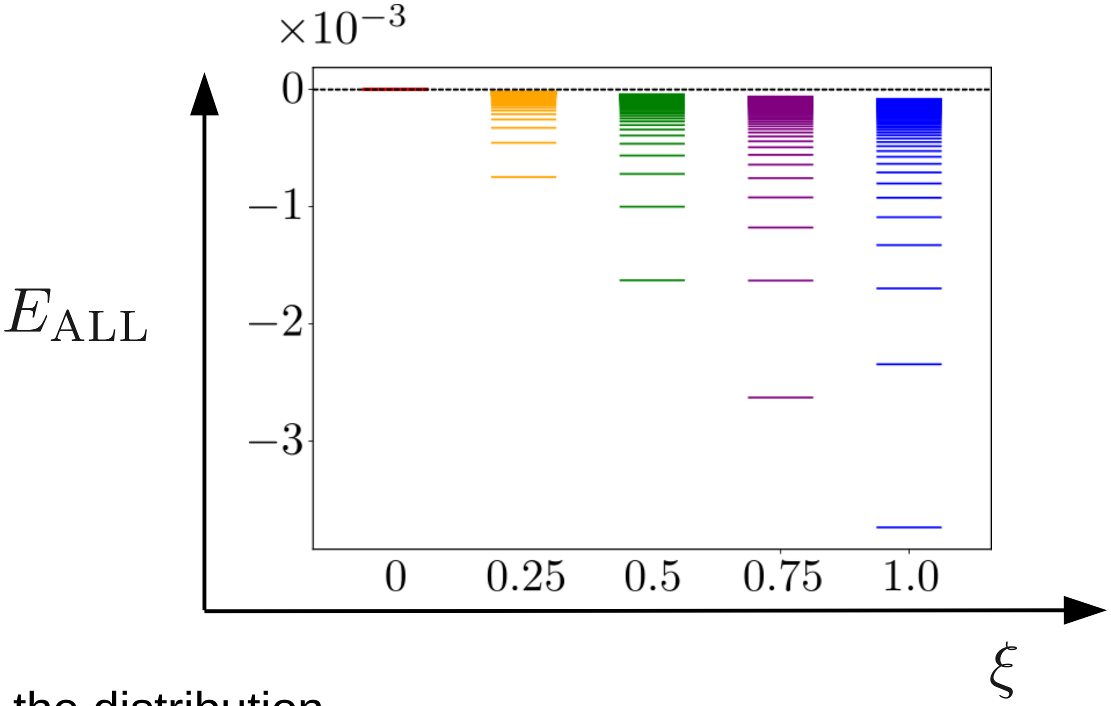

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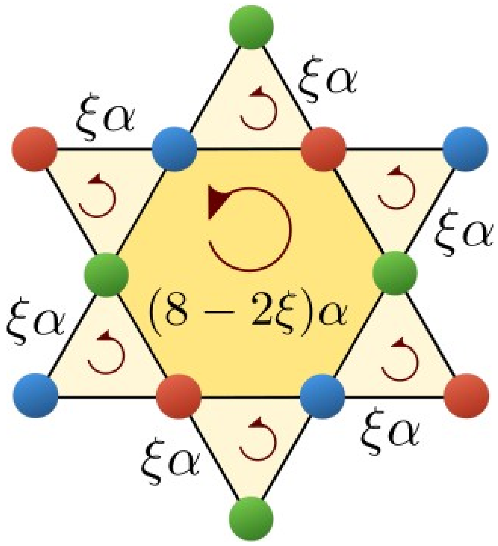

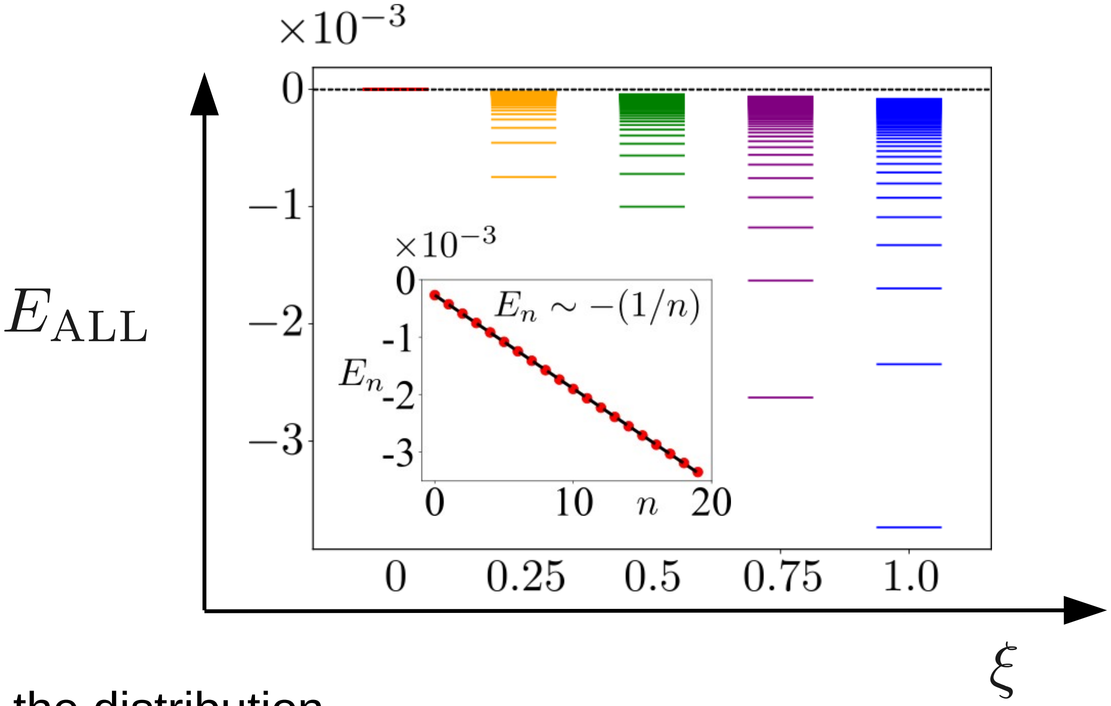

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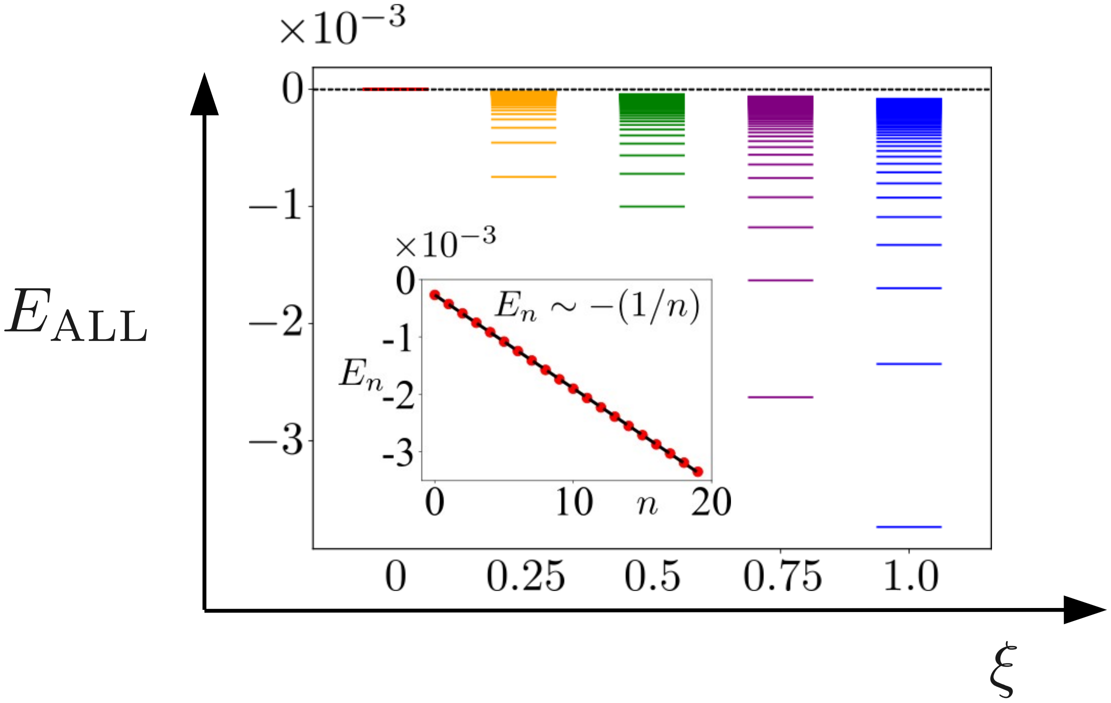
$$E_{\mathrm{ALL}} \sim -\xi/n$$

A collapse of the ALLs at $\xi = 0$



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A special point in the phase diagram

At $\xi = 0$, the Hamiltonian becomes positive semi-definite

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→ An emergent Supersymmetric algebra can be spelled out in terms of a generator

$$Q = \begin{pmatrix} & R \\ R^\dagger & \end{pmatrix}$$

and two partner Hamiltonians coming from the square of Q

$$R^\dagger R \longleftrightarrow RR^\dagger$$

These Hamiltonians are isospectral except for the zero modes – lattice SUSY

The number of zero modes are given by the Witten index.

Witten (1981), **KR** et al. (2024), Datta, **KR** (2025)

Lattice SUSY in the presence of magnetic fields

Our model demonstrates lattice SUSY in the presence of inhomogeneous fluxes

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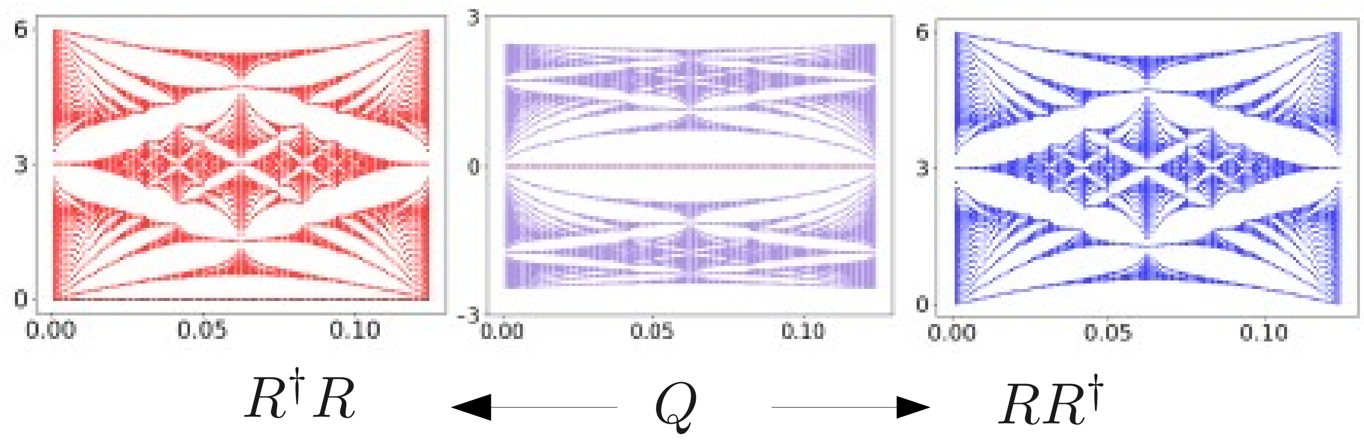

Datta, **KR** (2025)

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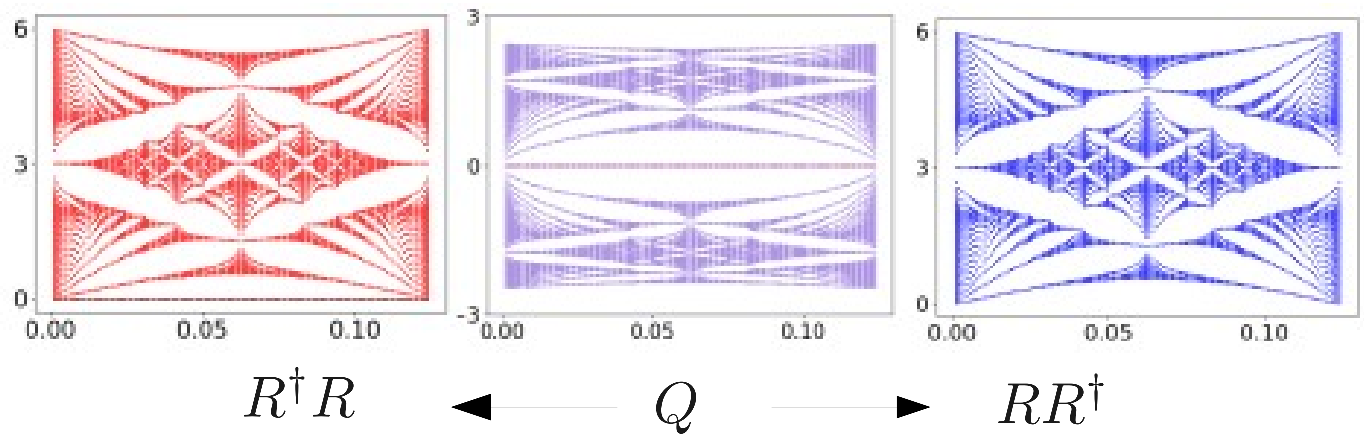

For a flux density $\Phi/\Phi_0 = p/q$, we have *q* number of ALLs at zero energy.

Lattice analog of the *Aharonov-Casher theorem* (1979) – **manifold of zero modes** (ALLs) with **degeneracy determined solely by the flux density**, originating from the same operator algebra

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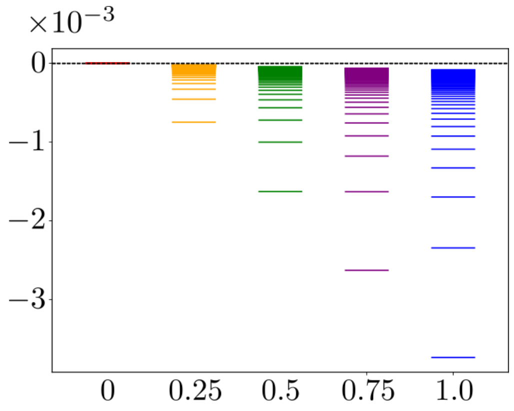
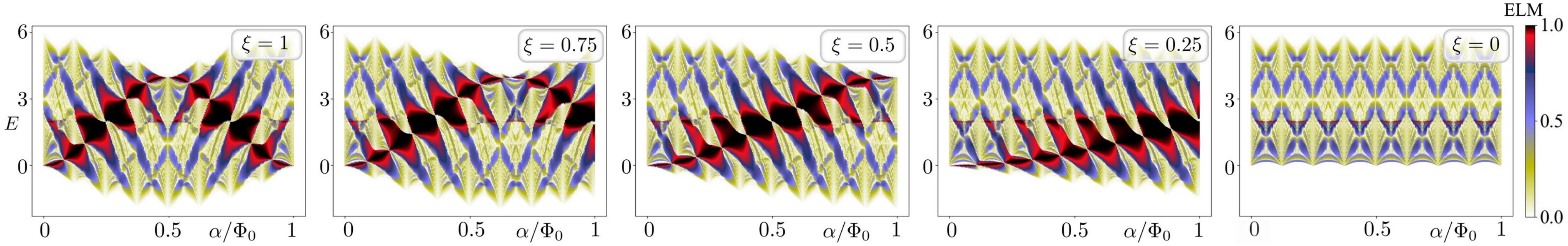
Evolution of the full spectrum





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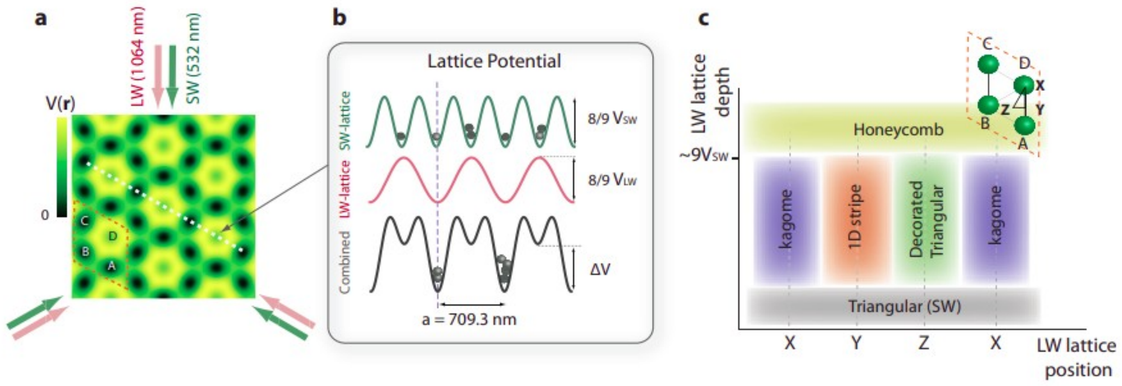
Material realization is difficult, can quantum simulators offer respite?

Kagome network has been simulated in a two-dimensional optical superlattice for ultracold ^{87}Rb atoms



Jo et al. (2012)

QHE (edge currents, Hall drift) was realized shortly afterward on a square geometry (2013).

An open problem to explore anomalous LLs (response of flat bands to flux inhomogeneity) in a quantum simulator

On optical lattices:

- Multiple plaquettes, coupled flux constraints
- bond-dependent gauge engineering complexity
- Scaling up is possible; controlled flux geometry is the frontier.

On superconducting quantum processors:

- Finite-size dominance, geometry fixed by hardware
- Control is feasible; scale and lattice geometry are the frontier.

What becomes possible in larger kagome qubit arrays?